A COMPARISON OF OPPORTUNISTIC TRANSMISSION SCHEMES WITH REDUCED CHANNEL INFORMATION FEEDBACK IN OFDMA DOWNLINK

Seokhyun Yoon, Oren Somekh*, Osvaldo Simeone** and Yeheskel Bar-Ness**

School of ECE, Dankook University, Seoul, Korea
*Dept. of EE, Princeton University, Princeton, NJ, USA
** CWCSPR, Dept. of ECE, New Jersey Institute of Technology, Newark, NJ, USA

ABSTRACT

In this paper, we consider downlink throughput performances of multiuser orthogonal frequency division multiplexing (multiuser OFDM) with reduced channel information feedback schemes. Specifically, two types of reduced feedback schemes, namely, 1-bit per sub-carrier and selective feedback scheme are considered and compared with each other in terms of average network throughput. For the latter, since the exact analysis is complicated, we resort to an approximate analysis. Simulations results will also be provided to verify the approximate analysis. Since the strict throughput comparison for given number of feedback bits per user is quite difficult, rather we compare their general behaviors in various system configurations with different system parameters, which can give us an insight into practical system design with those reduced feedback schemes.

I. INTRODUCTION

Multiuser diversity gain is now widely considered as a fundamental merit of OFDM based wireless access [2-4]. Basically, it is an opportunistic approach where each user is allowed to transmit data when the channel to that user becomes the best among others. With random beam-forming, this type of approach has been rigorously investigated in [5,6], where it was shown that the throughput can be increased asymptotically as loglog of the number of users. The difference between the setup in [5,6] and the one in [1-4] is that the former considers only flat fading channel, while in OFDM setup it has a multiple of orthogonal channels, usually refereed to as sub-carrier, where to achieve the capacity [7] the broadcasting station needs full knowledge on every users channel quality for every sub-carriers, resulting in prohibitive uplink feedback overhead.

To reduce the uplink feedback, various types of reduced feedback schemes were recently proposed at the expense of throughput reduction [8-15]. The reduced feedback schemes can be divided into two types, namely, 1-bit per sub-carrier scheme proposed in [9-13] and selective feedback scheme [14]. In the former, each user sends to the broadcasting station (BS) 1bit channel information per sub-carrier, resulting in total feedback equal to the number of sub-carrier by comparing the channel quality of each sub-carrier with a predetermined threshold. The threshold is set by BS to maximize the system throughput. In the latter, each user selects an integer number, M, of sub-carriers (or M sub-channels) whose channel are the best and sends to the BS the indices of the selected sub-channels and their channel quality values. In [11-13], it was shown that the 1bit per sub-carrier feedback scheme behaves the same as that with full channel side information, i.e., the system throughput increases as loglog of the number of users. Independent of those in [8-14], another rather straightforward scheme was considered in [15-17], where a set of adjacent sub-carriers form to a sub-channel to be used as a unit of resource allocation. In [15], it was shown that one can obtain the same throughput scaling even by feeding back only one representative value for a cluster of sub-carriers if the size of cluster meets certain condition. On the other hand, in [16] and [17], noticing that the channel quality of sub-carriers even in the same sub-channel may have different values the SNR distribution over a sub-channel (cluster) was modeled as Ricean and the channel quality information transmitted to the BS is represented by the two Rice parameters. The performances were evaluated for different coherence bandwidth to sub-channel bandwidth ratio.

In this paper, we focus on the comparison of the aforementioned two types of reduced feedback schemes in terms of average network throughput in multiuser OFDM downlink. Nevertheless, we will not make a 'strict' throughput comparison for given number of feedback bits per user. Rather, we compare their behaviors, pros and cons for various system configurations with different parameters. This can give us an insight into practical system design with different reduced feedback schemes.

In the next section, we first provide a simplified system model for analytical tractability, where we assume the channel between the broadcasting station (BS) and a terminal has finite order of frequency diversity. In section III, we discuss the 1-bit per sub-carrier scheme and its performance and, in Section IV, the selective feedback scheme; its operation and performances in terms of average throughput. Numerical/simulation results and the concluding remarks are provided in Section V and VI, respectively.

II. SYSTEM AND CHANNEL DESCRIPTION

In OFDM, the channel is divided into many sub-channels called sub-carriers, which comprises of a set of orthogonal complex exponential function basis. A guard time interval is inserted between every two consecutive OFDM symbols to avoid inter-symbol interference. By doing so, frequency selective inter-symbol interference channel looks like a set of parallel flat fading channel making signal processing at the receiver much easier. Usually, adjacent sub-carriers are highly correlated to each other so that the frequency spectrum over those is almost flat. Taking this into account, we define, though little bit ambiguous, a sub-band as a set of adjacent sub-carriers with high correlation and effective frequency diversity order as the number of sub-bands over the entire

1-4244-1144-0/07/$25.00 ©2007 IEEE.

signal bandwidth. In practice, the effective diversity order is much less than the total number of sub-carriers used since an OFDM symbol should be designed such that the symbol rate reduction due to the use of guard time interval is also minimized.

In this paper, we define a sub-channel as that of sub-bands, consisting of many sub-carriers, and assume that the number of sub-channel is equal to the effective frequency diversity order, which is assumed for simplicity an integer value. We further assume that the channel quality of sub-carriers belonging to a sub-channel are exactly the same and are uncorrelated with those belonging to different sub-channels. Then, denoting the frequency gain of the n^{th} sub-channel of the user k as $h_n^{(k)}$, the full channel side information in single-cell OFDM system with N sub-channels and K users consists of the set $H = \{h_n^{(k)}; n = 0,1,..,N\text{-}1, k = 1,..,K\}$ where we assume

$$h_n^{(k)} \overset{\text{i.i.d.}}{\sim} \mathcal{CN}(0,\sigma_s^2) \quad \forall k,n \,. \tag{1}$$

It is unrealistic in practical situation and, in [15-17], the statistical properties among sub-carriers have been investigated rigorously. However, it is certainly not our focus in this paper and with the simplified OFDM channel model we focus more on comparison of the two reduced feedback schemes mentioned before. For compact expression, we define channel quality as

$$\gamma_n^{(k)} \equiv | h_n^{(k)} |^2 \sim \frac{1}{\Gamma}\exp\left(-\frac{\gamma}{\Gamma}\right) \ \forall n,k \tag{2}$$

where Γ is the average SNR. We will use the same letter H for a realization of $\gamma_n^{(k)}$'s, i.e., $H = \{\gamma_n^{(k)}; n = 0,1,..,N\text{-}1, k = 1,..,K\}$

A. Performance with full Channel Side information

As a benchmark for the performance comparison, we briefly review the performance with full channel side information. When H is fully available at the BS, the system throughput is given by

$$R_{full\_CSI} = E_H\left[\max_{\mathbf{p}} \frac{1}{N}\sum_{n=0}^{N-1}\log(1+p_n\gamma_n^*)\right] \tag{3}$$

where from (2) $\gamma_n^* \equiv \max_{1\le k\le K}\gamma_n^{(k)} \sim dF_K(\gamma)/d\gamma$ with

$$F_K(\gamma) \equiv \Pr\{\max_{1\le k\le K}\gamma_n^{(k)} < \gamma\} = \left(1-\exp(-\gamma/\Gamma)\right)^K$$

Assuming $N \rightarrow \infty$ then the empirical distribution of $\gamma_n^{(k)}$ converges to its probability distribution and (2) can be calculated as [17];

$$R_{full\_CSI} = \max_{\lambda(\gamma)} \int_0^\infty \log(1+\lambda(\gamma)\gamma)dF(\gamma) \tag{4}$$

subject to $\int \lambda(\gamma)dF(\gamma) = P_T$

In (4), the maximization over $\lambda(\gamma)$ leads us to the well-known water-filling solution as follows

$$\lambda(\gamma) = \begin{cases} 1/\gamma_0 - 1/\gamma & \text{for } \gamma \ge \gamma_0 \\ 0 & \text{for } \gamma < \gamma_0 \end{cases} \tag{5}$$

where γ_0 is given by the solution of the constraint. As reported, in both [13] and [17], the impact of water-filling power distribution is in practice negligible compared to multiuser diversity gain and, sometimes (especially when SNR is high enough), uniform power allocation almost achieves the capacity making it possible to evaluate the rate in (3) with equal power allocation for high SNR region. Such power allocation will result in more compact expression for the achievable rate as reported in [11] and [12]. In fact, it simply gives $R_{full\_CSI} = \log\Gamma + \log\log K$ with $p_n = 1\ \forall n$. In this paper, however, the power distribution will be explicitly taken into account since it may have none-negligible impact under certain conditions of our interest.

III. 1BIT FEEDBACK PER SUB-CARRIER

In this section, we first briefly review the 1bit feedback scheme, where each user report to BS a total of N bits channel information, 1 bit for each sub-channel. Originally, the scheme was proposed and analyzed in [11-13] with slightly different system setup. In this paper, we will make it fit our framework under more practical assumptions. As such, the operation of the scheme will be as follows:

Terminal feedback: Each user, i.e., the k^{th}, compares the quality of each sub-channel with the pre-determined threshold, α_n, which is a network parameter announced by the BS. The result composed of 1bit information, $b_n^{(k)}$, per sub-channel is fed back to the BS; viz.

$$b_n^{(k)} = \begin{cases} 1 & \text{if } \gamma_n^{(k)} \ge \alpha_n \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Threshold vector is defined as $\boldsymbol{\alpha} = [\alpha_0, \alpha_1, \ldots, \alpha_{N-1}]$, meaning each sub-channel has different threshold. Later, however, we will assume same threshold for all sub-channels, which would be more realistic.

Resource allocation: Upon reception of the feedback from all the terminals, BS chooses for each sub-channel a terminal randomly (or in round robin fashion) among those with $b_n^{(k)} = 1$ and allocate power, p_n, such that the sum rate is maximized. The transmission rate, $R_n(p_n)$, is determined by choosing a pair of modulation size and code rate that can be supported for given power allocation and instantaneous channel quality. To ensure error free transmission, it must be less than the instantaneous capacity of the sub-channel for the selected user, saying κ^{th}, i.e., $R_n(p_n) \le C(\gamma_n^{(\kappa)}) \equiv \log_2(1+p_n\gamma_n^{(\kappa)})$. Since the transmitter does not know the exact value of $\gamma_n^{(k)}$, the rate should be given by $R_n = \log_2(1+p_n\alpha_n)$[1]. If no user has bit 1 for a certain sub-channel, the BS does not assign power for that sub-channel and other sub-channel will get more power and, hence, higher rate. Denoting for the n^{th}

[1] In practice, R_n is determined as the maximal combination of modulation size and code rate that can be supported within a target packet error probability. For analytical simplicity, however, we do not consider packet transmission error and assume that once we assign the rate such that $R_n \le C(\gamma_n^{(k)})$, no error will occur.

sub-channel the number of users whose $b_n^{(k)}=1$ as K_n, the instantaneous sum rate is given by

$$\max_{\mathbf{p}} \frac{1}{N}\sum_{n=0}^{N-1} 1_{K_n>0} \log(1+p_n\alpha_n)^{2} \quad (7)$$

where $\mathbf{p} = [p_0, p_1,\ldots, p_{N-1}]$ and $1_{K>0}$ is 1 if $K>0$ and 0 otherwise.

Performance: From (5), the system throughput is given as an average of (7) over all possible H, i.e.,

$$R_{1bit}(\boldsymbol{\alpha}) = E_H\left[\max_{\mathbf{p}} \frac{1}{N}\sum_{n=0}^{N-1} 1_{K_n>0}\cdot\log(1+p_n\alpha_n)\right] \quad (8)$$

subject to $\sum_{n=0}^{N-1} p_n = P_T$

Assuming constant threshold $\alpha_0=\alpha_1=\ldots=\alpha_{N-1}=\alpha$ and denoting the number of sub-channels for which $K_n>0$ as $N'\le N$, the power allocation will be given by

$$p_n = \begin{cases} P_T/N' & \text{if } K_n>0 \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

the same for all the sub-channels. From this, (8) now becomes

$$R_{1bit}(\alpha) = \sum_{n=0}^{N} \Pr\{N'=n\}\frac{n}{N}\log(1+\alpha P_T/n) \quad (10)$$

where $\Pr\{N'=n\} = \binom{N}{n} q^{N-n}(1-q)^n$ with

$$q = \Pr\{K_n=0\} = (1-\exp(-\alpha/\Gamma))^K$$

IV. Selective Feedback

In this section, we review the selective feedback scheme proposed in [14] in which the user sends to BS indices and quality values of some selected sub-channels. In addition, we also take the power distribution into account for possible improvement especially when there is many 'empty channels'. Instead of using the empty channel with average channel quality obtained from the preceding transmission on that sub-channel, we distribute the power to other none-empty sub-channels. Hence, the operation of this scheme will be as follows.

Terminal feedback: Each terminal selects M (predetermined number) best sub-channels out of N and feed back to BS, their index set, $I^{(k)}=\{i_m^{(k)};\ m=0,1,..,M-1\} \in [0,N-1]^M$, and the corresponding channel quality information, $\Theta^{(k)}=\{\gamma_n^{(k)};\ n\in I^{(k)}\} \in \mathcal{R}^M$. In practice, the channel quality information to be reported to BS is not a real number, but an index drawn from a look up table that maps a channel quality to a pair of modulation size and code rate. Preparing the look up table for proper operation of adaptive modulation and coding scheme would be another important issue in practice, that will not be considered in this paper.

Resource allocation: Upon reception of feedback from users, the BS schedules for each sub-channel the user with the best quality. Once the users are selected for each sub-channel, transmission powers are distributed such that the sum rate is maximized. If no channel information is available for a certain sub-channel, the BS does not schedule and assign no power for that sub-channel. Hence, similar to the previous scenario, other sub-channels will get more power and resulting in higher rate. Denoting the set of user indices who feed back the n^{th} sub-channel as being one of its M best sub-channels as $J_n=\{j_k;\ k=1,2,..,K'_n\}$ and the number of users who offered the n^{th} sub-channel as the cardinality of J_n, i.e., $K'_n=|J_n|$, the instantaneous transmission rate is given by

$$\max_{\mathbf{p}} \frac{1}{N}\sum_{n=0}^{N-1} 1_{K'_n>0} \log(1+p_n \max_{k\in J_n}\gamma_n^{(k)}) \quad (11)$$

Performance: The performance is obtained by taking expectation of (11) over all possible realization of $\{\gamma_n^{(k)}\}$, i.e.,

$$R_{select}(M) = E_H\left[\max_{\mathbf{p}} \frac{1}{N}\sum_{n=0}^{N-1} 1_{K_n>0} \log(1+p_n \max_{k\in J_n}\gamma_n^{(k)})\right] \quad (12)$$

subject to $\sum_{n=0}^{N-1} p_n = P_T$

Evaluation of (12) is quite tricky since (a) the random variable K'_n is not independent of each other and (b) the probability distribution of $\gamma_n^{(k)};k\in J_n$ is a mixture of the distributions of the channel quality of the M best sub-channels of a user. Hence, we try to resort an approximate approach by slightly modifying the operation.

An Approximation: An approximation can be made by making the statistics for each sub-channel uncorrelated. To this end, we make some changes on the operation of the selective feedback scheme. In the modified scenario, a threshold value is used, similar to that in 1bit feedback per sub-carrier scheme. But in this scenario, it is to determine for each sub-channel whether the real-valued channel information is fed back or not. Once the channel quality of a sub-channel to a certain user is over this threshold, its index and the channel quality information are sent to BS. In this scenario, the threshold could also be a network parameter, which should be determined by the BS. However, it is not determined to maximize the network throughput as in the previous section, but to make the average feedback rate, i.e., the average number of real-valued channel quality feedback per user to a desired value, M. Hence, for given threshold, α, the average number of sub-channels with channel quality greater than the threshold, $M(\alpha)$ is given by, for any n,

$$M(\alpha) = N\Pr\{\gamma_n^{(k)}>\alpha\} = N\exp(-\alpha/\Gamma)$$

making the threshold set to

$$\alpha = \Gamma\log(N/M) \quad (13)$$

for the average number M feedback (including channel quality and sub-channel index) out of N. Let us define

---

[2] We assume pilot symbol has uniform power allocation and the value is 1.

$$\tilde{\gamma}_n^{(k)} \equiv \begin{cases} \gamma_n^{(k)} & \text{if } \gamma_n^{(k)} > \alpha \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

with which we have

$$\Pr\{\tilde{\gamma}_n^{(k)} < x\} = \begin{cases} \Pr\{\gamma_n^{(k)} < x\} = 1 - e^{-\frac{x}{\Gamma}} & \text{for } x \geq \alpha \\ \Pr\{\gamma_n^{(k)} < \alpha\} = 1 - e^{-\frac{\alpha}{\Gamma}} & \text{for } x < \alpha \end{cases} \quad (15)$$

The performance of this modified scheme is then given by

$$R_{\text{modified}}(M) = E_H\left[\max_{\mathbf{p}} \frac{1}{N}\sum_{n=0}^{N-1} \log(1 + p_n \tilde{\gamma}_n^*)\right] \quad (16)$$

where $\tilde{\gamma}_n^* = \max_{1\leq k\leq K} \tilde{\gamma}_n^{(k)} \sim d\tilde{F}_K(\gamma)/d\gamma$ with, for any n,

$$\tilde{F}_K(x) \equiv \Pr\{\max_{1\leq k\leq K} \tilde{\gamma}_n^{(k)} < x\} = \begin{cases} \left(1 - e^{-\frac{x}{\Gamma}}\right)^K & \text{for } x \geq \alpha \\ \left(1 - e^{-\frac{\alpha}{\Gamma}}\right)^K & \text{for } x < \alpha \end{cases} \quad (17)$$

Equation (16) is much easier to evaluate than (12) since $\tilde{\gamma}_n^* \forall n$ are now uncorrelated to each other. As was done in (3), letting $N \to \infty$ such that the empirical distribution of $\tilde{\gamma}_n^*$ converges to its probability distribution of (17), then (16) becomes

$$R_{\text{modified}} = \max_{\lambda(a)} \int_{\alpha}^{\infty} \log(1 + \lambda(\gamma)\gamma) d\tilde{F}_K(\gamma) \quad (18)$$

Note that by setting $\alpha = 0$ we have $M = N$ and (17) simply becomes (3), the performance with full channel side information.

V. Numerical Results and Remarks

We now compare the two reduced feedback schemes in term of their throughput behaviors in various system configurations and their pros and cons. Fig.1 shows the performances of the two schemes for N=4 uncorrelated sub-channels, as a function of the number of users, K, for K=1, 2, 4, 8, 16, 32 and 64. For the 1bit feedback per sub-channel scheme, we depict the achievable throughput, while, for selective feedback scheme, we plotted both approximate analysis given by (18) (marks with dashed lines) and simulation results on (12) (solid lines), for M=1, 2 and 4, where M=4 corresponds to full channel side information (CSI) scenario. Even though there exist considerable gap between simulation results and the approximation, especially when the number of users in the system is relatively small, the pattern looks quite similar to each other. When the number of users is small the selective feedback scheme does not perform as well as the one with 1bit feedback per sub-carrier scheme. However, as the number of users increase, the performance of the former approaches to the performance with full CSI even with a small M, while the performance of the latter does not. The same pattern is depicted in Fig.2 with N=16, much larger effective frequency diversity order. In this scenario, selective feedback scheme is even worse, i.e., sending CSI for a few sub-channels (small M) is far not enough to achieve multiuser diversity gain, especially when the number of users, K, is smaller than the effective frequency diversity order. In such case, it may be possible to obtain more multiuser diversity gain by increasing M (i.e., the uplink feedback overheads).

VI. Concluding Remarks

In this paper, we provided a comparison of throughput characteristics of two reduced channel information feedback schemes with various design parameters. Disregarding the amount of feedback overheads the selective feedback scheme is preferable when the number of users is larger than the effective order of frequency diversity, in which case the scheme almost fully achieves multiuser diversity gain that can be achieved by full channel side information. While, the 1 bit feedback per sub-channel scheme, as reported in [11-13], behaves like the one with full channel side information, but with considerable sum-rate reduction. It can be improved only by increasing the number of feedback bits per sub-channel and, in practice, adjusting the threshold value would be another important issue especially when the difference in average received power (large-scale path loss) is very large among users.

References

[1] C. Y. Wong, R. S. Cheng, K. B. Letaief and R. D. Murch, "Multiuser OFDM with Adaptive Subcarrier, Bit and Power Allocation," *IEEE J. on Selected Areas in Comm.*, Vol.17, No.10, pp.1747-1758, Oct. 1999

[2] W. Rhee and J.M. Cioffi, "Increase in Capacity of Multiuser OFDM System Using Dynamic Sub-Channel Allocation", *Proc. of VTC*'00-Spring, Volume: 2 , pp.15-18 May 2000

[3] H. Yin, H. Liu, "An efficient multiuser loading algorithm for OFDM-based broadband wireless systems," *Proc. of Globecom* '00, Vol.1, pp.103-107, San Francisco, 2000.

[4] J. Jang and K. Lee, "Transmit Power Adaptation for Multiuser OFDM Systems," *IEEE Journal on Selected Areas in Comm.*, Vol.21, No.2, pp.171-178, Feb. 2003

[5] P. Viswanath, D.N.C. Tse and R. Laroia, "Opportunistic Beamforming using Dumb Antennas", *IEEE Trans. on Info. Theory*, Vol.48, No.6, pp.1277-1294, June 2002.

[6] M. Sharif and B. Hassibi, "On the Capacity of MIMO broadcast channels with partial side information", *IEEE Trans. on Information Theory*, Vol.51, No.2, pp.506-522, Feb. 2005.

[7] A.J. Goldsmith and M. Effros, "The Capacity Region of Broadcast Channels with Intersymbol Interference and Colored Gaussian Noise", *IEEE Trans. on Info. Theory*, Vol.47, No.1, pp.219-240, Jan. 2001

[8] P. Svedman, S.K. Wilson, L. Cimini and B. Ottersten, "A Simplified feedback and scheduling scheme for OFDM", *Proc. of VTC* 2005, May 17-19, 2004, Milan

[9] D. Gesbert and S. Alouini, "Selective multiuser diversity", *Proc. of Intl. Symp. on Sig. Proc. and Info. Tech. 2003* (*ISSPIT*'03), Dec. 2003, Darmstadt, Germany.

[10] D. Gesbert and S. Alouini, "How much feedback is multi-user diversity really worth", *Proc. of Intl. Symp. on Sig. Proc. and Info. Tech. 2003* (*ISSPIT*'03), Dec. 2003, Darmstadt, Germany.

[11] S. Sanayei, A. Nosratinia and N. Aldhahir, "Opportunistic dynamic subchannel allocation in multiuser OFDM networks with limited feedback", P*roc. of IEEE Info. Theory Workshop* 2004 (ITW04), Oct. 2004, San Antonio, Texas.

[12] S. Sanayei and A. Nosratinia, "Exploiting multiuser diversity with

only 1-bit feedback", *Proc. WCNC* 2005, March. 2005, Los Angeles, CA.

[13] Somekh, O. Simeone, A.M. Haimovich and Y. Bar-Ness, "Sum-Rate Analysis of OFDM Downlink Channels with 1-bit Feedback per Sub-carrier", *Proc. of CISS*' 06, pp. 306-311, Mar. 2006

[14] Z. Han and Y. Lee, "Opportunistic Scheduling with Partial Channel Information in OFDMA/FDD Systems", *Proc. Of VTC*'04 Fall, Vol.1, pp.511 – 514, Sept. 2004

[15] M.J. Fekhereddin, M. Sharif and B. Hassibi, "Throughput Analysis in Wideband MIMO broadcast channels with partial feedback", *Proc. of IEEE Workshop on SPAWC* 2005, pp.771-775

[16] S. Yoon, C. Suh, Y. Cho, DS Park and Y. Kim, " Orthogonal Frequency Division Multiple Access with An Aggregated Sub-channel Structure and Statistical Channel Quality Measurement", *Proc. of VTC* 2004 Fall, pp.1023-1027, Sep.26-29, Los Angeles, CA

[17] S. Yoon and C. Suh, "Performance of Orthogonal Frequency Division Multiple Access with Sub-banded Structure and Statistical Channel Quality Measurement", submitted to *IEEE Trans. on Wireless Communications*

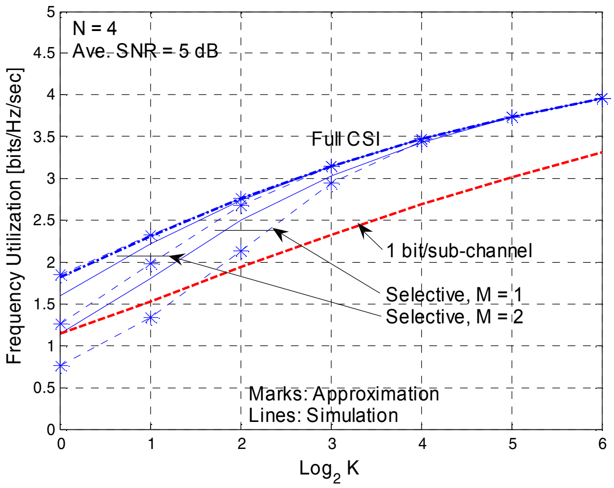

Figure1: Average network throughput with N=4

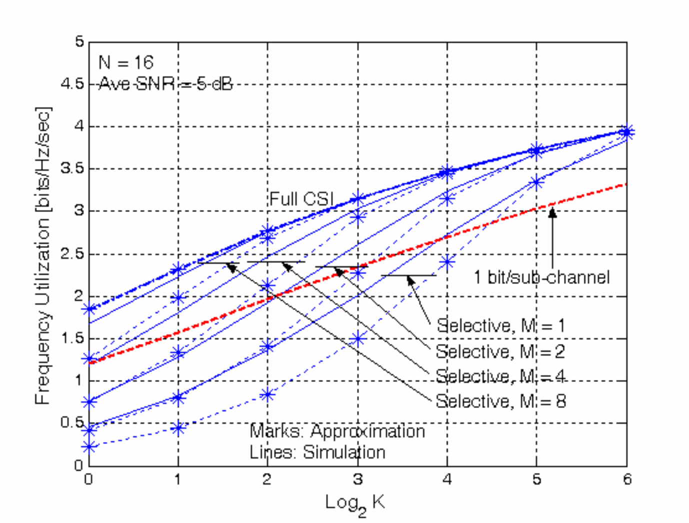

Figure 2: Average network throughput with N=16