On the Optimization of Two-way AF MIMO Relay Channel with Beamforming

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***Abstract*—This paper studies the joint design of transmit beamformers, receive combiners, and linear relaying matrix for a two-way amplify-and-forward (AF) relay system equipped with multiple-antennas at sources and relay. A single data stream is transmitted by each source. Due to the non-convexity of the optimization problem, finding a solution that maximizes the sum-rate appears to be intractable. Hence, a solution to the original problem is approximated via the iterative solution of three optimization problems, one for the transmit beamformer, one for the receive combiner, and one for the linear relaying matrix. Since the latter is non-convex, a suboptimal iterative procedure is proposed. Joint optimization is assumed to be performed at the relay, which designs the transceiver (thanks to perfect channel state information) and informs the sources of the transmit beamformers/receive combiners. Finally, an upper bound to the achievable sum-rate is provided. The proposed technique shows achievable sum-rate performance very close to the upper bound. Moreover, the algorithm converges to the final solution in a reasonable number of iteration.**

I. Introduction

Relay-based cooperative communications and MIMO systems are by now considered as key techniques to enhance system capacity by increasing coverage and reliability [1], [2]. A relay network of specific interest consists of two nodes communicating to one another via a relay (two-way relay channel). For this network, amplify-and-forward (AF) has been shown to be a promising solution [1]-[8]. Two-way relay methods with multiple-antennas at the relay only have been studied in [3][4]. The authors of [3] characterized the corresponding achievable rate region by providing an iterative algorithm that designs linear relay processing matrices that achieve Pareto-optimal points. A linear processing matrix that maximizes the sum-rate in the high signal-to-noise ratio (SNR) regime is provided in [4] via an iterative algorithm.

Studies of two-way AF relay system with multiple-antennas both at the source nodes and at the relay can be found in [5]-[8]. Zero-forcing (ZF) and minimum MSE (MMSE) based transceiver designs at the relay node were investigated in [5]. Furthermore, the optimization of the linear precoder in terms of minimizing the sum of MSE and maximizing sum-rate by using gradient descent algorithm were proposed in [6] and [7], respectively. In [7], precoding matrices at the sources are fixed (not optimized) to provide multiplexing without precoding gains. In [8], a two-way relay system where we have multiple communication pairs and a single relay is studied, and the relay processing matrix is optimized based on both ZF and MMSE criteria. Each user (source node) in [8] is equipped with predetermined transmit beamforming/receive combining vectors.

Based on the review above, to the best of our knowledge, the literature has not yet addressed the problem of jointly optimizing the transceivers at both sources and relay in multiple-antenna two-way AF relay system in terms of sum-rate. We focus on the case where each source transmits a single stream and the main contributions of this paper are as follows: *(i) Upper bound*: We extend the result in [1] to provide an upper bound on the achievable sum-rate of the system, based on the consideration of two appropriate non-interfering point-to-point channels; *(ii) Sum-rate maximizing technique*: We propose an approximate solution to the problem of sum-rate maximization with respect to the transmit beamformers, receive combiners, and the linear processing matrix for the two-way AF relay system.

This paper is organized as follows. The two-way AF relay model with multiple antennas and the beamforming at the sources and the relay is described in Section II-A; an upper bound on the sum-rate is derived in Section II-B. Section III proposes the jointly designed solution and Section IV provides numerical results on achievable sum-rate and convergence performances of the proposed techniques. Finally, Section V presents our conclusions.

Notation: A bold face letter denotes a vector or a matrix; $[\cdot]^T$, $[\cdot]^*$, $[\cdot]^\dagger$, $\mathtt{tr}(\cdot)$ the transpose, the conjugate transpose, the pseudo-inverse, and the trace of a vector or a matrix, respectively; $\mathtt{vec}(\boldsymbol{A})$ a vector which stacks the columns of a matrix $\boldsymbol{A}$; $\|\boldsymbol{A}\|_F$ the Frobenius norm of a matrix $\boldsymbol{A}$; $\|\boldsymbol{a}\|$ the 2-norm of a vector $\boldsymbol{a}$; $\min(x, y)$ the minimum element between x and y; $\boldsymbol{A} \otimes \boldsymbol{B}$ the Kronecker product of vectors or matrices $\boldsymbol{A}$ and $\boldsymbol{B}$; $\mathbb{E}[\cdot]$ an expected value of a vector or a matrix; $\boldsymbol{I}$ the identity matrix.

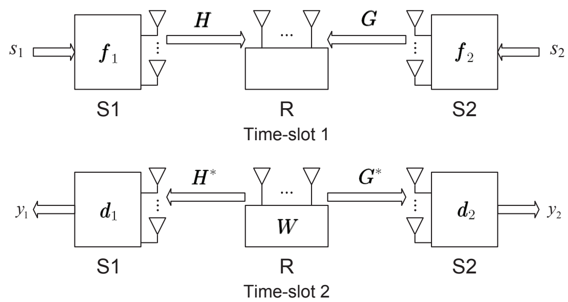

Fig. 1. Two-way relay system equipped with multiple antennas at the relay and the sources.

II. System Model

A. Signal Model

We consider a two-way relay channel consisting of two source nodes, S1 and S2, each with N_s antennas, and a relay node, R, equipped with N_r antennas, as shown in Fig. 1. We assume there is no direct link between the sources, i.e., due to large path loss. The transmission protocol uses two-consecutive time-slots. During the first time-slot (uplink), the two sources transmit to the relay, while in the second time-slot (downlink), the relay broadcasts to the source nodes. Channel reciprocity during the uplink and the downlink is assumed. The channel matrices $\boldsymbol{H}$ and $\boldsymbol{G}$ (see Fig. 1) keep constant over at least two time slots and are assumed to be perfectly known at the relay. The received signal at the relay node in the first time-slot is written as

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{f}_1 s_1 + \boldsymbol{G}\boldsymbol{f}_2 s_2 + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{r} \in \mathbb{C}^{N_r \times 1}$; $\boldsymbol{H}, \boldsymbol{G} \in \mathbb{C}^{N_r \times N_s}$ are the channel matrices from S1 and S2 to the relay, respectively; $\boldsymbol{f}_1, \boldsymbol{f}_2 \in \mathbb{C}^{N_s \times 1}$ are the transmit beamforming vectors at S1 and S2, respectively, with power constraints $\|\boldsymbol{f}_1\|^2 \le 1$ and $\|\boldsymbol{f}_2\|^2 \le 1$; s_1 and s_2 are the transmitted symbols from S1 and S2, respectively, with $\mathbb{E}[|s_1|^2] = P_1$ and $\mathbb{E}[|s_2|^2] = P_2$; and $\boldsymbol{n} \in \mathbb{C}^{N_r \times 1}$ is the noise vector at the relay with $\mathbb{E}[\boldsymbol{n}\boldsymbol{n}^*] = \sigma_{\mathsf{R}}^2 \boldsymbol{I}$. We emphasize that each node sends a single data stream, unlike [5]-[7]. The relay node applies AF operation to $\boldsymbol{r}$, constructing its transmitted signal as

$$\boldsymbol{x} = \boldsymbol{W}\boldsymbol{r}, \tag{2}$$

where $\boldsymbol{x} \in \mathbb{C}^{N_r \times 1}$ is the signal broadcasted from the relay to the source nodes and $\boldsymbol{W} \in \mathbb{C}^{N_r \times N_r}$ is the linear processing matrix at the relay. Transmit power at the relay is restricted to P_r, i.e.,

$$\begin{aligned} \mathtt{tr}\{\mathbb{E}(\boldsymbol{x}\boldsymbol{x}^*)\} = \mathtt{tr}\,\{&P_1 \boldsymbol{W}\boldsymbol{H}\boldsymbol{f}_1\boldsymbol{f}_1^*\boldsymbol{H}^*\boldsymbol{W}^* \\ &+ P_2 \boldsymbol{W}\boldsymbol{G}\boldsymbol{f}_2\boldsymbol{f}_2^*\boldsymbol{G}^*\boldsymbol{W}^* + \sigma_r^2 \boldsymbol{W}\boldsymbol{W}^*\} \le P_r \end{aligned} \tag{3}$$

With an assumption of the channel reciprocity during the uplink and the downlink, the received signals at S1 and S2 are expressed as

$$\begin{aligned} y_1 &= \boldsymbol{d}_1^*\boldsymbol{H}^*\boldsymbol{x} + \boldsymbol{d}_1^*\boldsymbol{z}_1 \\ &= \boldsymbol{d}_1^*\boldsymbol{H}^*\boldsymbol{W}(\boldsymbol{H}\boldsymbol{f}_1 s_1 + \boldsymbol{G}\boldsymbol{f}_2 s_2 + \boldsymbol{n}) + \boldsymbol{d}_1^*\boldsymbol{z}_1 \\ &= \boldsymbol{d}_1^*\boldsymbol{H}^*\boldsymbol{W}\boldsymbol{H}\boldsymbol{f}_1 s_1 + \boldsymbol{d}_1^*\boldsymbol{H}^*\boldsymbol{W}\boldsymbol{G}\boldsymbol{f}_2 s_2 + \boldsymbol{d}_1^*\boldsymbol{H}^*\boldsymbol{W}\boldsymbol{n} + \boldsymbol{d}_1^*\boldsymbol{z}_1, \end{aligned} \tag{4}$$

and

$$y_2 = \boldsymbol{d}_2^*\boldsymbol{G}^*\boldsymbol{W}\boldsymbol{H}\boldsymbol{f}_1 s_1 + \boldsymbol{d}_2^*\boldsymbol{G}^*\boldsymbol{W}\boldsymbol{G}\boldsymbol{f}_2 s_2 + \boldsymbol{d}_2^*\boldsymbol{G}^*\boldsymbol{W}\boldsymbol{n} + \boldsymbol{d}_2^*\boldsymbol{z}_2, \tag{5}$$

respectively, where $\boldsymbol{d}_1, \boldsymbol{d}_2 \in \mathbb{C}^{N_s \times 1}$ are the receive combining vectors at S1 and S2, respectively. $\boldsymbol{z}_1, \boldsymbol{z}_2 \in \mathbb{C}^{N_s \times 1}$ are the noise vectors at S1 and S2 where $\mathbb{E}[\boldsymbol{z}_1\boldsymbol{z}_1^*] = \sigma_{\mathsf{S1}}^2 \boldsymbol{I}$ and $\mathbb{E}[\boldsymbol{z}_2\boldsymbol{z}_2^*] = \sigma_{\mathsf{S2}}^2 \boldsymbol{I}$. The terms $\boldsymbol{d}_1^*\boldsymbol{H}^*\boldsymbol{W}\boldsymbol{H}\boldsymbol{f}_1 s_1$ and $\boldsymbol{d}_2^*\boldsymbol{G}^*\boldsymbol{W}\boldsymbol{G}\boldsymbol{f}_2 s_2$ in y_1 and y_2, respectively, are self-interference that can be removed under an assumption of perfect receive CSI. S1 and S2 are also assumed to know $\boldsymbol{d}_1^*\boldsymbol{H}^*\boldsymbol{W}\boldsymbol{G}\boldsymbol{f}_2$ and $\boldsymbol{d}_2^*\boldsymbol{G}^*\boldsymbol{W}\boldsymbol{H}\boldsymbol{f}_1$, respectively, for data detection. This can be obtained; e.g., training-based channel estimation as explained in [9] prior to data transmission. After self-interference cancellation, y_1 and y_2 become

$$\begin{aligned} y_1 &= \boldsymbol{d}_1^*\boldsymbol{H}^*\boldsymbol{W}\left(\boldsymbol{G}\boldsymbol{f}_2 s_2 + \boldsymbol{n}\right) + \boldsymbol{d}_1^*\boldsymbol{z}_1, \\ y_2 &= \boldsymbol{d}_2^*\boldsymbol{G}^*\boldsymbol{W}\left(\boldsymbol{H}\boldsymbol{f}_1 s_1 + \boldsymbol{n}\right) + \boldsymbol{d}_2^*\boldsymbol{z}_2. \end{aligned} \tag{6}$$

From (6), assuming standard Gaussian codebooks at the sources, we can express the achievable rate for the link from S1 to S2 as

$$R_{12} = \frac{1}{2}\log_2\left(1 + \frac{P_1 \boldsymbol{d}_2^*\boldsymbol{G}^*\boldsymbol{W}\boldsymbol{H}\boldsymbol{f}_1\boldsymbol{f}_1^*\boldsymbol{H}^*\boldsymbol{W}^*\boldsymbol{G}\boldsymbol{d}_2}{\sigma_{\mathsf{R}}^2 \boldsymbol{d}_2^*\boldsymbol{G}^*\boldsymbol{W}\boldsymbol{W}^*\boldsymbol{G}\boldsymbol{d}_2 + \sigma_{\mathsf{S2}}^2 \boldsymbol{d}_2^*\boldsymbol{d}_2}\right), \tag{7}$$

and that for the reverse link as

$$R_{21} = \frac{1}{2}\log_2\left(1 + \frac{P_2 \boldsymbol{d}_1^*\boldsymbol{H}^*\boldsymbol{W}\boldsymbol{G}\boldsymbol{f}_2\boldsymbol{f}_2^*\boldsymbol{G}^*\boldsymbol{W}^*\boldsymbol{H}\boldsymbol{d}_1}{\sigma_{\mathsf{R}}^2 \boldsymbol{d}_1^*\boldsymbol{H}^*\boldsymbol{W}\boldsymbol{W}^*\boldsymbol{H}\boldsymbol{d}_1 + \sigma_{\mathsf{S1}}^2 \boldsymbol{d}_1^*\boldsymbol{d}_1}\right). \tag{8}$$

Thus, the total sum-rate of the system is $R_{\text{sum}} = R_{12} + R_{21}$. Note that, in (7) and (8), $\boldsymbol{W}$ is required for computing R_{12} and R_{21}, while $\{\boldsymbol{f}_1, \boldsymbol{d}_2\}$ and $\{\boldsymbol{f}_2, \boldsymbol{d}_1\}$ are needed only for R_{12} and R_{21}, respectively. Throughout this paper, we assume, for simplicity, that $\sigma^2 = \sigma_{\mathsf{R}}^2 = \sigma_{\mathsf{S1}}^2 = \sigma_{\mathsf{S2}}^2$ and $P = P_1 = P_2$ and set the $\rho = {}^{P}/_{\sigma^2}$. The solution provided in this paper can be easily extended to an asymmetric scenario.

From (7) and (8), the sum-rate maximization problem of the two-way AF relay system transmitting single stream at each source is expressed as

$$\boxed{\begin{aligned} &\max_{\{\boldsymbol{d}_1, \boldsymbol{d}_2, \boldsymbol{f}_1, \boldsymbol{f}_2, \boldsymbol{W}\}} \quad R_{\text{sum}} \\ &\text{s.t. (3) and } \|\boldsymbol{f}_i\|^2 \le 1, i = 1, 2. \end{aligned}} \tag{9}$$

B. Sum-rate Upper Bound

In this section, we derive an upper bound on the sum-rate R_{sum} of the AF relay system described above. To this end, we consider, following [3], S1→R→S2 and S2→R→S1 as interference free communication links, as depicted in Fig. 2 and apply different linear processing matrices $\boldsymbol{W}_1^{\text{UB}} \in \mathbb{C}^{N_r \times N_r}$ and $\boldsymbol{W}_2^{\text{UB}} \in \mathbb{C}^{N_r \times N_r}$ to the two links. The total power of the relay, P_r, is allocated to support both directions of communication so as to maximize the total sum-rate, i.e.

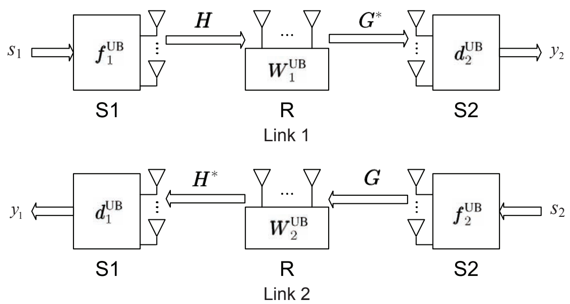

Fig. 2. Two-way relay system that provides upper bound.

$P_r = P_{\text{Link1}} + P_{\text{Link2}}$, where P_{Link1} and P_{Link2} are the transmit power at the relay for Link1 and Link2, respectively. The maximum rate for the one-way AF relay links S1→R→S2 and S2→R→S1 has been derived in [1]. Specifically, let $\boldsymbol{f}_1^{\mathrm{UB}}$ and let $\boldsymbol{d}_1^{\mathrm{UB}}$ be the beamforming and the combining vectors at S1; and $\boldsymbol{f}_2^{\mathrm{UB}}$ and $\boldsymbol{d}_2^{\mathrm{UB}}$ be those at S2. Moreover, let $\boldsymbol{H} = \boldsymbol{U}_1\boldsymbol{\Sigma}_1\boldsymbol{V}_1^*$ and $\boldsymbol{G} = \boldsymbol{U}_2\boldsymbol{\Sigma}_2\boldsymbol{V}_2^*$, where $\boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2 \in \mathbb{C}^{N_r \times N_s}$ are diagonal matrices with descending ordered singular-values of $\boldsymbol{H}$ and $\boldsymbol{G}$, respectively; $\boldsymbol{U}_1, \boldsymbol{U}_2 \in \mathbb{C}^{N_r \times N_r}$ and $\boldsymbol{V}_1, \boldsymbol{V}_2 \in \mathbb{C}^{N_s \times N_s}$ are unitary matrices with the left and right singular vectors of $\boldsymbol{H}$ and $\boldsymbol{G}$, respectively. From [1], $\boldsymbol{f}_1^{\mathrm{UB}} = \boldsymbol{q}_1$ and $\boldsymbol{d}_2^{\mathrm{UB}} = \boldsymbol{q}_2$, where $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ are the first column vectors of $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$, respectively, $\boldsymbol{W}_1^{\mathrm{UB}} = \gamma_1 \boldsymbol{q}_3 \boldsymbol{q}_4^*$, where $\boldsymbol{q}_3$ and $\boldsymbol{q}_4$ are the first column vectors of $\boldsymbol{U}_2$ and $\boldsymbol{U}_1$, respectively, and

$$\gamma_1 = \sqrt{\frac{P_{\text{Link1}}}{(P\omega_1^2 + \sigma^2)}},$$

with ω_1 being the largest singular value of $\boldsymbol{H}$. As a result, an upper bound of the achievable sum-rate, $R_{sum}^{\mathrm{UB}} = R_{12}^{\mathrm{UB}} + R_{21}^{\mathrm{UB}}$, can be expressed as

$$R_{sum}^{\mathrm{UB}} = \max_{\substack{P_{\text{Link1}}, P_{\text{Link2}} \\ \text{s.t. } P_r = P_{\text{Link1}} + P_{\text{Link2}}}} \left\{ \tfrac{1}{2}\log_2\left(1 + \tfrac{\gamma_1^2 P \omega_1^2 \omega_2^2}{\gamma_1^2 \sigma^2 \omega_2^2 + \sigma^2}\right) + \tfrac{1}{2}\log_2\left(1 + \tfrac{\gamma_2^2 P \omega_1^2 \omega_2^2}{\gamma_2^2 \sigma^2 \omega_1^2 + \sigma^2}\right) \right\}. \tag{10}$$

III. Sum-Rate Maximization Solution

In this section, we propose a suboptimal solution to the sum-rate maximization problem (9). The optimization problem (9) is not convex. To tackle it, we propose to approximate its solution by decomposing the original problem into the three simpler optimization problems that are iteratively solved. The corresponding optimization problems for the transmit/receive vectors are convex and easy to solve. The problem for the linear processing matrix $\boldsymbol{W}$ is still non-convex and an approximate solution is proposed. We propose to find a suboptimal solution by iterating the solution of the three problems below in an alternate maximization fashion:

$$\boxed{\begin{array}{l} \max\limits_{\boldsymbol{d}_1, \boldsymbol{d}_2} R_{\text{sum}} \text{ for given } \boldsymbol{f}_1, \boldsymbol{f}_2, \text{ and } \boldsymbol{W}. \\ \max\limits_{\boldsymbol{f}_1, \boldsymbol{f}_2} R_{\text{sum}} \text{ for given } \boldsymbol{d}_1, \boldsymbol{d}_2, \text{ and } \boldsymbol{W}, \text{ s.t. } \|\boldsymbol{f}_i\|^2 \le 1. \\ \max\limits_{\boldsymbol{W}} R_{\text{sum}} \text{ for given } \boldsymbol{f}_1, \boldsymbol{f}_2, \boldsymbol{d}_1, \text{ and } \boldsymbol{d}_2, \text{ s.t. (3).} \end{array}} \tag{11}$$

The optimal $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ in the first problem of (11) are well-known to be the MMSE filters $\hat{\boldsymbol{d}}_1 = (\boldsymbol{H}^*\boldsymbol{W}\boldsymbol{W}^*\boldsymbol{H} + \boldsymbol{I})^{-1}\boldsymbol{H}^*\boldsymbol{W}\boldsymbol{G}\boldsymbol{f}_2$ and $\hat{\boldsymbol{d}}_2 = (\boldsymbol{G}^*\boldsymbol{W}\boldsymbol{W}^*\boldsymbol{G} + \boldsymbol{I})^{-1}\boldsymbol{G}^*\boldsymbol{W}\boldsymbol{H}\boldsymbol{f}_1$. Moreover, the solution of the second problem in (11) leads easily to the matched filter (MF) solution $\hat{\boldsymbol{f}}_1 = \alpha_1 \boldsymbol{H}^*\boldsymbol{W}^*\boldsymbol{G}\boldsymbol{d}_2$ and $\hat{\boldsymbol{f}}_2 = \alpha_2 \boldsymbol{G}^*\boldsymbol{W}^*\boldsymbol{H}\boldsymbol{d}_1$ where α_1 and α_2 normalize $\|\hat{\boldsymbol{f}}_1\|$ and $\|\hat{\boldsymbol{f}}_2\|$, respectively, to 1.

The third problem in (11) is non-convex and we propose an algorithm that attempts to obtain a solution to the Karush-Kuhn-Tucker (KKT) conditions, which are necessary for optimality [10]. Specifically, by taking the first derivative for the Lagrangian function of the third problem in (11) with respect to the linear relaying matrix $\boldsymbol{W}$, the first KKT condition is given by $\nabla_{\boldsymbol{W}} L(\boldsymbol{W}) = \nabla_{\boldsymbol{W}} R_{\text{sum}} + \lambda \nabla_{\boldsymbol{W}}(P_r - \mathtt{tr}\{P\boldsymbol{W}\boldsymbol{H}\boldsymbol{f}_1\boldsymbol{f}_1^*\boldsymbol{H}^*\boldsymbol{W}^* + P\boldsymbol{W}\boldsymbol{G}\boldsymbol{f}_2\boldsymbol{f}_2^*\boldsymbol{G}^*\boldsymbol{W}^* + \sigma^2\boldsymbol{W}\boldsymbol{W}^*\}) = 0$, where λ is the Lagrange multiplier for the power constraint (3). By algebraic manipulation, we are able to write the above condition as

$$\boldsymbol{R}(\boldsymbol{w})\boldsymbol{w} = R_{\text{sum}}\boldsymbol{V}(\boldsymbol{w})\boldsymbol{w}, \tag{12}$$

where $\boldsymbol{w} = \mathtt{vec}(\boldsymbol{W})$. Matrices $\boldsymbol{R}(\boldsymbol{w})$ and $\boldsymbol{V}(\boldsymbol{w})$ are defined as

$$\boldsymbol{R}(\boldsymbol{w}) = (\rho\mu_2 + \nu_2)\boldsymbol{T}_1 + (\rho\mu_1 + \nu_1)\boldsymbol{T}_3, \tag{13}$$

and

$$\begin{aligned} \boldsymbol{V}(\boldsymbol{w}) &= \tfrac{1}{(\rho\mu_1\mu_2 + \mu_1\nu_2 + \nu_1\mu_2)} \\ &\times \{\nu_2\mu_1(\rho\mu_2 + \nu_2)\boldsymbol{T}_2 + \nu_1\mu_2(\rho\mu_1 + \nu_1)\boldsymbol{T}_4 + (\nu_1\nu_2)^2\lambda\boldsymbol{Q}_3\}, \end{aligned} \tag{14}$$

respectively, where $\mu_1 = \boldsymbol{w}^*\boldsymbol{T}_1\boldsymbol{w}$, $\mu_2 = \boldsymbol{w}^*\boldsymbol{T}_3\boldsymbol{w}$, $\nu_1 = \boldsymbol{w}^*\boldsymbol{T}_2\boldsymbol{w} + \boldsymbol{d}_2^*\boldsymbol{d}_2$, $\nu_2 = \boldsymbol{w}^*\boldsymbol{T}_4\boldsymbol{w} + \boldsymbol{d}_1^*\boldsymbol{d}_1$ and (15). Imposing also the second KKT condition, namely the power constraint, (3), the Lagrange multiplier λ can be obtained as

$$\lambda = \frac{\mu_1}{P_r\nu_1^2}\left(\rho\frac{\mu_2}{\nu_2} + 1\right)\boldsymbol{d}_2^*\boldsymbol{d}_2 + \frac{\mu_2}{P_r\nu_2^2}\left(\rho\frac{\mu_1}{\nu_1} + 1\right)\boldsymbol{d}_1^*\boldsymbol{d}_1, \tag{16}$$

which is obtained by multiplying $\boldsymbol{w}^*$ to both side of (12).

Notice that $\boldsymbol{R}(\boldsymbol{w})$ and $\boldsymbol{V}(\boldsymbol{w})$ in the KKT condition (12) depend on the unknown $\boldsymbol{w}$. If such dependence were removed, then clearly $\boldsymbol{w}$ could be found as the generalized eigenvector of $\boldsymbol{R}$ and $\boldsymbol{V}$. Since this is not the case, here we propose an iterative algorithm based on the power iteration technique [4] [11]. The algorithm works as described in **Algorithm 1**.

Algorithm 1

1: Initialize $\boldsymbol{W}_0$, λ_0, and $t \leftarrow 1$
2: **while** $\Gamma \ge \varepsilon$ or $t \le t_{\max}$ **do**
3: Compute $\boldsymbol{R}(\boldsymbol{w}_{t-1})$ and $\boldsymbol{V}(\boldsymbol{w}_{t-1})$ with λ_{t-1} from (13)-(15)
4: $\boldsymbol{w}_t = \sqrt{\gamma}\boldsymbol{a}_t$, $\boldsymbol{a}_t = \frac{\boldsymbol{V}(\boldsymbol{w}_{t-1})^\dagger \boldsymbol{R}(\boldsymbol{w}_{t-1})\boldsymbol{w}_{t-1}}{\|\boldsymbol{V}(\boldsymbol{w}_{t-1})^\dagger \boldsymbol{R}(\boldsymbol{w}_{t-1})\boldsymbol{w}_{t-1}\|} = \mathtt{vec}(\boldsymbol{A}_t)$, $\gamma = \frac{P_r}{\mathtt{tr}\{P\boldsymbol{A}_t\boldsymbol{H}\boldsymbol{f}_1\boldsymbol{f}_1^*\boldsymbol{H}^*\boldsymbol{A}_t^* + P\boldsymbol{A}_t\boldsymbol{G}\boldsymbol{f}_2\boldsymbol{f}_2^*\boldsymbol{G}^*\boldsymbol{A}_t^* + \sigma_r^2\boldsymbol{A}_t\boldsymbol{A}_t^*\}}$
5: Compute λ_t from (16) and $\Gamma = \|\boldsymbol{W}_t - \boldsymbol{W}_{t-1}\|_F^2$
6: $t \leftarrow t + 1$
7: **end while**
8: $\hat{\boldsymbol{W}} = \boldsymbol{W}_{t-1}$

$$
\begin{aligned}
&\boldsymbol{T}_1 = \left((\boldsymbol{H}\boldsymbol{f}_1)^T \otimes \boldsymbol{d}_2^*\boldsymbol{G}^*\right)^* \left((\boldsymbol{H}\boldsymbol{f}_1)^T \otimes \boldsymbol{d}_2^*\boldsymbol{G}^*\right), \boldsymbol{T}_2 = (\boldsymbol{I}^T \otimes \boldsymbol{d}_2^*\boldsymbol{G}^*)^*(\boldsymbol{I}^T \otimes \boldsymbol{d}_2^*\boldsymbol{G}^*),\\
&\boldsymbol{T}_3 = \left((\boldsymbol{G}\boldsymbol{f}_2)^T \otimes \boldsymbol{d}_1^*\boldsymbol{H}^*\right)^* \left((\boldsymbol{G}\boldsymbol{f}_2)^T \otimes \boldsymbol{d}_1^*\boldsymbol{H}^*\right), \boldsymbol{T}_4 = (\boldsymbol{I}^T \otimes \boldsymbol{d}_1^*\boldsymbol{H}^*)^*(\boldsymbol{I}^T \otimes \boldsymbol{d}_1^*\boldsymbol{H}^*);\\
&\boldsymbol{Q}_1 = \left((\boldsymbol{H}\boldsymbol{f}_1)^T \otimes \boldsymbol{I}\right)^* \left((\boldsymbol{H}\boldsymbol{f}_1)^T \otimes \boldsymbol{I}\right), \boldsymbol{Q}_2 = \left((\boldsymbol{G}\boldsymbol{f}_2)^T \otimes \boldsymbol{I}\right)^* \left((\boldsymbol{G}\boldsymbol{f}_2)^T \otimes \boldsymbol{I}\right), \boldsymbol{Q}_3 = P\boldsymbol{Q}_1 + P\boldsymbol{Q}_2 + \sigma^2\boldsymbol{I}.
\end{aligned} \tag{15}
$$

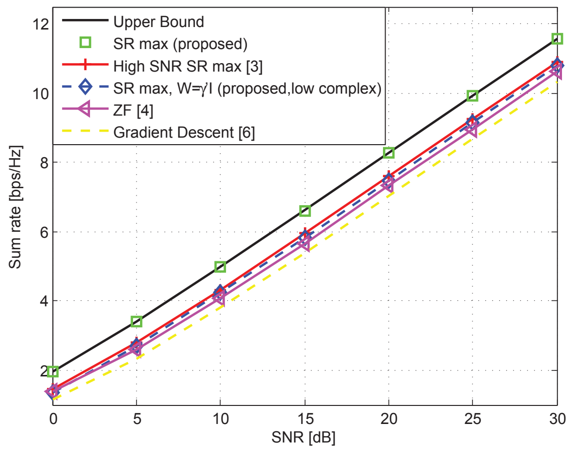

Fig. 3. Achievable sum-rate performances of the two-way AF relay systems with multiple-antennas, $N_r = N_s = 4$.

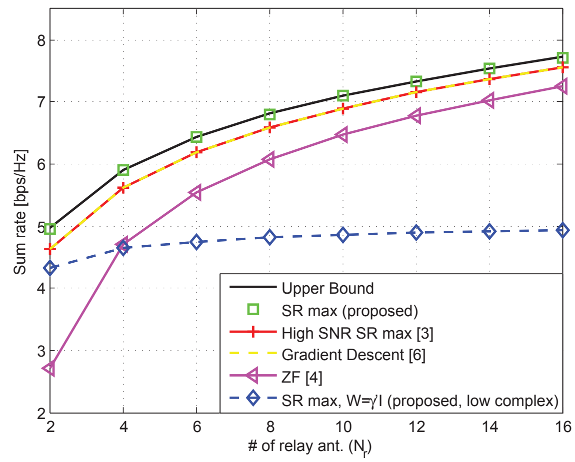

Fig. 4. Achievable sum-rate performance of the two-way AF relay systems with an increasing number of relay antennas, $N_s = 2$ and SNR=15dB.

Here, ε and $t_{\max}$ are parameters to control the accuracy and maximum number of iteration. Notice that Step 4 corresponds to a lower iteration steps. If the procedure detailed above converges, the solution, $\hat{\boldsymbol{W}}$, satisfies the first order necessary KKT conditions (12) and (16). Since the original problem is non-convex, however, this cannot guarantee optimality. The proposed technique is seen to converge and to provide a good suboptimal solution via the simulation results in Section IV compared to the upper bound (10).

IV. Numerical Results

In this section, we present simulation results of the achievable rates of the techniques proposed in this paper. Performance comparison is provided with respect to the techniques proposed in [4][5][7]. Since the technique of [4] assumes single antenna transmitters and those in [5][7] transmit multiple streams, for fairness, in all cases, we select the best two antennas (i.e., with largest channel gains) for transmission of a single stream from each source as in the proposed scheme. To assess the role of the linear precoding matrix on the achievable sum-rate performance, the trivial solution $\boldsymbol{W} = \gamma'\boldsymbol{I}$ with $\gamma' = \sqrt{\frac{P_r}{P\mathtt{tr}(\boldsymbol{H}\boldsymbol{f}_1\boldsymbol{f}_1^*\boldsymbol{H}^*)+P\mathtt{tr}(\boldsymbol{G}\boldsymbol{f}_2\boldsymbol{f}_2^*\boldsymbol{G}^*)+\sigma^2\mathtt{tr}(\boldsymbol{I})}}$, chosen so as to satisfy (3), is also considered.

We model the elements of the channel between each source and the relay as independent complex Gaussian random variables with zero mean and unit variance and average the rates to get achievable sum-rate performance. It is assumed that the sources and the relay deploy the same transmit power, i.e., $P_r = P$.

A. Sum-rate Performance of Two-way AF Relay Methods

Fig. 3 compares the achievable sum-rate performance of the various techniques for a two-way AF relay channel with $N_s = N_r = 4$. It is seen that the achievable sum-rate performance of the proposed technique (labeled as 'SR max') is very close to the upper bound. It is also observed that there is significant performance loss of by setting $\boldsymbol{W} = \gamma'\boldsymbol{I}$ ('SR max, $\boldsymbol{W} = \gamma'\boldsymbol{I}$'). Previous techniques also show inferior performance compared to the proposed technique, e.g., about 3dB difference between the proposed technique and the ZF-based method of [5] (with optimal antenna selection, as explained above).

Achievable sum-rate versus the number of relay antennas N_r is shown in Fig. 4 for SNR is 15 dB and $N_s = 2$. As N_r increases, while setting $\boldsymbol{W} = \gamma'\boldsymbol{I}$ shows increasing performance loss, the performance of ZF-based method approaches to the upper bound (given the ability of the relay to effectively cancel interference).

B. Convergence

In Fig. 5, the convergence property of the proposed sum-rate maximizing algorithm is presented in terms of average mean squared error (MSE). We compare the convergence behavior when $N_r = N_s = 2$, $N_r = Ns = 4$, and SNR=15dB. MSE of the system at each iteration can be obtained by the following calculation:

$$
\begin{aligned}
\mathrm{MSE}_t = &\left\|\hat{\boldsymbol{d}}_1 - \boldsymbol{d}_{1,t}\right\|^2 + \left\|\hat{\boldsymbol{d}}_2 - \boldsymbol{d}_{2,t}\right\|^2\\
&+ \left\|\hat{\boldsymbol{f}}_1 - \boldsymbol{f}_{1,t}\right\|^2 + \left\|\hat{\boldsymbol{f}}_2 - \boldsymbol{f}_{2,t}\right\|^2 + \left\|\hat{\boldsymbol{W}} - \boldsymbol{W}_t\right\|_F^2,
\end{aligned}
$$

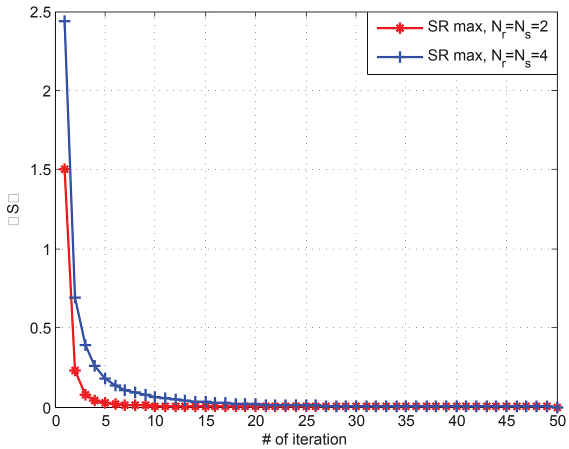

Fig. 5. Convergence of the proposed method when $N_r = N_s = 2$ and $N_r = N_s = 4$, SNR=15dB.

where t denotes the iteration number. Since we enforce the iteration number as 50, $\hat{\boldsymbol{d}}_1 = \boldsymbol{d}_{1,50}$ and it is same to the others. As the number of antennas at the sources and the relay increases, the algorithm converges slowly. The algorithm rapidly converges to the final solution in 10 iterations and it seems 30 iterations are enough to say the algorithm almost reaches the final solution. We cannot assure that the proposed algorithm always converges or not, however, it provides jointly designed system parameters in a reasonable number of iteration.

V. Conclusion

In this paper, we proposed near-sum rate optimal system designs for the two-way AF relay system equipped with multiple antennas at the sources and the relay for transmission of a single stream from each source. An upper bound to the achievable sum-rate was also provided. Numerical results confirmed that the proposed technique showed comparable achievable sum-rate performance, which was very close to the upper bound of the system. Moreover, the reasonable number of iteration was needed to reach steady state of the iterative algorithm.

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