PERFORMANCE ANALYSIS OF COLLABORATIVE HYBRID-ARQ INCREMENTAL REDUNDANCY PROTOCOLS OVER FADING CHANNELS

Igor Stanojev, Osvaldo Simeone and Yeheskel Bar-Ness

Center for Wireless Communications and Signal Processing
New Jersey Institute of Technology
Newark, New Jersey 07102-1982, USA
E-mail: {iss2, osvaldo.simeone.2, yeheskel.barness}@njit.edu

ABSTRACT

Future wireless communication systems are expected to rely heavily on spatial diversity for mitigation of fading impairments. In scenarios where practical constraints prevent the collocation of multiple antennas on a single user terminal, collaboration between single-antenna nodes becomes the only viable solution. Among cooperative schemes, Collaborative ARQ transmission protocols, prescribing cooperation only upon erroneous decoding by the destination, emerge as an interesting solution in terms of achievable spectral efficiency.

In this paper, an information theoretical approach is presented for assessing the performance of Collaborative Hybrid-ARQ protocol based on Incremental Redundancy. Upper and lower bounds for the expected number of retransmissions and the average throughput are derived in explicit form, for any number of relays. Numerical results are presented to supplement the analysis and give insight into the performance of the considered scheme.

1. INTRODUCTION

Impairments due multipath signal propagation on the performance of wireless communications systems can be efficiently mitigated with time, frequency or spatial diversity. To exploit spatial diversity, multiple-antenna technology has been thoroughly investigated and emerged as one of the most mature communications areas [1]. However, the need for smaller user terminals, which results in insufficient spacing for antenna collocation, tends to limit the practical implementation of this technology. Without compromising terminal dimensions, future wireless networks will therefore have to exploit their broadcast nature and rely on collaboration between single-antenna terminals for exploiting spatial diversity. Users cooperation has been investigated in [2], and then further addressed under practical limitations, such as the half-duplex constraint (see, e.g., [3]) or power allocation and consumption [4].

Most of the work on collaborative transmission assumes a fixed orthogonal medium access control mechanism, say TDMA, with typically one time-slot dedicated to the transmission by the source terminal and the other for the relay (see, e.g., [5], [6]). As recognized in [7], the reduction of the system throughput induced by this lack of flexibility can be mitigated if relays collaborate for transmission with source only *if needed*. In particular, collaboration can take place only during retransmission requested by the destination, whereby the relays and the source form a distributed antenna array and transmit space-time codeword to the destination. Clearly, this approach is an application of the Automatic Repeat ReQuest (ARQ) principle to a collaborative environment.

Performance analysis of Collaborative Hybrid-ARQ (HARQ) Type I and Chase Combining protocols has been presented in [8]. In this paper, we focus on the performance analysis of Collaborative HARQ schemes based on the Incremental Redundancy protocol (or code combining [9]). Lower and upper bounds on the system performance, namely the average number of retransmissions and the average throughput, are derived for any number of relays. Moreover, simulation results are provided to supplement the analysis.

2. SYSTEM OVERVIEW

2.1. Protocol Overview

According to the proposed Collaborative HARQ protocol, in the first time-slot the source S broadcasts a packet to destination D and any available relay R_i, $i = 1, ..., M$ (see fig. 1 for an example with $M = 2$). If Cyclic Redundancy Check (CRC) at the destination determines erroneous decoding, packet retransmission is requested by the destination via a Not Acknowledge (NACK) message. Then, relays, that have successfully decoded in the first time-slot (i.e., relay R_1 in example of fig. 1), signal their availability to the source and switch from receiving to transmitting

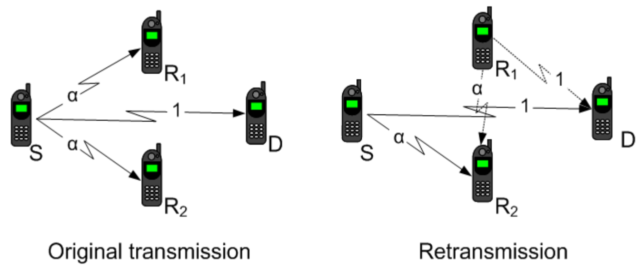

Fig. 1. Illustration of Collaborative ARQ with two relays, R_1 and R_2 ($M = 2$).

mode. The retransmission is performed by a distributed antenna array consisting in the source and activated relays, through joint transmission of a space-time codeword. Notice that, in conformity with the HARQ Incremental Redundancy (HARQ-IR) paradigm, the space-time codeword of the succeeding retransmissions contains new parity bits of the original packet. The destination, as well as any remaining receiving relays (i.e., relay R_2 in fig. 1), decode the data after appropriate code-combining with previously received codewords [9]. The procedure repeats until the CRC at the destination reveals successful detection and an Acknowledge (ACK) message is sent, or a predefined maximum number of retransmissions is reached.

2.2. System Model

Consider a system with $M+2$ single-antenna stations, consisting in a source S, destination D and M relays. A block Rayleigh fading model is assumed, where the channel gain $h_{ij}^{(n)}$ between terminals i and j at the nth transmission attempt (i.e., $(n-1)$th retransmission) stays constant during the transmission slot, but changes independently with each retransmission (i.e., with n). The channels between any two terminals are mutually independent circularly symmetric complex Gaussian variables with unit power. As illustrated in fig. 1, the average power received by the relay stations from the source and from any other relays is assumed to be larger than the average power received by the destination from any node by a factor $\alpha > 1$. This model accounts for a scenario where the source and the relays are relatively close to each other and at approximately the same distance from the destination. All nodes transmit with the same power P, and all receivers are impaired by the Gaussian noise with one-sided power spectral density N_0. To complete the set of assumptions, ACK and NACK messages are considered to be received reliably. Moreover, their transmission time, as well as propagation and processing delays are considered negligible as compared to the time needed for the packet transmission.

3. PERFORMANCE ANALYSIS

The average delay, i.e., the expected number of transmissions necessary for successful decoding at the destination, of any HARQ protocol is given by

$$E[T] = \sum_{n=1}^{\infty} nP\left\{T = n\right\}, \tag{1}$$

where the probability that exactly n attempts (i.e., $n-1$ retransmissions) are necessary, $P\left\{T=n\right\}$, reads

$$P\left\{T = n\right\} = \left[1 - p_e\left(n\right)\right] \prod_{k=1}^{n-1} p_e(k). \tag{2}$$

In (2), $p_e(n)$ denotes the probability that nth transmission is erroneously decoded at the destination, conditioned on the previous unsuccessful transmissions. The ratio $C_0/E[T]$, where C_0 $[nat/s/Hz]$ is the transmission rate, determines the throughput of the system. In the following, derivation of $p_e(n)$ is carried out. Moreover, as a result of this analysis, closed-form upper and lower bounds on the system performance are evaluated.

3.1. Derivation of $p_e\left(n\right)$

For HARQ protocols with memory, the probability of error in the nth attempt $p_e(n)$, conditioned on previous unsuccessful transmissions, reads

$$p_e(n) = \sum_{\mathcal{K}_n} p_R(k_1, ..., k_{n-1}) \times \tag{3}$$
$$P\left\{C_D(n; k_1, .., k_{n-1}) < C_0 | C_D(n-1; k_1, .., k_{n-2}) < C_0\right\}.$$

In (3), $p_R\left(k_1, ..., k_{n-1}\right)$ denotes the probability that k_1 relays have decoded successfully at the first transmission, k_2 in second (but not before) and so on, while $C_D(n; k_1, ..., k_{n-1})$ represents the achievable rate at the destination after n attempts given that k_i, $i = 1, .., n-1$, relays were activated exactly at ith attempt. The sum in (3) is to be carried out over the set $\mathcal{K}_n$ of tuples $(k_1, ..., k_{n-1})$ [1]

$$\mathcal{K}_n = \left\{ \left(k_1, ..., k_{n-1}\right) | \tilde{k}_n = \sum_{i=1}^{n-1} k_i \leq M \right\}. \tag{4}$$

Notice that we have defined as $\tilde{k}_n$ the total number of relays that have successfully decoded by the $(n-1)$th attempt and are therefore able to collaborate at the nth transmission. In other words, while k_n denotes the number of relays activated at nth attempt, $\tilde{k}_n$ includes all relays activated at attempts $1, 2, .., n-1$.

[1] It can be shown that cardinality of the set $\mathcal{K}_n$ is $|\mathcal{K}_n| = \sum_{i=0}^{M} \binom{n-2+i}{i}$.

For the second term of the product in (3), it is easy to show that

$$P\{C_D(n;k_1,..,k_{n-1})<C_0|C_D(n-1;k_1,..,k_{n-2})<C_0\} = \frac{P\{C_D(n;k_1,..,k_{n-1})<C_0\}}{P\{C_D(n-1;k_1,..,k_{n-2})<C_0\}}, \quad (5)$$

since the event of erroneous decoding in the $(n-1)$th attempt is included in the event of erroneous decoding in the nth attempt. Furthermore, according to the HARQ-IR protocol, the achievable rate at the destination after n transmissions is

$$C_D(n;k_1,..,k_{n-1}) = \sum_{j=1}^{n}\log\left(1+\left(\left|h_{SD}^{(j)}\right|^2+\sum_{q=1}^{\tilde{k}_j}\left|h_{R_qD}^{(j)}\right|^2\right)\frac{P}{N_0}\right), \quad (6)$$

where $h_{SD}^{(j)}$ and $h_{R_qD}^{(j)}$ denote respectively the channel gains between the source S and the destination D, and between the relay R_q and the destination D, in the jth transmission. In (6), the first summation describes the effect of code combining [9], while the second summation describes the diversity effect of space-time transmission from $\tilde{k}_j$ antennas. Notice that in enumerating M available relays, $R_1,...,R_M$, we have assumed without loss of generality that the indices of the active relays, i.e. the relays that have decoded successfully, precede those of inactive.

The first product term in (3), $p_R(k_1,...,k_n)$, can be expanded according to the chain rule:

$$p_R(k_1,k_2,...,k_n)=\prod_{i=1}^{n}p_R(k_i|k_1,...,k_{i-1}), \quad (7)$$

where $p_R(k_i|k_1,...k_{i-1})$ is the probability that k_i relays successfully decode at the ith attempt (but not before), given that k_j, $j=1,..,i-1$, relays were activated exactly at the jth attempt. Considering that at the time instant i there are $M-\tilde{k}_i$ receiving relays (which has not successfully decoded), and defining as $\bar{p}_R(i;k_1,..,k_{i-1})$ the probability that any of the receiving relays R_s, $\tilde{k}_i<s\leq M$, does not successfully decode in the ith trial, the terms of product in (7) can be expressed as

$$p_R(k_i|k_1,...k_{i-1}) = = P_{bin}\left(\bar{p}_R(i;k_1,..,k_{i-1}),M-\tilde{k}_i,k_i\right), \quad (8)$$

where $P_{bin}(p,N,n)=\binom{N}{n}p^{N-n}(1-p)^n$ represents the binomial distribution. Notice that in (8) we have exploited the fact that the activations of any relay R_s are independent events. The probability $\bar{p}_R(i;k_1,..,k_{i-1})$ reads

$$\bar{p}_R(i;k_1,..,k_{i-1}) = P\{C_{R_s}(i;k_1,..,k_{i-1})<C_0|C_{R_s}(i-1;k_1,..,k_{i-2})<C_0\} = = \frac{P\left\{C_{R_s}(i;k_1,..,k_{i-1})<C_0\right\}}{P\left\{C_{R_s}(i-1;k_1,..,k_{i-2})<C_0\right\}}, \quad (9)$$

where, similar to (5), $C_{R_s}(i;k_1,...,k_{i-1})$ denotes the rate achieved by the relay R_s after i attempts, given that k_j, $j=1,..,i-1$, relays were activated at jth attempt:

$$C_{R_s}(i;k_1,..,k_{i-1}) = \sum_{j=1}^{i}\log\left[1+\left(\left|h_{SR_s}^{(j)}\right|^2+\sum_{q=1}^{\tilde{k}_j}\left|h_{R_qR_s}^{(j)}\right|^2\right)\frac{\alpha P}{N_0}\right]. \quad (10)$$

According to the discussion above, in order to evaluate $p_e(n)$ in closed form, one should determine the outage probability $P\left\{C(n;k_1,..,k_{n-1})<C_0\right\}$ for both capacity of destination (6) and relays (10). While this appears to be not feasible, closed-form upper and lower bounds can be derived as explained below.

3.2. Performance bounds

A *lower bound* on the achievable rates (6) and (10) can be obtained by using the following known inequality. For the nonnegative values of x_j, $j=1,..,m$, and any positive integer m,

$$\sum_{j=1}^{m}\log(1+x_j)\geq\log(1+\sum_{j=1}^{m}x_j), \quad (11)$$

or, in the terms of (6) (or similarly for (10)):

$$C_D(n;k_1,..,k_{n-1}) \geq \log\left(1+\sum_{j=1}^{n}\left(\left|h_{SD}^{(j)}\right|^2+\sum_{q=1}^{\tilde{k}_j}\left|h_{R_qD}^{(j)}\right|^2\right)\frac{P}{N_0}\right). \quad (12)$$

Note that the right hand expression in (12) corresponds to the performance of soft (or Chase) packet combining [8]. In other words, the performance of Collaborative HARQ with Chase Combining (HARQ-CC) provides a lower bound on the performance of Collaborative HARQ-IR systems. Noticing that the equivalent channel power gain in (12) is a chi-square variable with

$$2\sum_{j=1}^{n}(1+\tilde{k}_j)=2(n+\sum_{j=1}^{n}\sum_{l=1}^{j-1}k_l)= =2(n+\sum_{j=1}^{n-1}k_j(n-j)) \quad (13)$$

degrees of freedom, we have the following bounds on the outage probability:

$$P\left\{C_D(n,k_1,..,k_{n-1})<C_0\right\}\leq F\left[\mu,2\left(n+\sum_{j=1}^{n-1}k_j(n-j)\right)\right], \quad (14)$$

and

$$P\{C_{R_s}(i, k_1, .., k_{i-1}) < C_0\} \leq \qquad (15)$$
$$F\left[\frac{\mu}{\alpha}, 2\left(i + \sum_{j=1}^{i-1} k_j(i-j)\right)\right],$$

where $\mu = 2\frac{e^{C_0}-1}{P/N_0}$ and $F(x, \nu)$ denotes the cumulative distribution function of a chi-square variable with ν degrees of freedom, taken at value x.

On the other hand, an *upper bound* on the achievable rates (6) and (10) can be found by exploiting the Jensen's inequality

$$\sum_{j=1}^{m} \log(1 + x_j) \leq m \log\left(1 + \sum_{j=1}^{m} \frac{x_j}{m}\right), \qquad (16)$$

which leads to

$$C_D(n; k_1, .., k_{n-1}) \leq \qquad (17)$$
$$n \log\left(1 + \sum_{j=1}^{n}\left(\left|h_{SD}^{(j)}\right|^2 + \sum_{q=1}^{\tilde{k}_l}\left|h_{R_qD}^{(j)}\right|^2\right)\frac{P}{nN_0}\right),$$

and the corresponding bound on (10). Finally, the probability of erroneous reception can be bounded as

$$P\{C_D(n, k_1, .., k_{n-1}) < C_0\} \geq \qquad (18)$$
$$F\left[\mu(n), 2\left(n + \sum_{j=1}^{n-1} k_j(n-j)\right)\right],$$

and

$$P\{C_{R_s}(i, k_1, .., k_{i-1}) < C_0\} \geq \qquad (19)$$
$$F\left[\frac{\mu(i)}{\alpha}, 2\left(i + \sum_{j=1}^{i-1} k_j(i-j)\right)\right].$$

where $\mu(n) = 2n\frac{e^{C_0/n}-1}{P/N_0}$.

Using the lower bounds (14)-(15) or the upper bounds (18)-(19), corresponding performance limits for both delay $E[T]$ or throughput $C_0/E[T]$ can be obtained according to the discussion above.

4. NUMERICAL RESULTS

Average throughput $C_0/E[T]$ versus the average signal to noise ratio $SNR = P/N_0$, for single-relay $(M = 1)$ Collaborative HARQ protocols, $C_0 = 2\ nat/s/Hz$ and $\alpha = 20dB$, is shown in fig. 2. For HARQ-IR, the simulated throughput along with the upper bound and the lower bound

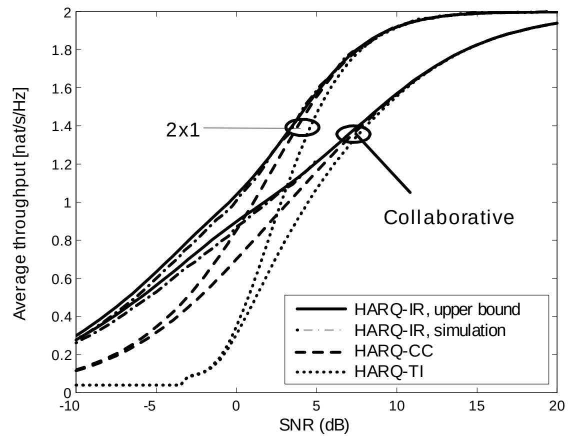

Fig. 2. Average throughput versus SNR for different HARQ systems ($C_0 = 2\ nat/s/Hz$, $\alpha = 20dB$).

(which corresponds to the performance of HARQ-CC) derived in the previous sections are presented. Another lower bound, performance of memoryless HARQ Type I (HARQ-TI) is also included (see also [8]). Finally, as a reference, the throughput of a 2×1 system (perfect collaboration) using the same HARQ protocols is also provided. It is seen that the upper bound matches well with the actual simulated system throughput. Furthermore, in the low SNR regime the average throughput of collaborative networks is comparable to that of a 2×1 system, while as the SNR increases, the performance of a collaboration tends to that of a 1×1 system (not shown in the figure for the sake of clarity). In other words, at lower SNR, where more retransmissions are needed, the initial advantage of double diversity degree achieved by the 2×1 model becomes less relevant. On the other hand, for large SNR, collaboration becomes less effective as the number of retransmissions decreases. Moreover, since the upper bound and the simulated throughput of HARQ-IR protocol match well, henceforth we will describe the performance of this protocol through its upper bound.

Fig. 3 shows the average throughput of single-relay HARQ-IR network $(M = 1)$ for different transmission rates C_0. Due to the characteristics of Code Combining, increasing the transmission rate, although it increases the number of retransmissions, does not imply a reduction of the average throughput. This behavior, as discussed in [8], is notably different from that of less powerful HARQ schemes, such as HARQ-TI or HARQ-CC. Notice that this result (due to our assumption) does not take into account the impact of signaling overhead due to retransmissions.

Finally, fig. 4 shows the performance of multi-relay Collaborative HARQ-IR with $M = 1, 2, ..., 10$, for transmission rate $C_0 = 5\ nat/s/Hz$. Moreover, the perfor-

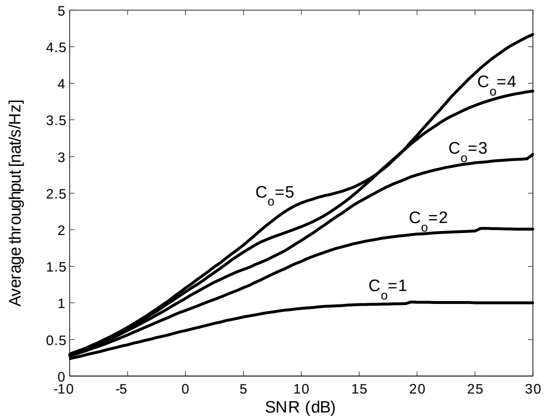

Fig. 3. Average throughput versus SNR for single-relay Collaborative HARQ-IR and different transmission rates C_0 $[nat/s/Hz]$ ($\alpha = 20dB$).

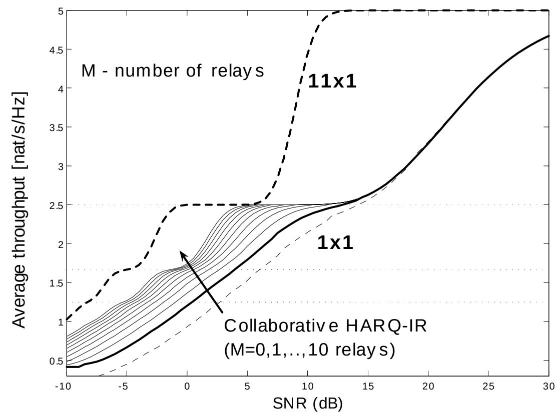

Fig. 4. Average throughput versus SNR for multi-relay Collaborative HARQ-IR system ($C_0 = 5\ nat/s/Hz$, $\alpha = 20dB$).

mance of a 11×1 (perfect collaboration among source and relays) and a 1×1 networks using HARQ-IR protocol are presented as references. While increasing the number of the relays yields relevant benefits in the low-SNR regime, performance drastically reduces to that of a 1×1 system for throughputs larger than $C_0/2$ (less than one retransmission).

5. CONCLUSION

In this paper, lower and the upper bounds on the performance of multi-relay Collaborative HARQ-IR protocol are provided. Extensive numerical results are presented to supplement analytical results and give an insight into system behavior.

6. ACKNOWLEDGMENT

This work was supported in part by Samsung Electronics Co., LTD, Korea.

7. REFERENCES

[1] A. Goldsmith, S. A. Jafar, N. Jindal and S. Vishwanath, "Capacity Limits of MIMO Channels," *IEEE Journ. Selected Areas Commun.*, vol. 21, no. 5, pp. 684-702, June 2003.

[2] A. Sendonaris, E. Erkip and B. Aazhang, "User cooperation diversity—Part I: System description," *IEEE Trans. Commun.*, vol. 51, pp. 1927-1938, Nov. 2003.

[3] J. Nicholas Laneman, David N. C. Tse and Gregory W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[4] S. Cui, A. J. Goldsmith, A. Bahai, "Energy-Efficiency of MIMO and cooperative MIMO Techniques in Sensor Networks," *IEEE Journ. Selected Areas Commun.*, vol. 22, no. 6, pp. 1089-1098, Aug. 2004.

[5] R. U. Nabar, H. Bölcskei and F. W. Kneubühler, "Fading relay channels: performance limits and space–time signal design," *IEEE Journ. Selected Areas Commun.*, vol. 22, no. 6, pp. 1099-1109, Aug. 2004.

[6] M. Janani, A. Hedayat, T. E. Hunter, A. Nosratinia, "Coded Cooperation in Wireless Communications: Space-Time Transmission and Iterative Decoding," *IEEE Trans. Signal Processing*, vol. 52, no. 2, pp. 362-371, Feb. 2004.

[7] B. Zhao and M. C. Valenti, "Practical Relay Networks: A Generalization of Hybrid-ARQ," *IEEE Journ. Selected Areas Commun.*, vol. 23, no. 1, pp. 7-18, Jan. 2005.

[8] I. Stanojev, O. Simeone, Y. Bar-Ness and C. You, "Performance Analysis of Multi-Relay Collaborative Hybrid-ARQ Protocols over Fading Channels", submitted to *IEEE Comm. Letters*.

[9] G. Caire and D. Tuninetti, "The throughput of Hybrid-ARQ Protocols for the Gaussian Collision Channel," *IEEE Trans. Inform. Theory*, vol. 47, pp. 1971-1988, July 2001