Delay-Tolerant Robust Communication on an Out-of-Band Relay Channel with Fading Side Information

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***Abstract*—This work considers a setting in which an encoder wishes to communicate with a decoder through a relay that is connected to the decoder via a finite-capacity link. Motivated by communication on the uplink of a cloud radio access cellular network, it is assumed that the relay compresses and forwards the received signal; moreover, the decoder has side information about the transmitted signal that is subject to fading whose realization is unknown to encoder and relay. A robust transmission and compression strategy is proposed that aims at minimizing the transmitted power under competitive rate optimality constraints. This contrasts with more conventional worst-case or average performance criteria. The transmission strategy is based on a broadcast coding and is parameterized by the maximum tolerable delay in terms of number of fading coherence blocks. Numerical results demonstrate the role of delay and the advantages of broadcast coding over the conventional single-layer transmission.**

I. Introduction

Consider the relay channel shown in Fig. 1, in which an encoder (ENC) wishes to communicate to a decoder (DEC) through a relay that is connected to the decoder via a finite-capacity digital link. Beside the digital information received from the relay on this link, the decoder also receives a signal Y that is correlated with the signal V received by the relay. Assume that encoder and relay know the channel state information (CSI) relative to the encoder-to-relay link, and thus to signal V, but are not aware of the CSI relative to the source-to-decoder link, and thus to signal Y. Assume also that the relay operates by compressing and forwarding the received signal V (see, e.g., [1]). We are interested in designing the transmission strategy at the encoder and compression at the relay in a way that is robust to the uncertainty on the CSI of the side information Y. A key aspect of interest is investigating the role of delay in coping with time-varying uncertain CSI.

Communication over relay channels is well studied with full or partial CSI (see, e.g., [2]). The issue of robust communication over fading relay channels was studied in [3][4]. In both works, a broadcast coding approach was adopted in which the encoder transmits a superposition of layers and the destination decodes as many layers as possible depending on the channel conditions. Reference [3] focuses on a time-division protocol, while [4] considers a two-hop model with no direct connection between encoder and decoder. Both references assume a strict delay constraint of one coherence block.

The relay model studied here is motivated by the uplink of cloud radio access networks [5][6], in which base stations act as relays and are connected to a central decoder via finite-capacity digital links. In this set-up, the side information Y of Fig. 1 represents the information that the central decoder in the "cloud" collects from the other base stations in the system (see [7] for further discussion on this point). We consider that the mobile station (i.e., the encoder) and the base station are typically informed about the CSI relative to the link between them (and thus to V in Fig. 1), while they are generally not aware of the CSI for the links to other BSs (and thus for Y in Fig. 1), and hence assume the CSI situation outlined above.

The scenario of interest, depicted in Fig. 1, was studied in [7][8] by considering a worst-case robustness optimality [7] and expected distortion [8] criteria under strict delay constraint. In this paper, we alleviate the strict delay constraint by allowing coding across a fixed number of fading coherence blocks. Moreover, we adopt competitive robustness constraints, rather than worst-case [7] or average [3][4][9] performance criteria (see, e.g., [10]). To this end, we adopt broadcast coding as in [3][4], in order to allow decoding of different rates of information depending on the channel conditions, and impose that a fraction of the rate achievable with full CSI can also be achieved in the CSI conditions under study. Having formulated the problem of minimizing the power under the said competitive optimality constraint, we propose an efficient solution that is shown to converge to a locally optimal point. Numerical results demonstrate the remarkable power savings achievable by the proposed strategy.

Notation: We adopt standard information-theoretic definitions for the mutual information $I(X;Y)$ between the random variables X and Y, and conditional mutual information $I(X;Y|Z)$ between X and Y conditioned on random variable Z. All logarithms are in base two unless specified. The transpose of a matrix or a vector is denoted by superscript t and Gaussian distribution with mean m and variance σ^2 is denoted by $\mathcal{N}(m,\sigma^2)$. Random variables are denoted with capital letters and their realizations with the corresponding

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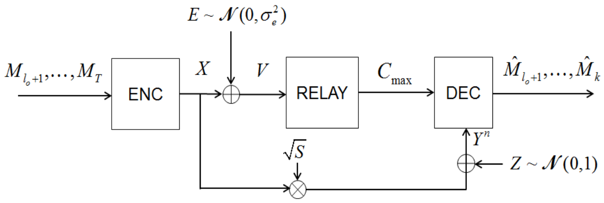

Figure 1. Block diagram for the system model.

lowercase letters. We use $p(x)$ to denote the distribution of random variable X and $p(x|y)$ for the conditional probability of X given Y.

II. System Model

We consider the system model in Fig. 1 in which an encoder communicates to a decoder via a relay, which is connected to the decoder via a finite-capacity link of capacity $C_{\max}$. As mentioned in Sec. I, we assume that the relay forwards soft information about the received signal to the decoder via compression. Since the cloud decoder also receives signals from the adjacent BSs, we assume that the decoder has the available side information Y to be discussed below. We are interested in designing the transmission strategy at the encoder and the compression at the relay across T slots, where T is the delay tolerated by the system.

The signal $V_{t,i}$ received by the relay in the ith channel use of the tth transmission block (or slot) is modeled as

$$V_{t,i} = X_{t,i} + E_{t,i}, \tag{1}$$

for $t = 1,\ldots,T$ and $i = 1,\ldots,n$, where $X_{t,i}$ represents the signal transmitted by the encoder and $E_{t,i}$ is the additive noise at the relay, which is independent and identically distributed (i.i.d.) according to $E_{t,i} \sim \mathcal{N}(0,\sigma_e^2)$. We assume that the block size n is large enough to allow the use of information-theoretic limits. Moreover, the transmitted signal $X_{t,i}$ is i.i.d. Gaussian distributed with $X_{t,i} \sim \mathcal{N}(0,P)$.

Assuming the encoder and the relay are not aware of the CSI relative to the source-to-decoder link, the side information $Y_{t,i}$ in the ith channel use of the tth block is given as

$$Y_{t,i} = \sqrt{S_t} X_{t,i} + Z_{t,i}, \tag{2}$$

for $t = 1,\ldots,T$ and $i = 1,\ldots,n$ with i.i.d. Gaussian noise $Z_{t,i} \sim \mathcal{N}(0,1)$. The fading coefficient $S_t \in \tilde{\mathcal{S}}$ is assumed to be constant during block t and to change independently across slots. The support $\tilde{\mathcal{S}} = \{\tilde{s}_1,\ldots,\tilde{s}_L\}$ is discrete with $\tilde{s}_1 < \tilde{s}_2 < \ldots < \tilde{s}_L$. We denote the probability mass function (pmf) of S_t by $p_l = \Pr[S_t = \tilde{s}_l]$ such that $\sum_{l=1}^{L} p_l = 1$. For simplicity of analysis, we focus here on the case $L = 2$, which corresponds to a binary quantization of the channel states as in [11]. We sometimes refer to $\tilde{s}_1$ and $\tilde{s}_2$ as the "bad" and "good" channel states, respectively.

It is remarked that the signal-to-noise ratio of the received signal (1) is assumed to be constant in all T transmission slots, unlike that of (2). This assumption can be justified in the cloud radio access scenario by considering that the encoder-to-relay link is typically the direct link between a mobile station and the local base station, and thus can be expected to remain static for a longer time than the inter-cell link corresponding to the side information Y. Extension of our results to the more general case in which (1) is subject to fading is conceptually straightforward but requires a more involved analysis.

The assumption $L = 2$ allows us to rank the quality of fading state vector $\mathbf{S} = [S_1,\ldots,S_T] \in \tilde{\mathcal{S}}^T$ according to the number of good channel states $\tilde{s}_2$. Specifically, define as $\mathcal{S}_k$ the set of all $\binom{T}{k}$ fading vectors $\mathbf{s}_k$ with k good channels. We also define

$$\mathbf{Y}_k = \mathrm{diag}(\mathbf{s}_k)^{\frac{1}{2}}\mathbf{X} + \mathbf{Z}, \tag{3}$$

for $k = 0,\ldots,T$ as the signal received across the T blocks if the fading vector $\mathbf{s}_k$, where $\mathbf{s}_k$ belongs to set $\mathcal{S}_k$, with $\mathbf{X} = [X_1,\ldots,X_T]^t$ and $\mathbf{Z} = [Z_1,\ldots,Z_T]^t$. Note that from now on we drop the dependence on i for simplicity of notation. Moreover, we introduce a parameter $l_o \in \{-1,0,\ldots,T-1\}$ that represents the allowed outage level. As detailed in Sec. III, the proposed system design does not impose any constraint for the states $\mathbf{s}_k$ with less than l_o+1 good states, i.e., for $\mathbf{s}_k \in \mathcal{S}_k$ with $k \le l_o$. Note that $l_o = -1$ corresponds to no outage.

We assume broadcast coding [12] in which the encoder sends $T - l_o$ independent messages $M_{l_o+1},\ldots,M_T$ and, if a fading vector $\mathbf{S} = \mathbf{s}_k \in \mathcal{S}_k$ is realized, the decoder is required to reliably decode only the subset of messages $M_{l_o+1},\ldots,M_k$ for $k \in \{l_o+1,\ldots,T\}$. Moreover, message M_j is uniformly distributed in the set $\{1,\ldots,2^{nTR_j}\}$ so that R_j is the rate of message M_j in bits per channel use. The rate decoded when $\mathbf{S} = \mathbf{s}_k \in \mathcal{S}_k$ for $k > l_o$ is then $R^k = \sum_{j=l_o+1}^{k} R_j$. We implement broadcast coding in the standard fashion, using superposition coding as

$$X_t = \sum_{j=l_o+1}^{T} X_t^j \tag{4}$$

where X_t^j is $\mathcal{N}(0,P_j)$. Each jth layer is allocated power P_j.

We assume the relay compresses the received signal $\mathbf{V} = [V_1,\ldots,V_T]^t$ and forwards it to the decoder over the link of capacity $C_{\max}$. By the Wyner-Ziv theorem, the digital description W_t of V_t produced by the relay can be recovered at the decoder for all fading realizations $\mathbf{S} = \mathbf{s}_k \in \mathcal{S}_k$ with $k > l_o$, if the test channel $p(\mathbf{w}|\mathbf{v})$ is selected so that the constraint

$$\frac{1}{T} I(\mathbf{V};\mathbf{W}|\mathbf{Y}_{l_o+1}) \le C_{\max}, \tag{5}$$

is satisfied where $\mathbf{W} = [W_1,\ldots,W_T]^t$.

III. Problem Definition

In order to introduce the problem of interest, we first define the *informed capacity* $C^k(\hat{P})$ as the maximum rate achievable when the current fading state $\mathbf{S} = \mathbf{s}_k \in \mathcal{S}_k$ is known at the encoder and the relay, and the encoder uses a given transmission power $\hat{P}$ in all blocks. The informed capacity can be computed as

$$C^k(\hat{P}) = \frac{k}{T} C_{\tilde{s}_2}(\hat{P}) + \frac{T-k}{T} C_{\tilde{s}_1}(\hat{P}), \tag{6}$$

for all k, where $C_{\tilde{s}_j}(\hat{P})$ represents the informed capacity for a single slot if the fading state is $S_1 = \tilde{s}_j$, $j = 1, 2$. This is calculated in [13] and is given as

$$C_{\tilde{s}_j}(\hat{P}) = \frac{1}{2}\log\left(1 + \frac{\hat{P}(\tilde{s}_j + q_j a_j)}{1 + a_j\sigma_e^2}\right), \quad (7)$$

for $j = 1, 2$ where $q_j = 1 + \tilde{s}_j\sigma_e^2$ and $a_j = (2^{2C_{\max}} - 1)(\sigma_e^2 + \hat{P}/(1 + \tilde{s}_j\hat{P}))^{-1}$. We recall that the informed capacity (7) is achieved by the Gaussian test channel $p(\mathbf{w}|\mathbf{v}) = p(w_1|v_1)$ given as

$$W_1 = \sqrt{a_j}V_1 + N_1, \quad (8)$$

where the compression noise is $N_1 \sim \mathcal{N}(0, 1)$ independent of the signal V_1.

We are interested in designing the system so that, for each possible state $\mathbf{s}_k$ affecting the side information in (3), a given fraction γ of the corresponding informed capacity (6) is achieved despite the absence of CSI about $\mathbf{s}_k$ at the encoder and relay. Specifically, we have the following definition.

Definition 1. The coding and compression strategies are said to satisfy the $(\hat{P}, l_o, \gamma)$*-competitive optimality constraints* if the following conditions are satisfied:

$$\frac{1}{T}R^k \geq \gamma C^k(\hat{P}), \text{ for all } k = l_o + 1, \ldots, T, \quad (9)$$

where $\gamma \in [0, 1]$ is a target fraction and $l_o \in \{-1, \ldots, T-1\}$ is the allowed outage level.

Constraints (9) impose that at any fading state $\mathbf{S} = \mathbf{s}_k \in \mathcal{S}_k$ with $k > l_o$, the achievable rate is at least a fraction γ of the capacity that would be achieved if the state had been known to the encoder and the relay. Note that these constraints are not imposed on the states in $\mathcal{S}_k$ with $0 \leq k \leq l_o$, consistently with the definition of l_o as the allowed outage level.

Due to the absence of CSI at the relay, we consider compression strategies based on the Gaussian test channel (8) with a constant compression gain a for all $t = 1, \ldots, T$. The backhaul constraint (5) can then be calculated as

$$\frac{l_o+1}{T}f_{\mathrm{bh}}(P, a, \tilde{s}_2) + \frac{T - l_o - 1}{T}f_{\mathrm{bh}}(P, a, \tilde{s}_1) \leq C_{\max}, \quad (10)$$

where the function $f_{\mathrm{bh}}(P, a, s)$ is defined as

$$f_{\mathrm{bh}}(P, a, s) = \frac{1}{2}\log\left(1 + a\left(\sigma_e^2 + \frac{P}{1 + Ps}\right)\right). \quad (11)$$

In this work, we aim at designing the power allocation $(P_{l_o+1}, \ldots, P_T)$ for the superposition (4) and the compression gain a so as to minimize the total transmission power $P = \sum_{j=l_o+1}^{T} P_j$ under the competitive optimality constraints (9) and the backhaul constraint (10). In the next section, we discuss the calculation of rates R^k in (9) and propose an iterative solution of the problem at hand. We remark that in [14] the problem at hand has been investigated for the special case $T = 1$ and by allowing for more general forms of compression based on successive refinement. It is emphasized that the solution proposed here allows for arbitrary delay T unlike the work in [14].

IV. BROADCAST CODING

Recall that with broadcast coding, if a fading state $\mathbf{S} = \mathbf{s}_k \in \mathcal{S}_k$ is realized with $k > l_o$, only the messages $M_{l_o+1}, \ldots, M_k$, and thus the codewords corresponding to $X_t^{l_o+1}, \ldots, X_t^k$ in (4), are required to be decoded. This is accomplished via successive decoding with ordering $M_{l_o+1} \to M_{l_o+2} \to \cdots \to M_k$ [12]. Defining $\mathbf{X}^j = [X_1^j, \ldots, X_T^j]^t$ and recalling that the decoder operates based on $\mathbf{W}$ and $\mathbf{Y}$, the rate $R_k = I(\mathbf{X}^k; \mathbf{W}, \mathbf{Y}_k | \mathbf{X}^{l_o+1}, \ldots, \mathbf{X}^{k-1})$ is thus achievable for message M_k, which can be computed as

$$\begin{aligned} &I(\mathbf{X}^k; \mathbf{W}, \mathbf{Y}_k | \mathbf{X}^{l_o+1}, \ldots, \mathbf{X}^{k-1}) \\ =&k f_{\mathrm{tr}}\left(P_k, \bar{P}_{k+1}, a, \tilde{s}_2\right) + (T-k) f_{\mathrm{tr}}\left(P_k, \bar{P}_{k+1}, a, \tilde{s}_1\right), \end{aligned} \quad (12)$$

with $\bar{P}_{k+1} = \sum_{j=k+1}^{T} P_j$ if $k < T$ and $\bar{P}_{k+1} = 0$ if $k = T$, and the function $f_{\mathrm{tr}}\left(P, \bar{P}, a, s\right)$ defined as

$$f_{\mathrm{tr}}\left(P, \bar{P}, a, s\right) = \frac{1}{2}\log\left(1 + P\frac{s + a(1 + s\sigma_e^2)}{1 + a\sigma_e^2 + \bar{P}(s + a(1 + s\sigma_e^2))}\right). \quad (13)$$

Overall, the power minimization problem is formulated as

$$\underset{P_{l_o+1}, \ldots, P_T, a \geq 0}{\text{minimize}} \sum_{j=l_o+1}^{T} P_j \quad (14a)$$

$$\begin{aligned} \text{s.t. } &\frac{k}{T} f_{\mathrm{tr}}\left(P_k, \bar{P}_{k+1}, a, \tilde{s}_2\right) + \\ &\frac{T-k}{T} f_{\mathrm{tr}}\left(P_k, \bar{P}_{k+1}, a, \tilde{s}_1\right) \geq \gamma C_k(\hat{P}), \\ &\text{for all } k = l_o + 1, \ldots, T, \end{aligned} \quad (14b)$$

$$\begin{aligned} &\frac{l_o+1}{T} f_{\mathrm{bh}}(\bar{P}_{l_o+1}, a, \tilde{s}_2) + \\ &\frac{T - l_o - 1}{T} f_{\mathrm{bh}}(\bar{P}_{l_o+1}, a, \tilde{s}_1) \leq C_{\max}, \end{aligned} \quad (14c)$$

where we defined the incremental capacity $C_k(\hat{P}) = C_k(\hat{P}) - C_{k-1}(\hat{P})$ for $k = l_o + 1, \ldots, T$ with $C_{l_o}(\hat{P}) = 0$.

We propose an iterative algorithm to solve the problem (14). We start by initializing $a^{(1)}$ as the solution of the equation (14c) obtained by imposing equality and substituting $f_{\mathrm{bh}}(\bar{P}_{l_o+1}, a, s)$ with the upper bound $f_{\mathrm{UB}}(a, s) \geq f_{\mathrm{bh}}(\bar{P}_{l_o+1}, a, s)$ where

$$f_{\mathrm{UB}}(a, s) = \frac{1}{2}\log\left(1 + a\left(\sigma_e^2 + \frac{1}{s}\right)\right). \quad (15)$$

Since this upper bound does not depend on the power variables $P_{l_o+1}, \ldots, P_T$, the so obtained value $a^{(1)}$ guarantees the constraint (14c) for any $P_{l_o+1}, \ldots, P_T$. Moreover, due to the monotonicity of the upper bound function $f_{\mathrm{UB}}(a, s)$ in (15) with respect to a, the solution $a^{(1)}$ is unique and can be obtained via bisection[1]. For fixed $a^{(1)}$, we then set $P_{l_o+1}^{(1)}, \ldots, P_T^{(1)}$ to satisfy the constraints (14b) with equality. This can be done successively with the ordering $P_T^{(1)} \to \ldots \to P_{l_o+1}^{(1)}$. Specifically, for each $P_k^{(1)}$ with $k \in \{l_o+1, \ldots, T\}$, we

[1]The range of a can be assumed to be between 0 and $(2^{2C_{\max}} - 1)/\sigma_e^2$.

Algorithm 1 Iterative algorithm for solving the problem (14)

Step 1. Compute an initial point $(P_{l_o+1}^{(1)}, \ldots, P_T^{(1)}, a^{(1)})$ as discussed in the text and set $l = 2$;
Step 2. Update $a^{(l)}$ such that the constraint (14c) is tight for fixed $P_{l_o+1}^{(l-1)}, \ldots, P_T^{(l-1)}$ via bisection;
Step 3. Update $P_{l_o+1}^{(l)}, \ldots, P_T^{(l)}$ such that the constraints (14b) are tight for fixed $a^{(l)}$ via bisection;
Step 4. Stop the iterations if a convergence criterion is satisfied and go back to Step 2 with $l \leftarrow l+1$ otherwise.

solve (14b) via bisection search[2] using the monotonicity of the function $f_{\mathrm{tr}}\left(P_k, \bar{P}_{k+1}, a, s\right)$ with respect to P_k. As a result, we have an initial point $(P_{l_o+1}^{(1)}, \ldots, P_T^{(1)}, a^{(1)})$ satisfying the constraints (14b) and (14c) with equality and inequality, respectively. With this initial point, the proposed algorithm proceeds as per Algorithm 1.

A. Properties of Algorithm 1

The following lemmas provide results on the intermediate solutions and convergence of Algorithm 1.

Lemma 1. *Algorithm 1 guarantees the feasibility of each intermediate solution* $(P_{l_o+1}^{(l)}, \ldots, P_T^{(l)}, a^{(l)})$.

Proof: The proof appears in Appendix A. ■

Lemma 2. *The proposed algorithm in Algorithm 1 converges to a locally optimal point for problem (14).*

Proof: The result follows since, as shown in Appendix A, we have $P_j^{(l)} \leq P_j^{(l-1)}$ for all $j = l_o+1, \ldots, T$ and any iteration l, and thus a monotonically decreasing objective function. ■

B. Single-Layer Transmission

For comparison with broadcast coding, we now discuss a conventional single-layer transmission strategy in which the encoder maps the messages $M_{l_o+1}, \ldots, M_T$ into an individual codeword, rather than using $T - l_o$ layers as in (4). In order to satisfy the competitive optimality constraints (9) with this approach, one must enforce that the informed capacity $C^T(\hat{P})$ corresponding to the best state $\mathbf{s}_T \in \mathcal{S}_T$ be achieved also when the worst-case fading states $\mathbf{s}_{l_o+1} \in \mathcal{S}_{l_o+1}$ are realized. This results in the constraint

$$\frac{l_o+1}{T} f_{\mathrm{tr}}\left(P, 0, a, \tilde{s}_2\right) + \frac{T-l_o-1}{T} f_{\mathrm{tr}}\left(P, 0, a, \tilde{s}_1\right) \geq \gamma C^T(\hat{P}). \tag{16}$$

The power minimization problem for the single-layer transmission can thus be formulated as (14) with the constraint (14b) replaced with (16). This problem can be solved by employing Algorithm 1 with straightforward modification.

V. Numerical Results

Here, we present numerical results to assess the role of the delay T and the advantages of broadcast coding over single-layer approach. To ease the interpretation of the results, we tie the outage level l_o to more conventional outage probability. To this end, given the allowed outage probability P_o, we compute the outage level l_o as the maximum value of l in the outage set $\mathcal{L}_o = \{-1\} \cup \{l | \sum_{k=0}^{l} \Pr[\mathbf{S} \in \mathcal{S}_k] \leq P_o,\ l = 0, \ldots, T-1\}$ where the probability $\Pr[\mathbf{S} \in \mathcal{S}_k]$ is given as $\Pr[\mathbf{S} \in \mathcal{S}_k] = \binom{T}{k} p_2^k p_1^{T-k}$ by assumptions in Sec. II.

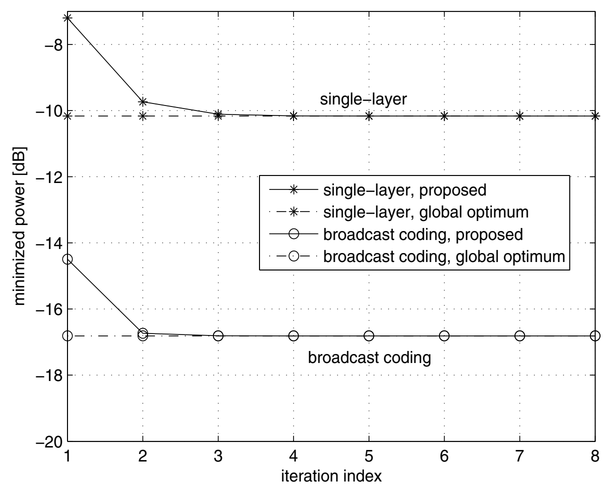

Figure 2. Minimized power versus the number of iterations with $\gamma = 0.2$, $C_{\max} = 1$, $\hat{P} = 1$, $p_1 = 0.1$, $T_{\max} = 10$, $P_o = 0.001$, $[\tilde{s}_1, \tilde{s}_2] = [1, 10]$ and $\sigma_e^2 = 0.1$.

We first show an example of the fast convergence of the proposed algorithm to the global optimum. Specifically, Fig. 2 shows the minimum power obtained with Algorithm 1 versus the iteration index l as compared to the global optimum obtained via global optimization tools (here exhaustive search) for $\gamma = 0.9$, $C_{\max} = 3$, $\hat{P} = 1$, $p_1 = 0.1$, $[\tilde{s}_1, \tilde{s}_2] = [1, 100]$ and $\sigma_e^2 = 0.1$. It is seen that 2-3 iterations are sufficient for convergence to the global minimum power, although Lemma 2 only guarantees local optimality.

In Fig. 3, we show the minimum power P versus the maximum allowed delay $T_{\max}$[3] for $\gamma = 0.9$, $C_{\max} = 3$, $\hat{P} = 1$, $p_1 = 0.1$, $[\tilde{s}_1, \tilde{s}_2] = [1, 100]$ and $\sigma_e^2 = 0.1$. It is observed that the performance gain of broadcast coding over the single-layer approach grows with the maximum delay $T_{\max}$, as broadcast coding is better suited to leverage the increased degree of freedom afforded by a larger delay.

In Fig. 4, the minimum power is shown versus the target fraction γ in the competitive optimality constraints (9) for $T_{\max} = 10$, $C_{\max} = 2$, $\hat{P} = 1$, $P_o = 0.01$, $[\tilde{s}_1, \tilde{s}_2] = [1, 100]$ and $\sigma_e^2 = 0.1$. As the target fraction γ increases, the performance gain of the broadcast coding over the single-layer transmission becomes more pronounced. This implies that, as the system requirement becomes stricter, one needs to employ more flexible transmission schemes than the single-layer approach.

[2] The range of P_k can be assumed to be between 0 and $(2^{2\gamma C_k(\hat{P})} - 1)(1 + \bar{P}_{k+1}\tilde{s}_1)/\tilde{s}_1$.

[3] For a fixed $T_{\max}$, the value $T \leq T_{\max}$ that minimizes the power is selected.

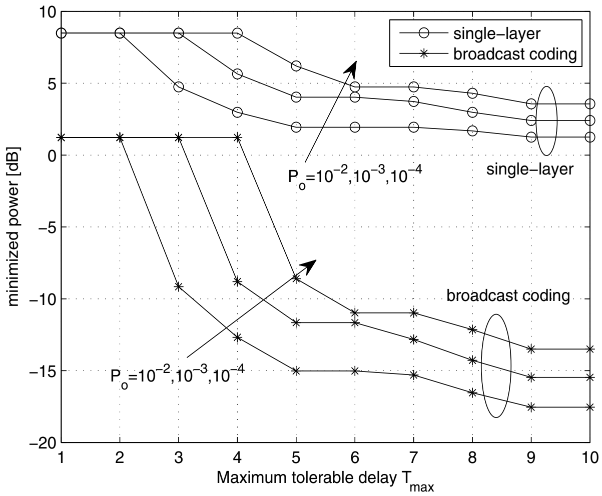

Figure 3. Minimized power versus the tolerable delay $T_{\max}$ with $\gamma = 0.9$, $C_{\max} = 3$, $\hat{P} = 1$, $p_1 = 0.1$, $[\tilde{s}_1, \tilde{s}_2] = [1, 100]$ and $\sigma_e^2 = 0.1$.

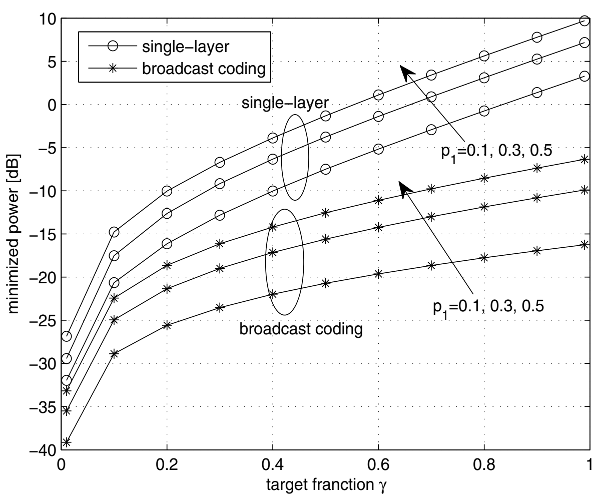

Figure 4. Minimized power versus the target fraction γ with $T_{\max} = 10$, $C_{\max} = 2$, $\hat{P} = 1$, $P_o = 0.01$, $[\tilde{s}_1, \tilde{s}_2] = [1, 100]$ and $\sigma_e^2 = 0.1$.

VI. Conclusions

We addressed the problem of power minimization under delay-tolerant and competitive rate constraints and backhaul capacity limitations for a relay channel model motivated by cloud radio access network applications. Via numerical results, we have assessed the role of the delay and the effectiveness of broadcast coding over the single-layer scheme.

Appendix A
Proof of Lemma 1

We have to prove that a tuple $(P_{l_o+1}^{(l)}, \ldots, P_T^{(l)}, a^{(l)})$ satisfies both (14b) and (14c) at each iteration l. By leveraging the monotonic property of the functions $f_{\rm tr}\left(P, \bar{P}, a, s\right)$ and $f_{\rm bh}(P, a, s)$ with respect to the arguments $(P, \bar{P}, a)$ and (P, a), respectively, we first show that $a^{(l)} \geq a^{(l-1)}$ and $P_j^{(l)} \leq P_j^{(l-1)}$ for all $j = l_o+1, \ldots, T$ and for all iterations l. Specifically, function $f_{\rm bh}(P, a, s)$ is monotonically increasing with respect to P and with respect to a. Therefore, we have $a^{(2)} < a^{(1)}$, since the updated $a^{(2)}$ is obtained by imposing equality in the condition (14c) for the given $(P_{l_o+1}^{(1)}, \ldots, P_T^{(1)})$, while $a^{(1)}$ is obtained by imposing the same condition but using the upper bound (15). Given the mentioned monotonicity properties, it follows that $a^{(2)} < a^{(1)}$. Moreover, function $f_{\rm tr}\left(P, \bar{P}, a, s\right)$ is monotonically decreasing in $\bar{P}$ and increasing in P and a. It follows that we have $P_j^{(2)} < P_j^{(1)}$ for all $j = l_o + 1, \ldots, T$ since Step 2 requires to decrease P_j for all $j = l_o + 1, \ldots, T$ in order to satisfy (14b) with equality for the given $a^{(2)} > a^{(1)}$. Continuing this argument for $l > 2$, we obtain $a^{(l)} \geq a^{(l-1)}$ and $P_j^{(l)} \leq P_j^{(l-1)}$ for all $j = l_o + 1, \ldots, T$. Proof is concluded by noting that, since $a^{(l)}$ satisfies (14c) with $P_{l_o+1}^{(l-1)}, \ldots, P_T^{(l-1)}$ with equality as imposed by Step 2 of Algorithm 1, it also satisfies (14c) with the reduced $P_{l_o+1}^{(l)}, \ldots, P_T^{(l)}$ given the monotonicity of the function $f_{\rm bh}(P, a, s)$ with respect to P. Moreover, the condition (14b) is satisfied with equality by the definition of the proposed update in Step 3 of Algorithm 1.

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