Stability analysis of a cognitive multiple access channel with primary QoS constraints

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***Abstract*— In this paper, a cognitive radio scenario composed of one primary (licensed) node and M secondary (unlicensed) nodes coexisting in the same spectral resource is considered. According to the commons model of cognitive radio, the secondary nodes are required to guarantee Quality-of-Service (QoS) constraints on the primary activity. Assuming a collision channel model, the stability region of the arrival rates at the secondary queues is investigated for given primary throughput and QoS constraints defined in terms of average delay of primary packets. Inner and outer bounds on the stability region are derived. The analysis is carried out at first for the case $M = 2$ and then generalized for any number of secondary nodes ($M > 2$), and is based on the concept of dominant systems. The results shed light on the impact of detection errors at the secondary nodes and of different levels of QoS requirements of the primary user to the achievable (stability) rate region of the secondary nodes. Numerical results suggest that the derived inner bound is a tight approximation of the real stability region.**

I. INTRODUCTION

Cognitive radio has been recently proposed as a technology that aims at guaranteeing the coexistence of primary licensed users and secondary unlicensed users in the same spectral resource [1]. In this paper, we focus on the cognitive commons model [2], which is characterized by the facts that primary nodes are oblivious to the secondary activity and that the transmissions of the secondary nodes are required to be *transparent* to the primary activity. In [3], a simple cognitive commons model scenario composed of one primary and one secondary node transmitting to two separate receivers over a slow fading channel has been considered. Therein, the maximum stable throughput of the secondary transmitter with and without relaying capability has been investigated through the concept of dominant system, explicitly introduced in [4].

In this paper, we consider a scenario characterized by a primary node and M secondary nodes operating over a collision channel for transmission to a common receiver (e.g., access point, see fig. 1). While the primary node is allowed to access the bandwidth at any time (that is, at any time-slot), the secondary nodes seek opportunity for transmission by exploiting the idle time-slots of the primary transmitter. Moreover, multiple access to the channel by the secondary nodes is ruled by a random access policy. Due to errors in sensing the primary activity by the secondary users, interference to the primary transmission is unavoidable (see, e.g., [1]-[3]) and appropriate mechanisms should be put in place in order to guarantee Quality-of-Service (QoS) constraints on the primary activity. Here we focus on QoS requirements defined in terms of maximum average delays of primary packets. By leveraging the concept of dominant systems, we derive inner and outer bounds on the region of the average arrival rates at the secondary nodes for which all the queues in the system remain stable under given primary throughput and QoS constraints.

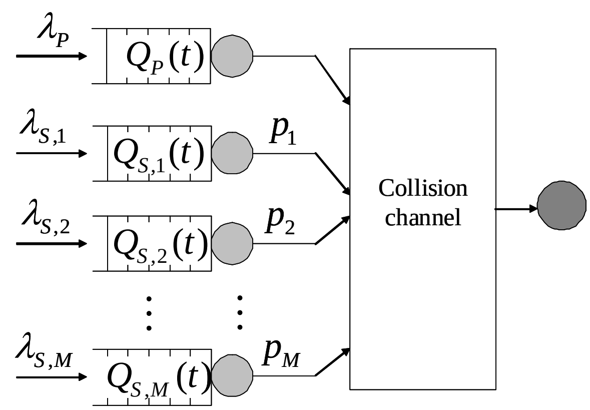

Fig. 1. Cognitive scenario with one primary node and M secondary nodes transmitting over a collision channel to a common receiver.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider the cognitive scenario in fig. 1, where a primary licensed node P and M secondary nodes S_i ($i \in \mathcal{M} = \{1, 2, ..., M\}$) transmit in the same spectral resource to a common receiver (e.g., access point). We refer to this system as $\Omega^{(M)}$ in the following. Each terminal is equipped with infinite-length buffers, where the incoming packets are stored. The packets arrival processes at each node are independent and i.i.d. Bernoulli processes with mean λ_{P} [packets/slot] for the primary user and $\lambda_{\mathrm{S},i}$ [packets/slot] for the ith secondary node ($i \in \mathcal{M}$). Let $Q_{\mathrm{P}}(t)$ be the stochastic process referring to the number of packets stored in the queue of the primary node and, similarly, let $Q_{\mathrm{S},i}(t)$ refer to the number of packets stored in the queue of the secondary node S_i. Time is slotted and all the packets have the same length, equal to one time slot (the average arrival rates, thus, correspond to the probabilities of an arrival at a given node in a given time slot). We employ the standard definition of stability for a queue $Q(t)$ as in, e.g., [7], that is, a queue $Q(t)$ is stable if and only if its probability of being empty does not vanish as time progresses:

$$\lim_{t\to+\infty} \Pr[Q(t)=0] > 0. \quad (1)$$

As for the physical layer, a collision channel is considered. This means that any packet is correctly received by the destination if and only if only one transmission takes place in a given time-slot. Moreover, according to the paradigm of cognitive radio, the primary should be oblivious to the secondary and the activity of the secondary nodes must be compliant with QoS constraints on the primary transmission. In order to respect this principle, here we assume that the primary node P attempts transmission whenever it has packets in its queue, while any secondary node S_i ($i \in \mathcal{M}$), at each time slot, senses the channel and, if no primary activity is detected, transmits a packet (if it has any in its queue) with probability p_i (random access). Due to inevitable errors, the secondary transmitter S_i can correctly detect the activity of the primary user with a probability $P_{d,i}$ (probability of detection), while it can detect primary activity even in an idle slot and, consequently, miss an opportunity for transmission with a probability $P_{fa,i}$ (probability of false alarm) [1]-[3]. This implies that, in a given *idle* time-slot, any secondary node S_i, if its queue is not empty, attempts the transmission of a packet with probability

$$\theta_i = p_i(1-P_{fa,i}). \quad (2)$$

The outcome of the detection at any secondary node can be considered as independent from the other ones. The QoS constraint on the primary activity is specified as

$$D_{\mathrm{P}}^{(M)}(\mathbf{p}) \leq D_{\max}, \quad (3)$$

where $D_{\mathrm{P}}^{(M)}(\mathbf{p})$ is the average delay of primary packets in system $\Omega^{(M)}$(see Sec. III-A), that depends on the transmission probabilities $\mathbf{p}=[p_1,p_2,...,p_M]^T$. Notice that the maximum delay constraint $D_{\max}$ on the primary activity implies the stability of the primary queue $Q_{\mathrm{P}}(t)$.

In this paper, our aim is to find inner and outer bounds to the (stability) region $\mathcal{S}^{(M)}(D_{\max})$ of the average arrival rates $\{\lambda_i\}_{i\in\mathcal{M}}$ at the secondary nodes $\{\mathrm{S}_i\}_{i\in\mathcal{M}}$ for which at least one combination of the transmission probabilities $\mathbf{p}$ exists that guarantees stability of the secondary queues $\{Q_{\mathrm{S},i}\}_{i\in\mathcal{M}}$, under the specified QoS constraint $D_{\max}$ and given system parameters $[\lambda_{\mathrm{P}}, \{P_{d,i}\}_{i\in\mathcal{M}}, \{P_{fa,i}\}_{i\in\mathcal{M}}]$.

III. Stability analysis

According to the definition above, the stability region $\mathcal{S}^{(M)}(D_{\max})$ can be expressed as:

$$\mathcal{S}^{(M)}(D_{\max}) = \Bigg\{ \bigcup_{p_i, i\in\mathcal{M}} \tilde{\mathcal{S}}^{(M)}(\mathbf{p}) \mid p_i \in [0,1] \text{ with } i\in\mathcal{M},\ D_{\mathrm{P}}^{(M)}(\mathbf{p}) \leq D_{\max} \Bigg\}, \quad (4)$$

where $\tilde{\mathcal{S}}^{(M)}(\mathbf{p})$ is the stability region of the average arrival rates at the secondary nodes for given transmission probabilities $\mathbf{p}$.

Various bounds to the stability region of M interacting queues in a random access environment have been found in [4]-[7], and the exact region identification has been achieved only for the cases $M=2$ [4], [6] and $M=3$ [5]. However, the cognitive radio scenario at hand differs significantly from the homogeneous random access model considered in [4]-[6], since the secondary queues interact not only among themselves but also with the primary queue $Q_{\mathrm{P}}(t)$, which, in addition, requires delay constraints for the transmission of its packets.

An outer bound on the stability region $\mathcal{S}^{(M)}(D_{\max})$ can be trivially obtained by assuming that the primary queue $Q_{\mathrm{P}}(t)$ is always empty (or, equivalently, $\lambda_{\mathrm{P}}=0$), which leads exactly to the results in [4] and [5]. For example, for the special case of $M=2$, this outer bound is analytically characterized by the relationship $\sqrt{\lambda_{\mathrm{S},1}}+\sqrt{\lambda_{\mathrm{S},2}}=1$, see [4]. In the remaining part of this section, after obtaining an analytical expression for the primary delay constraints in Sec. III-A, we derive inner bounds on the region $\mathcal{S}^{(M)}(D_{\max})$ for the cases $M=2$ and $M>2$ in Sec. III-B and III-C, respectively.

The following analysis is based on the concept of dominant systems, which, in general, allow to obtain sufficient conditions for stability of a system of interacting queues: by construction, if a dominant system is stable, then the original system is [4]. Applying this idea to our system, here we introduce the following class of dominant systems:

$$\bar{\boldsymbol{\Omega}}^{(M)} = \left\{\bar{\Omega}_{\mathcal{V}}^{(M)}\right\}_{\mathcal{V}\subseteq\mathcal{M}}, \quad (5)$$

where $\bar{\Omega}_{\mathcal{V}}^{(M)}$ is any system which differs from the original system $\Omega^{(M)}$ for two facts: (*i*) in any time-slot occupied by the primary transmission, every secondary user $\{\mathrm{S}_i\}_{i\in\mathcal{M}}$, if failing the detection, transmits a (possibly dummy) packet with probability p_i even if its queue is empty; (*ii*) in any time-slot left idle by the primary user, a subset $\mathcal{V}$ of the M secondary nodes continues to transmit (possibly dummy) packets with probability θ_i defined in (2) even if their queues are empty. Since transmission of dummy packets does not decrease the queue sizes but can still cause collisions, any system belonging to the class $\bar{\boldsymbol{\Omega}}^{(M)}$ is a dominant system with respect to the original system $\Omega^{(M)}$: any average arrival rates set $\{\lambda_{\mathrm{S},i}\}_{i\in\mathcal{M}}$ which can be supported (with given QoS constraint $D_{\max}$ on the primary) in any system $\bar{\Omega}_{\mathcal{V}}^{(M)}$ can also be supported in $\Omega^{(M)}$. In particular, it should be noticed that the average delay $\overline{D}_{\mathrm{P}}^{(M)}(\mathbf{p})$ experienced by the primary packets in systems belonging to the class $\bar{\mathbf{\Omega}}^{(M)}$ (to be derived in the next section) is an upper bound on $D_{\mathrm{P}}^{(M)}$ so that the QoS constraint

$$\overline{D}_{\mathrm{P}}^{(M)}(\mathbf{p}) \leq D_{\max} \tag{6}$$

in systems $\bar{\mathbf{\Omega}}^{(M)}$ implies also (3) in $\Omega^{(M)}$. In other words, the stability region of any system in $\bar{\mathbf{\Omega}}^{(M)}$ for given QoS constraint $D_{\max}$ provides an inner bound on the stability region $\mathcal{S}^{(M)}(D_{\max})$ on $\Omega^{(M)}$.

A. Average primary delays

As discussed in the previous section, the activity of the secondary nodes must guarantee the QoS condition $D_{\mathrm{P}}^{(M)} \leq D_{\max}$ on the average delay $D_{\mathrm{P}}^{(M)}$ experienced by the primary packets. The analysis of the average delay $D_{\mathrm{P}}^{(M)}$ in system $\Omega^{(M)}$ is very difficult since it requires, in principle, to find the stationary distribution of an $M+1$-dimensional Markov chain in which the state is defined by the vector of queue sizes $\left[Q_{\mathrm{P}}(t), \{Q_{\mathrm{S},i}\}_{i \in \mathcal{M}}\right]$. The task is complicated by the boundaries transition anomalies of the $M+1$-dimensional Markov chain, which are caused by the interaction of the queues [4]. However, as explained above, here we restrict the focus to study the class of dominant systems $\bar{\mathbf{\Omega}}^{(M)}$ defined in (5). Interestingly, in any system belonging to the class $\bar{\mathbf{\Omega}}^{(M)}$, the primary queue therein can be treated as an isolated discrete Markov process, uncoupled from the stochastic processes $\{Q_{\mathrm{S},i}\}_{i \in \mathcal{M}}$ accounting for evolution of the ith secondary queue. Therefore, the average delay $\overline{D}_{\mathrm{P}}^{(M)}$ in the modified system is easily calculated as:

$$\overline{D}_{\mathrm{P}}^{(M)}(\mathbf{p}) = \frac{1-\lambda_{\mathrm{P}}}{\bar{\mu}_{\mathrm{P}}(\mathbf{p}) - \lambda_{\mathrm{P}}}, \tag{7}$$

where $\bar{\mu}_{\mathrm{P}}(\mathbf{p})$ is the average departure rate from queue $Q_{\mathrm{P}}(t)$ in systems $\bar{\mathbf{\Omega}}^{(M)}$. The latter can be obtained by considering that the primary node can successfully transmit a packet if and only if the packet experiences no collision over the channel. This event happens when either all the secondary nodes correctly detect primary activity (which happens with probability $\prod_{i=1}^{M} P_{d,i}$), or when any subset of secondary nodes $\mathcal{V}$ fails detection but decides not to attempt transmission (which happens with probability $\prod_{j \in \mathcal{V}} (1-P_{d,j})(1-p_j)$). Therefore, we have the following:

$$\bar{\mu}_{\mathrm{P}}(\mathbf{p}) = \prod_{i=1}^{M} P_{d,i} + \sum_{\mathcal{V} \subseteq \mathcal{M}} \left(\prod_{j \in \mathcal{V}} (1-P_{d,j})(1-p_j) \right) \times \left(\prod_{j \notin \mathcal{V}} P_{d,j} \right). \tag{8}$$

An important related quantity is the probability of a transmission opportunity for the secondary nodes in systems belonging to the class $\bar{\mathbf{\Omega}}^{(M)}$, which coincides with the probability that the primary user has no packets stored in its queue and reads:

$$\Pr[Q_{\mathrm{P}}(t)=0] = \bar{\eta} = (1 - \lambda_{\mathrm{P}}/\bar{\mu}_{\mathrm{P}}(\mathbf{p})). \tag{9}$$

B. Inner bounds on $\mathcal{S}^{(2)}(D_{\max})$

An inner bound on the stability region $\mathcal{S}^{(2)}(D_{\max})$ (that is, in the case $M=2$) is derived in the following by considering the class of dominant systems $\bar{\mathbf{\Omega}}^{(2)}$ and building on the analysis in [4]. In particular, we focus on systems $\bar{\Omega}_1^{(2)}$ and $\bar{\Omega}_2^{(2)}$ according to (5), with $\mathcal{V}=\{1\}$ and $\mathcal{V}=\{2\}$, respectively. Considering the dominant system $\bar{\Omega}_1^{(2)}$, it follows from Loynes' theorem [8] that stability of the secondary queue $Q_{\mathrm{S},2}(t)$ is guaranteed if [4]:

$$\lambda_{\mathrm{S},2} < \bar{\mu}_{\mathrm{S},2}^{1} = \theta_2(1-\theta_1)\bar{\eta}. \tag{10}$$

The equation above simply states that a sufficient condition for the stability of queue $Q_{\mathrm{S},2}(t)$ in the considered dominant system $\bar{\Omega}_1^{(2)}$ is that its average arrival rate $\lambda_{\mathrm{S},2}$ is smaller than its average departure rate $\bar{\mu}_{\mathrm{S},2}^{1}$, obtained as the product of the probability θ_2 that the secondary queue $Q_{\mathrm{S},2}(t)$ attempts transmission in an idle time-slot, the probability $(1-\theta_1)$ that the secondary queue $Q_{\mathrm{S},1}(t)$ does not transmit and the probability $\bar{\eta}$ that the primary queue $Q_{\mathrm{P}}(t)$ is empty (9). Similarly, the secondary user S_1 has an average departure rate equal to $\theta_1\bar{\eta}$ if queue $Q_{\mathrm{S},2}(t)$ is empty or equal to $\theta_1(1-\theta_2)\bar{\eta}$ if queue $Q_{\mathrm{S},2}(t)$ is non-empty, so that the condition

$$\lambda_{\mathrm{S},1} < \left(\theta_1 \left(1 - \frac{\lambda_{\mathrm{S},2}}{\bar{\mu}_{\mathrm{S},2}^{1}} \right) + \theta_1(1-\theta_2)\frac{\lambda_{\mathrm{S},2}}{\bar{\mu}_{\mathrm{S},2}^{1}} \right) \bar{\eta} \tag{11}$$

guarantees stability of queue $Q_{\mathrm{S},2}(t)$. To sum up, from consideration of the dominant system $\bar{\Omega}_1^{(2)}$, the conditions (10) and (11) provide an inner bound on the stability region $\tilde{\mathcal{S}}^{(2)}(\mathbf{p})$. Moreover, following the same approach for the dominant system $\bar{\Omega}_2^{(2)}$, we obtain further sufficient conditions for stability:

$$\begin{cases} \lambda_{\mathrm{S},1} < \bar{\mu}_{\mathrm{S},1}^{2} = \theta_1(1-\theta_2)\bar{\eta} \\ \lambda_{\mathrm{S},2} < \left(\theta_2 \left(1 - \frac{\lambda_{\mathrm{S},1}}{\bar{\mu}_{\mathrm{S},1}^{2}} \right) + \theta_2(1-\theta_1)\frac{\lambda_{\mathrm{S},1}}{\bar{\mu}_{\mathrm{S},1}^{2}} \right) \bar{\eta} \end{cases} \tag{12}$$

In conclusion, an inner bound on the stability region $\tilde{\mathcal{S}}^{(2)}(\mathbf{p})$ is defined by considering at the same time equations (10), (11), (12), and an inner bound on the stability region $\mathcal{S}^{(2)}(D_{\max})$ is then obtained from (4). Each point on the boundary of the inner bound to stability region $\mathcal{S}^{(2)}(D_{\max})$ can be obtained by solving the following optimization problem: fixed $\lambda_{\mathrm{S},1} = \bar{\lambda}_{\mathrm{S},1}$, maximize $\lambda_{\mathrm{S},2}$ (or viceversa) with respect to the transmission probabilities $\mathbf{p}$ under the constraint (6).

C. Inner bounds on $\mathcal{S}^{(M)}(D_{\max})$

In this section, we consider the case $M>2$. From the same considerations of the dominant systems (5), following a similar approach as in the previous section, we can find that the region $\mathcal{S}^{(M)}(\mathbf{p})$ of the arrival rates guaranteeing stability of the secondary queues given a choice of the transmission probabilities $\mathbf{p}$ is inner-bounded by the following conditions (see Appendix for details):

$$\lambda_{\mathrm{S},i} < \frac{\theta_i}{(1-\theta_i)} \max_{1 \leq k \leq M} \left[\bar{\eta} \prod_{j \neq k} (1-\theta_j) - \lambda_{\mathrm{S},k} \right], \quad \text{for } i \in \mathcal{M}. \tag{13}$$

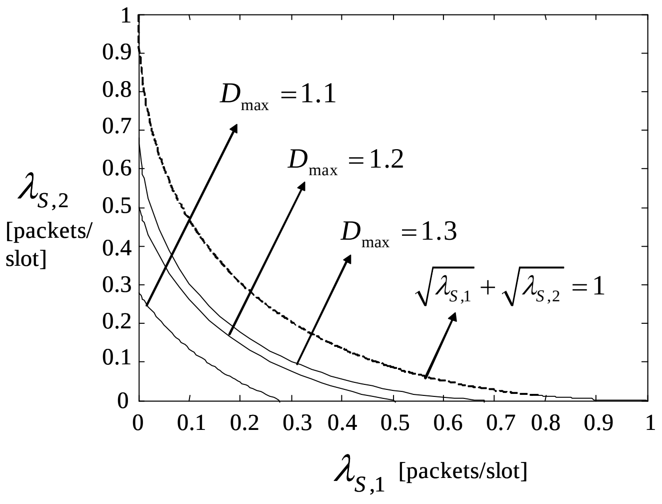

Fig. 2. Inner bounds (solid lines) on the stability region $\mathcal{S}^{(2)}(D_{\max})$ for different values of the maximum allowed delay $D_{\max}$ for the primary transmissions. The outer bound (obtained from [4]) is also shown in dashed line ($\lambda_{\mathrm{P}} = 0.2$, $P_{d,1} = 0.8$, $P_{d,2} = 0.8$, $P_{fa,1} = 0.02$, $P_{fa,2} = 0.02$).

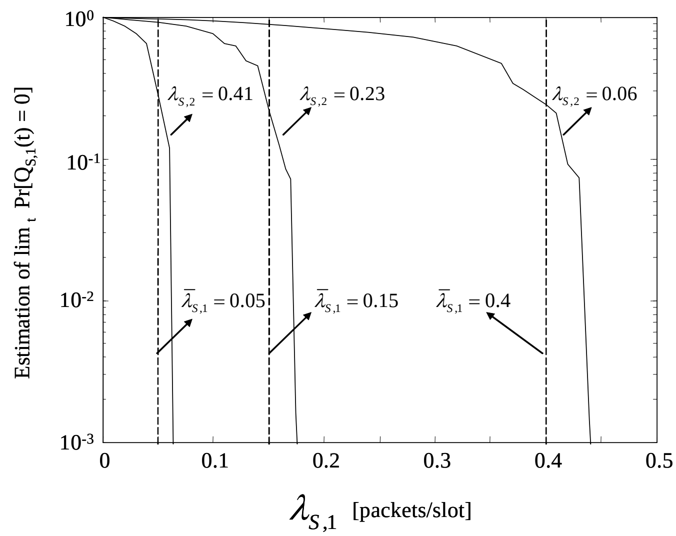

Fig. 3. Numerical estimation of the quantity $\lim_{t\to+\infty} \Pr[Q_{\mathrm{S},1}(t) = 0]$ versus the average arrival rate $\lambda_{\mathrm{S},1}$ for the original system $\Omega^{(M)}$ where the vector of the probabilities of transmission $\mathbf{p}$ is selected so as to guarantee that queue $Q_{\mathrm{S},2}(t)$ is stable and that the primary delay constraint (6) is satisfied. As a reference, the figure also shows the quantity $\bar{\lambda}_{\mathrm{S},1}$ (maximum stable value of $\lambda_{\mathrm{S},1}$ according to our inner bound) in dashed line (system parameters selected as in fig. 2, with $D_{\max} = 1.3$).

Finally, the inner bound on the stability region $\mathcal{S}^{(M)}(D_{\max})$ can be found as (4).

IV. Numerical results

In order to get insight into the performance of the system, we first focus on a system with $M = 2$ secondary users. Fig. 2 shows the obtained inner bounds on the stability region $\mathcal{S}^{(2)}(D_{\max})$ defined as (4) for different values of the maximum primary average delay $D_{\max}$ specified by the QoS constraints. System parameters are selected as follows: $\lambda_{\mathrm{P}} = 0.2$, $P_{d,1} = 0.8$, $P_{d,2} = 0.8$, $P_{fa,1} = 0.02$, $P_{fa,2} = 0.02$. As a reference, the outer bound corresponding to the case of absence of primary user discussed in Sec. III is also shown. The inner bounds suggest that QoS delay constraint $D_{\max}$ has a significant impact on the achievable secondary rates region $\mathcal{S}^{(2)}(D_{\max})$.

In order to validate the inner bound on $\mathcal{S}^{(2)}(D_{\max})$ derived in this paper, fig. 3 shows a numerical estimation of the quantity $\lim_{t\to+\infty} \Pr[Q_{\mathrm{S},1}(t) = 0]$ versus the average arrival rate $\lambda_{\mathrm{S},1}$ for the original system $\Omega^{(M)}$ where the vector of the probabilities of transmission $\mathbf{p}$ is selected so as to guarantee that queue $Q_{\mathrm{S},2}(t)$ is stable and that the primary delay constraint (6) is satisfied [1]. We recall that, from (1), $\lim_{t\to+\infty} \Pr[Q_{\mathrm{S},1}(t) = 0]$ is a measure of stability. The other system parameters are the same as in fig. 2, with $D_{\max} = 1.3$. As a reference, fig. 3 also shows the quantity $\bar{\lambda}_{\mathrm{S},1}$ which is the maximum stable value of $\lambda_{\mathrm{S},1}$ according to our inner bound. As it is clear from fig. 2, outer bounds are far away and, for the sake of intelligibility of the graph, not shown. As we can see, the estimated asymptotic probability of an empty queue is always greater than zero for $\lambda_{\mathrm{S},1} < \bar{\lambda}_{\mathrm{S},1}$ and tends to vanish as $\lambda_{\mathrm{S},1}$ is larger than $\bar{\lambda}_{\mathrm{S},1}$. By recalling the definition of queue stability given in (1), these results suggest that stability of queue $Q_{\mathrm{S},1}(t)$ is guaranteed if $\lambda_{\mathrm{S},1}$ is smaller than $\bar{\lambda}_{\mathrm{S},1}$, whereas for $\lambda_{\mathrm{S},1}$ slightly larger than the bound $\bar{\lambda}_{\mathrm{S},1}$ the secondary queue tends to instability. This lends evidence to the fact that the derived inner bounds on $\mathcal{S}^{(2)}(D_{\max})$ provide a tight approximation of the real stability region of the system.

We now consider a scenario with $M = 3$ secondary users. Fig. 4 shows the inner bound on the stability region $\mathcal{S}^{(3)}(D_{\max})$ for selected values of $\lambda_{\mathrm{S},3}$, given the following choice for the system and channel parameters: $\lambda_{\mathrm{P}} = 0.2$, $P_{d,1} = 0.8$, $P_{d,2} = 0.8$, $P_{d,3} = 0.8$, $P_{fa,1} = 0.02$, $P_{fa,2} = 0.02$, $P_{fa,3} = 0.02$, $D_{\max} = 1.6$. The figure highlights the trade-off among the rates achievable by the secondary nodes.

[1]Given a choice for ($\lambda_{\mathrm{S},1}$, $\lambda_{\mathrm{S},2}$), we estimate this probability by: (*i*) fixing vector $\mathbf{p}$; (*ii*) evaluating via Monte Carlo simulations the average of the quantity $(1/(N-K))\sum_{t=K}^{N} \mathbf{1}[Q_{\mathrm{S},1}(t) = 0]$ (where $\mathbf{1}$ is the indicator function) and the average primary delay $D_{\mathrm{P}}^{(M)}(\mathbf{p})$; (*iii*) repeating (*i*) and (*ii*) to search the probability simplex of $\mathbf{p}$; (*iv*) selecting the maximum value of considered quantity under the condition that the corresponding $D_{\mathrm{P}}^{(M)}(\mathbf{p}) \leq D_{\max}$ (here we have selected $N = 10^4$ and $K = N/2$).

V. Conclusions

In this paper, the considered cognitive scenario is composed of one primary licensed node and M secondary unlicensed nodes coexisting on the same spectral resource and operating over a collision channel. Inner and outer bounds on the stability region of the average arrival rates at the secondary nodes under constraints of maximum delay for the primary packets have been derived. Numerical simulations have shown that the found inner bounds represent a tight approximation of the real stability region.

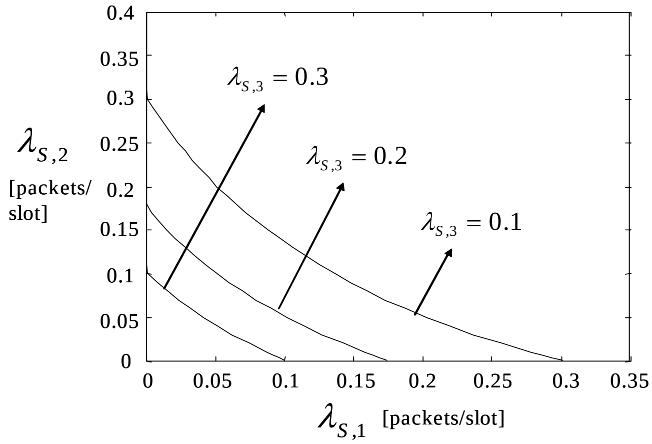

Fig. 4. Inner bounds on the stability region $\mathcal{S}^{(3)}(D_{\max})$ of the average arrival rates $\lambda_{S,1}$ and $\lambda_{S,2}$ for selected values of $\lambda_{S,3}$ ($\lambda_P = 0.2$, $P_{d,1} = 0.8$, $P_{d,2} = 0.8$, $P_{d,3} = 0.8$, $P_{fa,1} = 0.02$, $P_{fa,2} = 0.02$, $P_{fa,3} = 0.02$, $D_{\max} = 1.6$).

VI. Appendix: Derivation of (13)

Here we derive the inner bounds on the region $\tilde{\mathcal{S}}^{(M)}(\mathbf{p})$ for $M > 2$ stated in (13). Towards this end, we extract from the class $\bar{\mathbf{\Omega}}^{(M)}$ the system $\bar{\Omega}_{\mathcal{K}}^{(M)}$ so that, with reference to relation (5), $\mathcal{V} = \{K \leq i \leq M\}$. Our goal is to find a sufficient condition for the stability of the dominant system $\bar{\Omega}_{\mathcal{K}}^{(M)}$, which will also be sufficient for the stability of the original system $\Omega^{(M)}$. In particular, following a similar discussion as in [7], according to Loynes' theorem, the mth secondary node is stable in the dominant system $\bar{\Omega}_{\mathcal{K}}^{(M)}$ if:

$$\lambda_{S,m} < \theta_m \bar{\eta} P_E^{(K)} \prod_{\substack{j \neq m \\ K \leq j \leq M}} (1-\theta_j), \tag{14}$$

where $P_E^{(K)}$ is defined as the probability that the first $K-1$ secondary users do not transmit in a given idle time-slot and reads:

$$P_E^{(K)} = \Pr[Q_{S,1}(t) = 0] \Pr[\text{queues from 2 to } K-1 \text{ do not transmit} \mid Q_{S,1}(t) = 0] + \Pr[Q_{S,1}(t) \neq 0] \times \Pr[\text{queues from 1 to } K-1 \text{ do not transmit} \mid Q_{S,1}(t) \neq 0]$$

and the term $(1-\theta_j)$, for j such that $K \leq j \leq M$, is the probability that the secondary queue $Q_{S,j}(t)$ does not transmit given that it is not empty, which is an under-estimation of the overall (non conditional) non-transmission probability of queue $Q_{S,j}(t)$. A lower bound on $P_E^{(K)}$ can be found as:

$$P_E^{(K)} \geq \Pr[Q_{S,1}(t) = 0] \prod_{j=2}^{K-1} (1-\theta_j) + \Pr[Q_{S,1}(t) \neq 0] \times \prod_{j=1}^{K-1} (1-\theta_j), \tag{15}$$

since $\Pr[\text{queues from 2 to } K-1 \text{ do not transmit} \mid Q_{S,1}(t) = 0] \geq \prod_{j=2}^{K-1}(1-\theta_j)$ and $\Pr[\text{queues from 1 to } K-1 \text{ do not transmit} \mid Q_{S,1}(t) \neq 0] \geq \prod_{j=1}^{K-1}(1-\theta_j)$. Moreover, by Little's theorem, we have that $\Pr[Q_{S,1}(t) = 0] = 1 - \lambda_{S,1}/\bar{\mu}_{S,1}$, where $\bar{\mu}_{S,1}$ is the average service rate of the secondary node S_1 and satisfies the following inequality:

$$\bar{\mu}_{S,1} \geq \theta_1 \bar{\eta} \prod_{j=2}^{M} (1-\theta_j), \tag{16}$$

Using (16), we can consequently rewrite (15) as:

$$P_E^{(K)} \geq \prod_{j=2}^{K-1} (1-\theta_j) - \frac{\lambda_{S,1}}{\bar{\eta} \prod_{j=K}^{M} (1-\theta_j)} \tag{17}$$

By using (17), a sufficient condition for the secondary queue $Q_{S,m}(t)$ in system $\bar{\Omega}_{\mathcal{K}}^{(M)}$ to be stable is:

$$\lambda_{S,m} < \frac{\theta_m}{(1-\theta_m)} \left(\bar{\eta} \prod_{j \neq 1} (1-\theta_j) - \lambda_{S,1} \right) \tag{18}$$

Without loss of generality, we assume that:

$$\bar{\eta} \prod_{j \neq 1} (1-\theta_j) - \lambda_{S,1} = \max_{1 \leq k \leq M} \left[\bar{\eta} \prod_{j \neq k} (1-\theta_j) - \lambda_{S,k} \right] \tag{19}$$

Therefore, by exploiting the stochastic dominance of the considered system $\bar{\Omega}_{\mathcal{K}}^{(M)}$ on the original system $\Omega^{(M)}$, with a similar discussion as in [7], we obtain that, given the probabilities of transmission $\mathbf{p}$ and the system parameters $[\lambda_P, \{P_{d,i}\}_{i \in \mathcal{M}}, \{P_{fa,i}\}_{i \in \mathcal{M}}]$, the secondary queues in system $\Omega^{(M)}$ are stable if condition (13) is satisfied.

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