ON THE THROUGHPUT REGION OF SINGLE AND TWO-WAY MULTI-HOP FADING NETWORKS WITH RELAY PIGGYBACKING

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ABSTRACT

One and two-way communication strategies are studied in a two-hop model in which the intermediate node (relay) piggybacks data packets intended for the end-users. Assuming quasi-static fading channels, memoryless processing at all the nodes and no latency constraints, the region of simultaneously achievable throughputs for the two end-users and the relay, measured in reliably transmitted bits per second per Hz, is investigated. For both one-way and two-way transmission, outer and inner bounds to the throughput region are derived. The considered achievable strategies involve type-I HARQ, Decode-and-Forward and joint or superposition encoding at the relay. The trade-off between the user and relay throughputs is analyzed and studied via numerical simulations, yielding insight into optimal design choices.

1. INTRODUCTION

In wireless networks operating over quasi-static fading channels, link failures are of non-ergodic nature. In order to guarantee reliable data delivery, this calls for retransmission strategies, also referred to as HARQ (Hybrid Automatic Repeat reQuest). The problem of designing HARQ strategies on multihop fading networks is significantly different from the counterpart in the wired world, due to the unique features of radio propagation (e.g., broadcasting, fast variability), and has attracted interest in recent years (see [1] [2] and references therein). This work is motivated by two relevant issues that have been mostly overlooked in previous activity in this area, namely: (*i*) intermediate nodes in a multihop network may have locally generated data to communicate to other nodes (e.g., control signals); (*ii*) different routes (chains of nodes) may be used in a bidirectional fashion by the two-end users.

We focus on a two-hop fading network over quasi-static fading channels, as shown in Fig. 1, and address the design of memoryless encoding/ decoding strategies at the nodes that guarantee reliable data delivery via HARQ. As detailed below, memoryless processing prevents encoders and decoders to combine packets received in different slots, thus reducing memory requirement and simplifying system design. This approach is typically referred to as type-I HARQ in point-to-point channels. We consider two scenarios: (*a*) *one-way communications*: the end-user T_a communicates with T_b via the relay T_r, which also has locally generated data for T_b; (*b*) *two-way communications*: both end-users T_a and T_b have data for one another

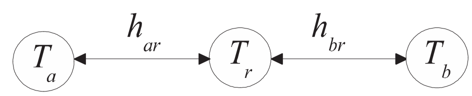

Fig. 1. Two-hop relay network over quasi-static fading channels for either one-way or two-way communications.

and the relay T_r is interested in communicating common information to both T_a and T_b. Scenario (*a*) was studied in [2] in the special case where the relay has no private data for T_b, and for a more general topology, under a Gilbert-Elliot channel model. Also related is the work [3] where a discrete and Gaussian (unfaded) relay channel was investigated in the presence of a relay with a private message. Scenario (*b*) without relay data was considered in [4], where strategies based on physical layer network coding were first proposed (see also [5]). Scenario (*b*) was finally studied over unfaded channels in [6] in the presence of relay data.

We formulate the problem at hand, along with the definition of throughput region, in Sec. 2. Based on this, we first tackle the one-way communications model in Sec. 3, deriving outer and inner bounds to the corresponding throughput region, and then address the two-way case in Sec. 4. Finally some numerical results and conclusions are provided in Sec. 5.

2. SYSTEM MODEL

We consider the two-hop network in Fig. 1, where all the nodes work in *half-duplex* mode. When considering *one-way* communications in Sec. 3, terminal T_a is assumed to communicate with terminal T_b with the help of the relay T_r, which has also a message of its own to transmit to the terminal T_b. With *two-way* communications studied in Sec. 4, instead, both T_a and T_b have data for each other, and the relay broadcast common data to both T_a and T_b as in [6]. The application at hand is assumed to be insensitive to delays, and retransmissions (HARQ) are exploited to achieve reliable (zero-error) data delivery. To enable HARQ, packet transmission is assumed to be acknowledged via reliable ACK/NACK messages. Terminal(s) and the relay have infinite backlogs of data intended for the given destination(s). Time is slotted, with slots (also referred to as packets) of n channel uses. The (frequency-flat) channel h_{ij} between nodes i and j is characterized by Rayleigh fading, $|h_{ij}|^2 \sim \exp(1)$ and changes independently slot by slot. Only *channel state information* at the receiver's side is assumed. In the following, we focus the description on the two-way model and point out the differences with the one-way case only when not clear from the context.

Each time slot is assigned to either *uplink* (source to relay) or *downlink* (relay to destination). For uplink time-slots, the signal received at the relay node is given by the $n \times 1$ vector $\mathbf{y}_r$:

$$\mathbf{y}_r = h_{ar}\mathbf{x}_a + h_{br}\mathbf{x}_b + \mathbf{n}_r, \quad (1)$$

where $\mathbf{n}_r \sim \mathcal{CN}(0, \mathbf{I})$ is Additive White Gaussian Noise (AWGN), and $\mathbf{x}_a$, $\mathbf{x}_b$ represent the codewords transmitted by T_a and T_b respectively, with the convention that $\mathbf{x}_i = \mathbf{0}$ if user T_i is not transmitting. Packet $\mathbf{x}_a$ ($\mathbf{x}_b$) is assumed to convey a fixed-rate message $W_a \in [1, 2^{nR_a}]$ ($W_b \in [1, 2^{nR_b}]$) of nR_a (nR_b) bits, taken from the infinite backlog of the user T_a (T_b). In other words, $\mathbf{x}_i$ is taken from a codebook of 2^{nR_i} codewords via an encoding function $\mathbf{x}_i = f_i(W_i)$, $i \in \{a, b\}$. For downlink, the signal received at terminal T_i is:

$$\mathbf{y}_i = h_{ri}\mathbf{x}_r + \mathbf{n}_i, \quad (2)$$

where $\mathbf{x}_r$ is the codeword transmitted by the relay and $i \in \{a, b\}$. The signal $\mathbf{x}_r$ depends on a relay message $W_r \in [1, 2^{nR_r}]$ of fixed rate R_r, taken from the relay backlog, and on the packets received from the users (see below for details). We assume the standard power constraint $\|\mathbf{x}_i\|^2 \leq nP_i$, $i \in \{a, b, r\}$.

In the interest of simplifying system design and analysis, we focus on *memoryless* operation at the nodes, extending the standard type-I HARQ strategy for point-to-point channels. In particular, we assume that: (**a.1**) decoding at the users takes place based on only *one* received packet via decoding functions $(\hat{W}_b, \hat{W}_r) = g_a(\mathbf{y}_a)$ and $(\hat{W}_a, \hat{W}_r) = g_b(\mathbf{y}_b)$, as in type-I HARQ (i.e., no packet combining is allowed). Notice that for one-way communications $g_a(\cdot)$ does not apply; (**a.2**) a new message W_a, W_b or W_r is generated only after the previously generated message has been correctly received at the intended destination(s) (i.e., no combining of different message "generations" is allowed); (**a.3**) the relay constructs its downlink packet $\mathbf{x}_r$ as a function of its message W_r and at most one packet $\mathbf{y}_r$ (1) received from each user (i.e., no combination of different packets from the same user is allowed). Finally, we enforce the assumption that the relay can only *piggyback* own data on the user's data, thus preventing the relay from using all the slots for downlink. This amounts to: (**a.4**) the relay cannot send a new message W_r if the current message W_r has been correctly delivered but not the current users' messages W_a and W_b. It is emphasized that the piggybacking assumption is appropriate for systems in which the major role of the relay is forwarding users' data, while common data (such as control information) is sent with lower priority.

2.1. Throughput Region

We focus on the long-term throughputs achievable by the users and by the relay. Given transmission rates (R_a, R_b, R_r) (recall the discussion above), the users' sum-throughput, measured in reliably transmitted bits per second per Hz, is defined as:

$$\eta_u = \lim_{M\to\infty} \frac{1}{M} \sum_{m=1}^{M} R_a I_a[m] + R_b I_b[m], \quad (3)$$

where $I_i[m]$, for $i \in \{a, b\}$, is an indicator function of a successful decoding event for time slot m, defined as: $I_i[m] = 1$ if T_j *decodes a packet from* T_i *in time slot* m, for $i \neq j$, with $j \in \{a, b\}$, and $I_i[m] = 0$ otherwise. For the one-way case, user T_a's throughput is defined as

$$\eta_a = \lim_{M\to\infty} \frac{1}{M} \sum_{m=1}^{M} R_a I_a[m]. \quad (4)$$

The relay throughput is similarly defined as:

$$\eta_r = \lim_{M\to\infty} \frac{1}{M} \sum_{m=1}^{M} R_r I_r[m], \quad (5)$$

where $I_r[m] = 1$ if m *is the first slot by which both* T_a *and* T_b *have decoded the current message* W_r *during time slot* m. For the one-way model, only decoding at T_b applies. A user relay throughput pair (η_u, η_r) for two-way communications and (η_a, η_r) for one-way communications is said to be *achievable* if there exists a transmission strategy satisfying the assumptions above and (non-negative) rates (R_a, R_b, R_r) for which (3)-(5) hold. The *throughput region* is the set of all achievable pairs (η_u, η_r) (two-way) or (η_a, η_r) (one-way). In the rest of the paper, we focus on a *symmetric system*, characterized by $R_a = R_b \triangleq R_u$ and $P_a = P_b = P_r \triangleq P$. Extension to a non-symmetric scenario is possible but will not be further pursued here.

3. ONE-WAY COMMUNICATIONS

In this section, we consider the one-way communications model and first derive an outer bound on the throughput region in Sec. 3.1 and then propose achievable strategies in Sec. 3.2. Comparison of achievable regions and outer bound is provided in Sec. 5 via numerical results.

3.1. Outer Bound on the Throughput Region

Proposition 1. The throughput region for the one-way communications model is included in the union of all pairs (η_a, η_r) that satisfy

$$2\eta_a + \eta_r \leq R^* \exp\left(-\frac{2^{R^*} - 1}{P}\right) \quad \text{(6a)}$$

$$\eta_r \leq \max_{R_r} \frac{R_r}{1 + \exp\left(\frac{2^{R_r}-1}{P}\right)} \quad \text{(6b)}$$

with $R^* = \mathcal{W}_0(P)/\log(2)$, where $\mathcal{W}_0(\cdot)$ is the Lambert $\mathcal{W}$ function main branch[1].

Proof: See Appendix A.

3.2. Achievable Throughput Regions

In this section we propose two protocols, which provide achievable throughput regions and differ in the strategy adopted in downlink. In uplink, for both schemes, user T_a transmits its current message W_a using a standard "Gaussian codebook"[2] $\mathbf{x}_a(W_a)$ and, following (a.1), retransmits the same codeword until it is correctly received at the relay as $W_a = g_r(\mathbf{y}_r)$. For downlink, in the first scheme, the relay performs *joint encoding* of the user's and the relay's current messages W_a and W_r, by transmitting a codeword $\mathbf{x}_r(W_a, W_r)$ from a Gaussian codebook of size $2^{n(R_a+R_r)}$. The relay then keeps transmitting until T_b correctly decodes both messages, i.e., $(W_a, W_r) = g_b(\mathbf{y}_b)$. Note that, because of the joint encoding, T_b is forced to decode a message with rate $R_a + R_r$, and no single message W_a or W_r can be separately retrieved (unless both are). This joint encoding protocol can be studied via the Markov

[1] $\mathcal{W}_0(\cdot)$ is defined as the inverse of the function $f(w) = we^w$.

[2] A Gaussian codebook is randomly generated with each letter selected independently according to a circularly symmetric Gaussian distribution with same power P.

model of Fig. 2-(a), where the state is given by the current content of the relay's buffer B_r (messages that have been correctly decoded at T_b are dropped), namely: 1) state S_0 ($B_r = \{\varnothing\}$): the relay does not contain any message and T_a transmits in uplink to deliver a new message W_a to the relay; 2) state S_{ar} ($B_r = \{W_a, W_r\}$). Notice that in transitioning from S_0 to S_{ar}, after having decoded W_a, the relay extracts a new message W_r from its backlog, and transmits in downlink, as explained above, until T_b correctly decodes both messages.

In the second scheme, in downlink, the relay employs *superposition encoding* by encoding the two messages W_a and W_r onto two *different* codewords, say $\mathbf{x}'_r(W_a)$ and $\mathbf{x}''_r(W_r)$. These codewords are taken from Gaussian codebooks of rates R_a and R_r, respectively, and power P, and are superimposed as

$$\mathbf{x}_r = \sqrt{\beta}\mathbf{x}'_r(W_a) + \sqrt{(1-\beta)}\mathbf{x}''_r(W_r), \quad (7)$$

where $\beta \in [0, 1]$ defines the power allocation between the two codewords. On the one hand, superposition coding is more flexible than joint encoding since it allows the decoder, when failing to decode *both* message, to at least possibly decode only *one* of the two messages. However, on the other hand, superposition coding is more likely to result in failed decoding for both messages than joint encoding. This can be seen by viewing the received signal (2) given (7) as a Multiple Access Channel (MAC) and considering its capacity region [7]. In the proposed protocol, if T_b decodes only one of the two messages, in the following time-slot the relay allocates all the power to the other message (i.e., $\beta = 0$ or $\beta = 1$ in (7)). The protocol based on superposition encoding can then be described following the Markov model of Fig. 2-(b), where the states are: 1) state S_0 ($B_r = \{\varnothing\}$); 2) state S_{ar} ($B_r = \{W_a, W_r\}$); 3) state S_a ($B_r = \{W_a\}$); 4) state S_r ($B_r = \{W_r\}$). States S_a and S_r are reached when, after a downlink transmission in state S_{ar}, the destination T_b decodes only W_r or W_a respectively. As explained above, in such states, the relay transmits until the message in the buffer is correctly received.

It is finally noted that the two protocols proposed above satisfy the memoryless (a.1)-(a.3) and relay piggybacking (a.4) assumptions formulated in the previous section.

3.2.1. Throughput Analysis

The user's and relay's throughputs (4) and (5), respectively, for the two strategies discussed above can be obtained by finding the steady-state probabilities of the Markov chains in Fig. 2. The transition probabilities in the Markov models are given by the outage probabilities $p_{out,UL} = p_{out}(R_a, P)$, $p_{out,DL} = p_{out}(R_a + R_r, P)$, $p_{out,DLa} = p_{out,UL}$ and $p_{out,DLr} = p_{out}(R_r, P)$ where we have defined

$$p_{out}(R, P) = \Pr\left\{R > C\left(P|h|^2\right)\right\} = 1 - \exp\left(-\frac{2^R-1}{P}\right), \quad (8)$$

with $C(x) = \log_2(1 + x)$ being the capacity of an AWGN channel, as the outage probability for transmission of a Gaussian codebook of rate R and power P over a unit-power Rayleigh fading channel h. For superposition encoding (Fig. 2-(b)), we also need to define the probability that only one message, e.g., W_r, gets decoded in state S_{ar}:

$$p^{SUP}_{out,a} = \Pr\left\{R_a > C\left(P\beta|h_{rb}|^2\right), R_r \leq C\left(\frac{P(1-\beta)|h_{rb}|^2}{1+P\beta|h_{rb}|^2}\right)\right\}, \quad (9)$$

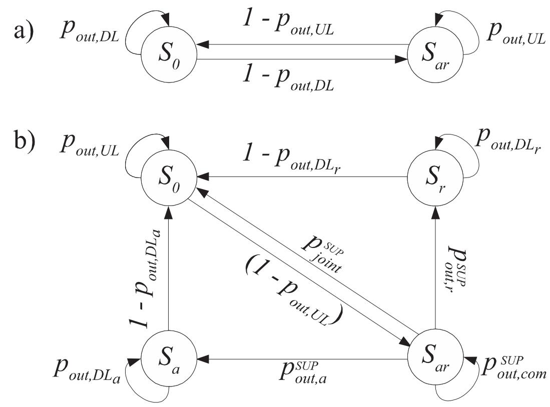

Fig. 2. One-way communications: Markov chains representing: a) the joint encoding protocol; b) the superposition encoding protocol.

and $p^{SUP}_{out,r}$ is similarly defined, and the probability that no message gets decoded when transmitting in state S_{ar}:

$$\begin{aligned} p^{SUP}_{out,com} = \Pr\Big\{ & R_a > C\left(\frac{P\beta|h_{rb}|^2}{1+P(1-\beta)|h_{rb}|^2}\right), \\ & R_r > C\left(\frac{P(1-\beta)|h_{rb}|^2}{1+P\beta|h_{rb}|^2}\right), \\ & R_a + R_r > C\left(P|h_{rb}|^2\right)\Big\}. \end{aligned} \quad (10)$$

As a result, the probability of joint decoding is given by: $p^{SUP}_{joint} = 1 - p^{SUP}_{out,com} - p^{SUP}_{out,r} - p^{SUP}_{out,a}$. It is remarked that (9) and (10) follow from the capacity region of a Gaussian MAC (see, e.g., [7]).

As mentioned above, the throughputs can then be calculated by finding the steady-state probabilities of the Markov chains in Fig. 2. Namely, it can be seen that, for the joint encoding scheme, the throughputs for user and relay are given by:

$$\eta^{JE}_a = R_a\pi^{JE}_{ar}(1 - p_{out,DL}) \text{ and } \eta^{JE}_r = R_r\pi^{JE}_{ar}(1 - p_{out,DL}), \quad (11)$$

respectively, where π^{JE}_{ar} is the steady-state probability of state S_{ar} of the chain in Fig. 2-(a). For the superposition scheme instead we have:

$$\eta^{SUP}_a = R_a\left(\pi^{SUP}_{ar}(p^{SUP}_{joint} + p^{SUP}_{out,r}) + \pi^{SUP}_a(1 - p_{out,DLa})\right), \quad (12)$$

$$\eta^{SUP}_r = R_r\left(\pi^{SUP}_{ar}(p^{SUP}_{joint} + p^{SUP}_{out,a}) + \pi^{SUP}_r(1 - p_{out,DLr})\right), \quad (13)$$

where π^{SUP}_i represent the steady-state probabilities for state S_i of the Markov chain in Fig. 2-(b). We omit the cumbersome expressions of such probabilities for lack of space.

4. TWO-WAY COMMUNICATIONS

In this section, we consider the two-way communications model. First, we extend the outer bound to the throughput region of Sec. 3.1 to the two-way scenario in Sec. 4.1 and then propose achievable strategies in Sec. 4.2. Comparison of achievable regions and outer bound is provided in Sec. 5 via numerical results.

4.1. Outer Bound on the Throughput Region

Proposition 2. The throughput region for the one-way communications model is included in the union of all pairs (η_u, η_r) that satisfy

$$\eta_u + \eta_r \quad \leq \quad R^* \exp\left(-\frac{2^{R^*}-1}{P}\right) \tag{14a}$$

$$\eta_r \quad < \quad \max_{R_r} \frac{R_r}{1+\exp\left(\frac{2^{R_r}-1}{P}\right)} \tag{14b}$$

for some (non-negative) rates (R_a, R_b, R_r), with $R^* = \frac{\mathcal{W}_0(P)}{\log(2)}$.

Proof: The proof follows directly from Proposition 1 since the throughput per user attainable in a one-way scenario, given a certain relay throughput, is an upper bound to the throughput per user attainable in a two-way model.

4.2. Achievable Throughput Region

In this section we propose two protocols, which provide achievable throughput regions and differ in the strategy adopted in uplink. In downlink, for both protocols, the relay performs *joint encoding* of the users and relay's messages, similarly to Sec. 3.2. In particular, the codeword $\mathbf{x}_r$ is selected from a Gaussian codebook of size $2^{n(2R_u+R_r)}$ and indexed as $\mathbf{x}_r(W_a, W_b, W_r)$. It is noted that each user, say T_a, knows its own message W_a, and thus can decode the downlink transmission by seeking in a codebook of size $2^{n(R_u+R_r)}$. For uplink, in the first strategy, referred as *Single Decode-and-Forward* (SDF), users T_a and T_b transmit to the relay in different slots, while in the second strategy, referred as *Joint Decode-and-Forward* (JDF), users T_a and T_b transmit simultaneously and the relay performs joint decoding of the users' messages.

Description of both SDF and JDF protocols can be given in terms of the Markov model in Fig. 3, with states: 1) state S_0 ($B_r = \{\varnothing\}$): the relay's buffer does not contain any message, so that in the SDF protocol the relay keeps polling T_a until it correctly decodes a message W_a, while in the JDF the relay keeps polling both users until it simultaneously decodes both users messages; 2) state S_a ($B_r = \{W_a\}$): this state applies only to SDF and is reached when the relay correctly receives a message W_a starting from state S_0 [3]; 3) state S_{abr} ($B_r = \{W_a, W_b, W_r\}$ and both users have not correctly decoded the downlink transmission): this state is reached for SDF after that the relay decodes W_b while in state S_a, whereas with JDF it is reached after successful joint decoding in state S_0; 4) State S_{ur} ($B_r = \{W_a, W_b, W_r\}$ and one of the two users has correctly decoded the downlink transmission): this state is reached by both SDF and JDF whenever during the downlink transmission one of the two users does not decode the jointly encoded message[4].

4.2.1. Throughput Analysis

Similarly to Sec. 3.2.1, the throughput region achieved by the proposed schemes can be found by calculating the steady-state probabilities of the Markov chains in Fig. 3. The transition probabilities are $p_{out,UL} = p_{out}(R_u, P)$ and $p_{out,DL} = p_{out}(R_u + R_r, P)$ (see (8)), while the joint decoding probability $p_{joint}^{JDF} = 1 - p_{out,com}^{JDF} - p_{out,a}^{JDF} - p_{out,b}^{JDF}$ for the JDF follows from the capacity region of a Gaussian MAC, where:

$$p_{out,com}^{JDF} = \Pr\left\{2R_u > C\left(\left(P\left|h_{rb}\right|^2 + |h_{rb}|^2\right)\right)\right\}, \tag{15}$$

is the probability that no messages are decoded at the relay, while the single outage probability for the user T_a is given by:

$$p_{out,a}^{JDF} = \Pr\left\{R_u > C\left(P\left|h_{ar}\right|^2\right), R_u \leq C\left(\frac{P|h_{br}|^2}{1+P\beta|h_{br}|^2}\right)\right\}, \tag{16}$$

and similarly for user T_b follows $p_{out,b}^{JDF}$. These probabilities are calculated in closed-form in [8]. It can be seen that, defining π_i^j the steady-state probability of state S_i for protocol j, we have the following. For the SDF scheme, the users sum-throughput is:

$$\eta_u^{SDF} = R_u(1 - p_{out,DL})(2\pi_{abr}^{SDF} + \pi_{ur}^{SDF}), \tag{17}$$

while the relay's throughput is:

$$\eta_r^{SDF} = R_r(1 - p_{out,DL})\left[\pi_{abr}^{SDF}(1 - p_{out,DL}) + \pi_{ur}^{SDF}\right]. \tag{18}$$

Similarly, for the JDF scheme we obtain:

$$\eta_u^{JDF} = R_u(1 - p_{out,DL})(2\pi_{abr}^{JDF} + \pi_{ur}^{JDF}), \tag{19}$$

and for the relay:

$$\eta_r^{JDF} = R_r(1 - p_{out,DL})\left(\pi_{abr}^{JDF}(1 - p_{out,DL}) + \pi_{ur}^{JDF}\right). \tag{20}$$

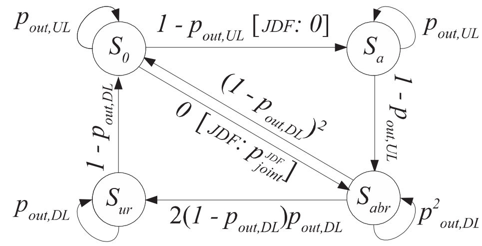

Fig. 3. Two-way communications: Markov chain representing SDF and JDF protocols. Square brackets indicate the transition probabilities for the JDF protocol when different from the SDF.

5. NUMERICAL RESULTS AND FINAL REMARKS

Fig. 4 shows the achievable throughput regions for one-way communications using the joint and superposition encoding (for $\beta = 0$, 1 and $1/2$) protocols proposed in Sec. 3.2, along with the outer bound derived of Proposition 1, for $P = 3$ and 15dB. It is noted that the superposition encoding strategy with $\beta = 0$, 1 corresponds to a Time Division (TD) scheme in which the relay transmits messages W_a and W_r in different time-slots. It is interesting to remark that TD attains the outer bound when $\eta_a = \eta_r$. This is similar to known results for Gaussian (unfaded) MAC channels [7]. Moreover, the throughput region obtained with the joint encoding attains the outer bound for either $\eta_a = 0$ or $\eta_r = 0$. It is also noted that all the pairs (η_a, η_r) contained in the *convex hull* of the two achievable throughput regions can be obtained by time sharing between the two techniques [7]. Therefore, it can be easily seen from Fig. 4 that time-sharing

[3] The asymmetry between T_a and T_b is simply due to the arbitrary choice of start polling user T_a when in state S_0.

[4] There is no need to distinguish between the two users by the assumption of symmetry.

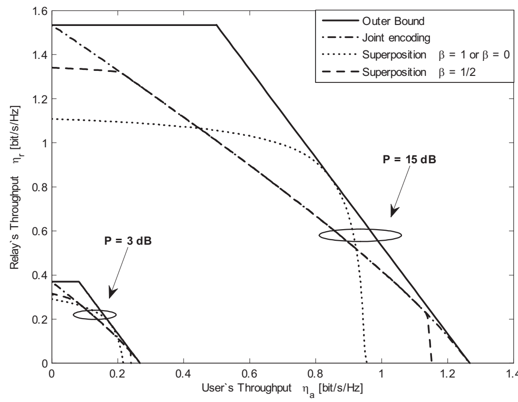

Fig. 4. One-way communications: achievable throughput regions for the joint and superposition encoding (for $\beta = 0$, 1 and $1/2$) protocols, along with the outer bound of Proposition 1 ($P = 3$ and 15dB).

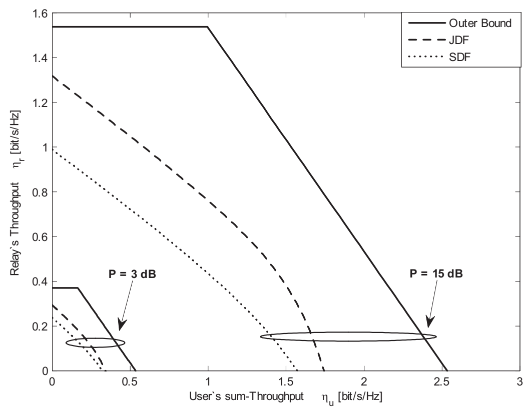

Fig. 5. Two-way communications: achievable throughput regions for SDF and JDF protocols, along with the outer bound of Proposition 2 ($P = 3$ and 15dB).

allows the outer bound to be attained for all pairs with $\eta_a \geq \eta_r$. The throughput regions for two-way communication are plotted in Fig. 5, along with the outer bound of Proposition 2. As expected, JDF provides relevant gains over SDF, especially for values of η_a not too large. It is also noted that the achievable regions are much smaller than the outer bound, when confronted with the one-way case. It can be inferred that the bottleneck of two-way communications is then mainly determined by the uplink channel. In order to alleviate the problem, one could use more sophisticated techniques that exploit structured codes at the users' side [9]. However, implementing such techniques over fading channels is quite challenging and, at the very least, require channel state information also at the transmitter (see also [5]). A full investigation of this point is left as future work.

A. PROOF OF PROPOSITION 1

We derive (6a) using cut-set arguments similarly to [10]. Due to the half-duplex constraint, we can use the uplink channel for a given fraction $\theta \in [0, 1]$ of the time, and the downlink channel for the remaining fraction $(1 - \theta)$. Consider now the cut between T_a and T_r: the user throughput η_a is limited by the throughput that can be delivered to the relay. Moreover, it is optimal for terminal T_a to use standard random "Gaussian codebooks" when transmitting, since this maximizes the uplink capacity, and to perform retransmission according to type-I HARQ due to (a.1). It follows that $\eta_a \leq \theta \max_{R_a} R_a/N_a(R_a)$ where $N_a(R_a)$ is the average number of retransmissions needed for reliable transmissions. We have $N_a(R_a) = \exp\left(\frac{2^{R_a}-1}{P}\right)$ since

$\Pr\left\{R_a \leq C\left(P\left|h_{ar}\right|^2\right)\right\} = \exp\left(-\frac{2^{R_a}-1}{P}\right)$ is the probability of successful decoding at the relay, where $C(x) = \log_2(1 + x)$ is the capacity over an AWGN channel. Maximizing over R_a, we get $\eta_a \leq \theta R^* \exp\left(-\frac{2^{R^*}-1}{P}\right)$. Similarly, considering the cut between T_r and T_b, the sum-throughput $\eta_a + \eta_r$ is upper bounded by $\eta_a + \eta_r \leq (1-\theta) R^* \exp\left(-\frac{2^{R^*}-1}{P}\right)$. Eliminating θ, upper bound (6a) follows. The upper bound (6b) is obtained similarly by setting rate $R_a = 0$. In fact, in the resulting network, it is clearly optimal for the relay to transmit towards the user T_b using Gaussian codebooks and type-I HARQ. It is noted that the addition of 1 in the denominator of (6b) is a technical consequence of assumption (a.4) in that the relay, before transmitting a new message W_r, has to acquire a new user message W_a, which requires one time-slot.

B. REFERENCES

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