A General Account of Argumentation with Preferences - Erratum

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Abstract

We show that the ASPIC+ elitist set comparison relation, as defined in [3], is not reasonable inducing and hence cannot guarantee normative rationality for structured argumentation. Rather, one should revert to Prakken’s original strict elitist set comparison, as defined in [4], in order to guarantee that instantiations of Dung’s framework satisfy rationality postulates. We also show that by reverting to [4]’s elitist set comparison, Dung’s counter-example [2] is avoided.

1 The Elitist Set Comparison Relation in [3] is not Reasonable Inducing

Recall that the property of reasonable inducing for a given set comparison relation ⊴ over the defeasible elements in arguments, is necessary to ensure that preference relations ≼ over arguments, defined on the basis of ≤, are normatively rational.1

In what follows, \(P_{\text{fin}}(X)\) denotes the set of all finite subsets of a set \(X\), and the symbol “\(\subseteq\) fin” means “is a finite subset of”, so \(U \in P_{\text{fin}}(X) \iff U \subseteq_{\text{fin}} X\). Recall that for a set comparison relation \(\leq, \Gamma \preceq \Gamma' \iff [\Gamma \leq \Gamma', \Gamma' \not\leq \Gamma]\).

Definition 1.1. (From [3] page 376, Definition 22]) Given \(\langle P, \leq \rangle\) a preset (pre-ordered set), a set comparison \(\leq \subseteq [P_{\text{fin}}(P)]^2\) is reasonable inducing iff

1. \(\leq\) is transitive.

2. For any \(\Gamma_0, \Gamma_1, \ldots, \Gamma_n \subseteq P\) (for \(n \geq 1\), if

\[
\bigcup_{i=1}^{n} \Gamma_i \preceq \Gamma_0 \text{ then } (1.1)
\]

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1In the sense that when using preference relations to determine which attacks succeed as defeats, \(\Gamma\)’s rationality postulates are satisfied.
(a) \((\exists 1 \leq i \leq n) \Gamma_i \preceq \Gamma_0\) and
(b) \((\exists 1 \leq i \leq n) \Gamma_0 \not\preceq \Gamma_i\).

We now show with a counterexample, that the elitist set comparison relation as defined in [3, page 375, Definition 19] is not reasonable inducing. Note that in Appendix A (page 6), we precisely locate the error in the proof [3, page 376, Proposition 21] that [3]'s elitist set comparison relation is reasonable inducing.

**Lemma 1.1.** The proposition “if \(\bigcup_{i=1}^{n} \Gamma_i \preceq_{\text{Elit}} \Gamma_0\) then \((\exists 1 \leq i \leq n) \Gamma_0 \not\preceq_{\text{Elit}} \Gamma_i\)” is false, i.e. criterion 2(b) of definition 1.1 fails for \(\preceq_{\text{Elit}}\).

**Proof.** The counterexample is as follows: let \(\langle P, \leq \rangle\) be an arbitrary preset\(^2\) such that \(a, b, c, d \in P\). Let \(\Gamma_0 = \{c, d\}, \Gamma_1 = \{a\}, \Gamma_2 = \{b\}\) so \(\Gamma_1 \cup \Gamma_2 = \{a, b\}\). Let \(\leq\) be such that \(a \approx c\), \(a < d\), \(d \leq b\) and \(c \not\approx b\). Notice

\[
\Gamma_1 \cup \Gamma_2 \preceq_{\text{Elit}} \Gamma_0 \iff [\Gamma_1 \cup \Gamma_2 \preceq_{\text{Elit}} \Gamma_0 \text{ and } \Gamma_0 \not\preceq_{\text{Elit}} \Gamma_1 \cup \Gamma_2] \\
\iff [(a, b) \preceq_{\text{Elit}} \{c, d\} \text{ and } \{c, d\} \not\preceq_{\text{Elit}} \{a, b\}] \\
\iff \text{true as } [a \approx c \Rightarrow a \leq c \text{ and } a < d \Rightarrow a \leq d] .
\]

\[
\{a, b\} \preceq_{\text{Elit}} \{c, d\} \iff [(a \leq c, a \leq d) \text{ or } (b \leq c, b \leq d)] .
\]

\[
\iff \text{true because } d \leq b .
\]

Therefore, we have found a situation where \(\Gamma_1 \cup \Gamma_2 \preceq_{\text{Elit}} \Gamma_0, \Gamma_0 \preceq_{\text{Elit}} \Gamma_1\) and \(\Gamma_0 \preceq_{\text{Elit}} \Gamma_2\) are all true.

**Corollary 1.2.** \(\preceq_{\text{Elit}}\) is not reasonable inducing.

**Proof.** Immediate from Definition 1.1 and Lemma 1.1.

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2 The Elitist Set Comparison in [4] is Reasonable Inducing

Consider the strict version of the elitist order, as originally proposed by Prakken in [4, page 109]. We will show that it is reasonable inducing. First recall that given a preset \(\langle P, \leq \rangle\) its strict counterpart preorder is \(a < b \iff [a \leq b, b \not\leq a]\), which is a strict partial order.

\(^2\)For such a preset we write \(x \approx y\) iff \(x \leq y\) and \(y \leq x\). We write \(x < y\) iff \(x \leq y\) and \(y \not\leq x\).
**Definition 2.1.** Let $\langle P, \leq \rangle$ be a preset and form its strict poset (partially-ordered set) $\langle P, < \rangle$. Define the **strict elitist set comparison** $\triangleleft_{Eli}'$ on $P_{fin}(P)$ as

$$\Gamma \triangleleft_{Eli}' \Gamma' \iff (\exists x \in \Gamma) (\forall y \in \Gamma') x < y.$$  \hspace{1cm} (2.1)

Its **non-strict counterpart** is

$$\Gamma \triangledown_{Eli}' \Gamma' \iff [\Gamma = \Gamma' \text{ or } \Gamma \triangleleft_{Eli}' \Gamma'].$$  \hspace{1cm} (2.2)

**Corollary 2.1.** $\triangleleft_{Eli}'$ is irreflexive.

**Proof.** Assume for contradiction that $\Gamma \triangleleft_{Eli}' \Gamma$, which is equivalent to

$$(\exists x \in \Gamma) (\forall y \in \Gamma) x < y.$$  

Let $x_0 \in \Gamma$ be the witness to $\exists x$, which means $(\forall y \in \Gamma) x_0 < y$, and one can instantiate $\forall y$ to $y = x_0$, which means $x_0 < x_0$ and hence a contradiction. Therefore, $\triangleleft_{Eli}'$ is irreflexive. \hfill $\square$

**Lemma 2.2.** The strict elitist set comparison (Definition 2.1) is reasonable inducing.

**Proof.** Following Definition 1.1, we have:

1. Transitivity:

$$\Gamma \triangleleft_{Eli}' \Gamma' \triangleleft_{Eli}' \Gamma'' \iff (\exists x \in \Gamma) (\forall y \in \Gamma') x < y \text{ and } (\exists y \in \Gamma'') \forall z \in \Gamma'' y < z \iff (\forall y \in \Gamma') x_0 < y < z \iff x_0 < y_0 \text{ and } (\forall z \in \Gamma'') y_0 < z \iff (\forall z \in \Gamma'') x_0 < y_0 < z \iff (\forall z \in \Gamma'') x_0 < z \iff (\exists x \in \Gamma) (\forall z \in \Gamma'') x < z \iff \Gamma \triangleleft_{Eli}' \Gamma''.$$  

where in the third line $x_0 \in \Gamma$ is the witness to the first $\exists$, and $y_0 \in \Gamma'$ is the witness to the second $\exists$. Therefore, $\triangleleft_{Eli}'$ is transitive.

2. Definition 1.1 property 2(a): first recall that for a family of sets $\{A_i\}_{i \in I}$ and a first order unary predicate $P(x)$, we have that

$$\left( \exists x \in \bigcup_{i \in I} A_i \right) P(x) \iff (\exists i \in I) (\exists x \in A_i) P(x).$$  \hspace{1cm} (2.3)
Then,
\[ n \bigcup_{i=1}^{n} \Gamma_i \not\preceq_{Eli}^\prime \Gamma_0 \]
\[ \iff \left( \exists x \in \bigcup_{i=1}^{n} \Gamma_i \right) \left( \forall y \in \Gamma_0 \right) x < y \]
\[ \iff \left( \exists 1 \leq i \leq n \right) \left( \exists x \in \Gamma_i \right) \left( \forall y \in \Gamma_0 \right) x < y \text{ by Equation 2.3} \] (2.4)
\[ \iff \left( \exists 1 \leq i \leq n \right) \left( \exists x \in \Gamma_i \right) \left( \forall y \in \Gamma_0 \right) x < y \text{ by Definition 2.1} \]
\[ \implies \left( \exists 1 \leq i \leq n \right) \Gamma_i \preceq_{Eli} \Gamma_0 \] .

Therefore, \( \preceq_{Eli}^\prime \) satisfies the first property.

3. Definition 1.1, property 2(b): let \( 1 \leq i_0 \leq n \) be the witness to the first \( \exists \) in Equation 2.4, and \( x_{i_0} \in \Gamma_{i_0} \) be the witness to the second \( \exists \) in Equation 2.4. Hence Equation 2.4 reads as:
\[ \left( \forall y \in \Gamma_0 \right) x_{i_0} < y \] . (2.5)

Now assume for contradiction that
\[ \left( \forall 1 \leq i \leq n \right) \Gamma_0 \not\preceq_{Eli}^\prime \Gamma_i \implies \Gamma_0 \not\preceq_{Eli} \Gamma_{i_0} \]
\[ \iff \left( \exists x \in \Gamma_0 \right) \left( \forall y \in \Gamma_{i_0} \right) x < y \]
\[ \iff \left( \forall y \in \Gamma_{i_0} \right) x_0 < y \text{ ,} \] (2.6)
where \( x_0 \in \Gamma_0 \) in Equation 2.6 is the witness to \( \exists \) in the previous line.

Now instantiate \( y \in \Gamma_0 \) in Equation 2.5 to \( x_0 \), and instantiate \( y \in \Gamma_{i_0} \) in Equation 2.6 to \( x_{i_0} \). Therefore, we have
\[ x_{i_0} < x_0 \text{ and } x_0 < x_{i_0} \text{ ,} \] (2.7)
which is a contradiction. Therefore, \( \left( \exists 1 \leq i \leq n \right) \Gamma_0 \not\preceq_{Eli}^\prime \Gamma_i \) and \( \preceq_{Eli}^\prime \) satisfies the second property.

This means the strict elitist set comparison is reasonable inducing. \( \square \)

3 Dung’s Counterexample to Normative Rationality

We now show how [2, Example 5.1] is repaired under the strict elitist set comparison as defined in [4], and Definition 2.1.

Example 1. [2, Example 5.1] Let \( \mathcal{L} = \{a_i\}_{i=1}^{4} \) be closed under (syntactic) negation, the contrary function \( - \) denote symmetric negation \( \neg \), \( \mathcal{K} = \emptyset \), \( \mathcal{R}_d = \{ (\top \Rightarrow a_i) \}_{i=1}^{4} \) and
\[ \mathcal{R}_s = \{ (a_1, a_2, a_3 \rightarrow \neg a_4), (a_2, a_3, a_4 \rightarrow \neg a_1), (a_3, a_4, a_1 \rightarrow \neg a_2), (a_4, a_1, a_2 \rightarrow \neg a_3) \} \]
such that the preorder $\preceq$ is such that $d_1 \approx d_2$ and $d_3 \approx d_4$ only (reflexivity and transitivity are implicit). This instantiation is well-defined [3, page 369, Definition 12]. The arguments are $A_i := [\top \Rightarrow a_i]$ for $1 \leq i \leq 4$, and

\[ B_i := [A_1, A_2, A_4 \rightarrow \neg a_4], \quad B_1 := [A_2, A_3, A_4 \rightarrow \neg a_1], \]
\[ B_2 := [A_3, A_4, A_1 \rightarrow \neg a_2], \quad B_3 := [A_4, A_1, A_2 \rightarrow \neg a_3]. \]

The strict elitist set comparison (Definition 2.1) gives:

\[ \{d_1, d_2, d_3\} \not\preceq_{Eli} \{d_4\}, \quad \{d_2, d_3, d_4\} \not\preceq_{Eli} \{d_1\}, \]
\[ \{d_3, d_4, d_1\} \not\preceq_{Eli} \{d_2\}, \quad \{d_4, d_1, d_2\} \not\preceq_{Eli} \{d_3\}, \]

because (e.g.) there is no defeasible rule in $\{d_1, d_2, d_3\}$ that is strictly less than $d_4$. If the witness were $d_3$, say, then $d_3 \not\preceq d_4$, but $d_4 \not\preceq d_3$ as well (rather than $d_4 \preceq d_3$), so $d_3 \not\prec d_4$. Therefore, under the strict elitist set comparison, we have $B_i \not\prec A_i$ (here, $\prec$ denotes the argument preference and not the preorder on defeasible rules), hence $B_i \nLeftarrow A_i$ ($B_i$ defeats $A_i$) for $1 \leq i \leq 4$. The possible sets of justified arguments are $\{A_1, A_2, A_3, B_4\}$, $\{A_1, A_2, B_3, A_4\}$, $\{A_1, B_2, A_3, A_4\}$ and $\{B_1, A_2, A_3, A_4\}$, whose conclusion sets are consistent.

4 Conclusion

We conclude that Prakken’s original elitist set comparison (Definition 2.1) should be used instead of the elitist set comparison from [3, page 375, Definition 19] in all future instantiations of ASPIC$^+$, in order to avoid Dung’s counter-example in [2], and ensure that ASPIC$^+$ instantiations are normatively rational in the sense that [1]’s rationality postulates are satisfied.

References


A  The Error of the Initial Proof

We explain why the proof of [3, page 376, Proposition 21] is incorrect by locating the error.

Lemma A.1. (The following statement, [3, page 376, Proposition 21], may not be true) \( \preceq_{Eli} \) is reasonable inducing.

Proof. (The following proof, from [3, page 390, Proposition 21], is incorrect)

We know that \( \preceq_{Eli} \) is transitive, and from Equation 2.3 (page 3), \( \preceq_{Eli} \) satisfies property 2(a) of reasonable inducing (Definition 1.1). Assume for contradiction that property 2(b) is false, i.e.

\[
(\forall 1 \leq i \leq n) \Gamma_0 \preceq_{Eli} \Gamma_i
\]

\[\iff (\forall 1 \leq i \leq n) (\exists x \in \Gamma_0) (\forall y \in \Gamma_i) x \leq y \]

\[\implies (\exists x \in \Gamma_0) (\forall y \in \Gamma_1) x \leq y \text{ (by choosing } i = 1), \]

\[\iff (\forall y \in \Gamma_1) x_1 \leq y , \quad (A.1)\]

where \( x_1 \in \Gamma_0 \) is the witness to \( \exists \). Now from the assumption of strictly less than in the set comparison relation (Equation 1.1), we have

\[
\Gamma_0 \not\preceq_{Eli} \bigcup_{i=1}^{n} \Gamma_i \iff (\forall x \in \Gamma_0) \left( \exists y \in \bigcup_{i=1}^{n} \Gamma_i \right) x \not\leq y
\]

\[\iff (\forall x \in \Gamma_0) (\exists 1 \leq i \leq n) (\exists y \in \Gamma_i) x \not\leq y \text{ by Equation 2.3}
\]

\[\implies (\exists 1 \leq i(x_1) \leq n) (\exists y \in \Gamma_{i(x_1)}) x_1 \not\leq y , \quad (A.2)\]

where in the last step we have instantiated \( x \) under \( \forall \) to \( x_1 \in \Gamma_0 \), which is the witness to Equation A.1.

(INCORRECT STEP) Assume the witness to \( \exists \) in Equation A.2 is 1, i.e. \( i(x_1) = 1 \). Of course, there is no guarantee that the witness to the first \( \exists \) in Equation A.2 is the same as the instantiation of the first \( \forall \) in “\((\forall 1 \leq i \leq n) \Gamma_0 \preceq_{Eli} \Gamma_1\)”.

Running with this, we have from Equations A.1 and A.2

\[
(\forall y \in \Gamma_1) x_1 \leq y \text{ and } (\exists y \in \Gamma_1) x_1 \not\leq y . \quad (A.3)
\]

Therefore, by instantiating the first quantifier \( \forall \) to the witness of the second quantifier \( \exists \), calling it \( y_0 \in \Gamma_1 \), we have

\[
x_1 \leq y_0 \text{ and } x_1 \not\leq y_0 , \quad (A.4)
\]

which is a contradiction. \( \square \)