Half-Duplex Gaussian Diamond Relay Channel with Interference Known at One Relay

Kağan Bakanoğlu†, Elza Erkip† and Osvaldo Simeone‡
†Dept. of ECE, Polytechnic Institute of NYU, kbakan01@students.poly.edu, elza@poly.edu
‡Dept. of ECE, NJIT, osvaldo.simeone@njit.edu.

***Abstract*—A diamond relay channel in the presence of interference which is non-causally available only at one relay is considered. The interference signal may have structure, for example it could come from another source communicating with its own destination. However, the external interferer is not willing to adjust its communication strategy to minimize the interference and is considered to be fixed. Two approaches are possible to mitigate the interference: exploiting the structure or treating it as unstructured. Using these approaches, bounds for the Gaussian half-duplex diamond relay channel based on two transmission time patterns are established. The importance of exploiting the interference structure and transmission time patterns are discussed.**

I. Introduction

In wireless networks, interference affects terminals participating in the same communication session in different ways. This gives rise to a number of key design challenges, which have been well studied studied in the context of medium access control protocols. An example is the so called exposed terminal problem, in which a node is incorrectly prevented from transmitting to its receiver when it overhears the transmission of another node that, however, does not affect the receiver.

A recent line of work has started to address the issue of locality of interference from an information-theoretic standpoint. A critical aspect of interference, from a physical layer standpoint, is that the interfering signal is not a purely random noisy waveform, but instead it has the *structure* provided by the specific codebook, it is selected from [1]. In [1][2] the transmitter is able to learn the interference non-causally, while the receiver is not. In this case, the main design issue is whether the encoder should exploit the structure of the interference by boosting reception of the latter at the receiver so as to enable interference decoding and stripping. Alternatively, the encoder could simply treat the interference as *unstructured* by using standard Gelfand-Pinsker (GP) [3] or Dirty Paper Coding (DPC) [4] precoding techniques. It is recalled that GP and DPC precoding are capacity-achieving for state-dependent memoryless channels with "unstructured", i.e., independent identically distributed (i.i.d.), state sequences in the case of discrete alphabet [3] and with arbitrary state sequences for Additive White Gaussian Noise (AWGN) channels [4][5][6], respectively.

The design choice between structured and unstructured approaches to interference is also dealt with in [7][8]. In [7], a relay node can learn an interfering signal by listening to the ongoing transmissions and convey such information to the interfered decoder. Instead, in [8] a multiple access channel is considered in which only one of the two transmitters is

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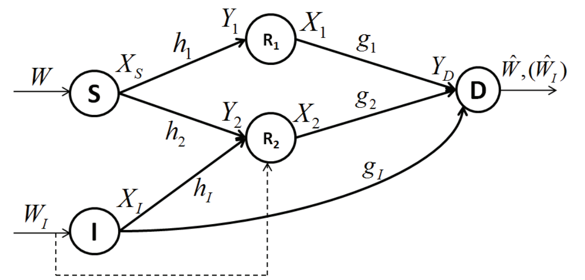

Figure 1. Diamond relay channel with an external interference non-causally known at one relay.

aware of the interfering sequence (in a non-causal fashion). In this latter model, the optimal unstructured transmission strategy was derived in [9] and consists of generalizations of GP (for discrete alphabets) and DPC (for AWGN channels), that are referred to as Generalized GP (GGP) and Generalized DPC (GDPC), respectively. The key difference between GGP/GDPC and GP/DPC is that in the former the terminal that is aware of the interference may also spend part of its power cancelling the state (interference) at the decoder for the benefit of the encoder that does not have state information.

In this work, we tackle the investigation of the impact of the locality of interference in a baseline scenario for networks with multiple relays. The model, illustrated in Fig. 1, consists of a single source communicating to a single destination via two half-duplex relays also known as diamond relay channel. An interferer affects only reception at one of the two relays and at the destination, and can be measured at the affected relay in a non-causal fashion. Such a model may be useful for designing cooperative wireless networks with some terminals equipped with cognition capabilities. It is remarked that non-causal knowledge of the interfering sequence may occur, for instance, if the interferer performs retransmissions (HARQ) and the interference was estimated from a previous retransmission. The model at hand, in the absence of the interference, has been widely studied, even though capacity results are very limited. In particular, reference [10] shows that a specific scheduling of the transmissions from source and relays[1] is optimal under certain symmetric conditions on the channels, that are generalized in [11]. Here, we propose achievable schemes that combine the scheduling strategies in [10][11] with interference management techniques that either exploit the structure of the interference or treat it as unstructured. Finally, we provide numerical results to obtain insight into optimal scheduling and interference management techniques.

[1] Scheduling is necessary due to the half-duplex nature of the relays.

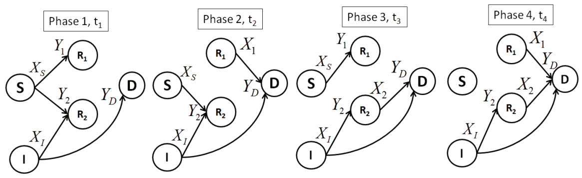

Figure 2. All transmission phases.

II. System model

We consider a diamond relay channel consisting of a source S, two half-duplex relays (R_1,R_2), a destination D and an external interferer I where the interference is available non-causally to R_2, as depicted in Fig. 1. There is no direct link between the source and the destination. The source wishes to transmit a rate-R message W, uniformly distributed in the set $\{1, \cdots, 2^{nR}\}$, to the destination with the help of the two relays in n channel uses. Denoting the symbol transmitted by S, R_1 and R_2 at a given channel use as X_S, X_{R_1} and X_{R_2}, respectively, and the symbol transmitted by the interferer as X_I, the signals received at the relays, if in receive mode, and destination are

$$\begin{aligned} Y_{R_1} &= h_1 X_S + N_1 \\ Y_{R_2} &= h_2 X_S + h_I X_I + N_2 \\ Y_D &= g_1 X_{R_1} + g_2 X_{R_2} + g_I X_I + N_D, \end{aligned} \tag{1}$$

where N_1, N_2 and N_D are independent Gaussian random variables with zero mean and unit variance. Channel gains for S-R_i, R_i-D, I-R_2 and I-D links are denoted by h_i, g_i, h_I and g_I respectively which are real, fixed and known to all nodes. Notice that the interferer is assumed to be symbol and frame synchronous with the source and relays. Since the relays are half-duplex, whenever the relay R_i is receiving, its input satisfies $X_{R_i} = 0$, whereas, when transmitting, we have $Y_{R_i} = 0$. As in [10], we assume per-symbol power constraints on S, R_1 and R_2 given by P_S, P_1 and P_2, respectively, so that $E[|X_S|^2] \leq P_S$ and similarly for the relays. This precludes the possibility to allocate the power over time as in the case where power constraints are on a per-block basis.

Due to half-duplex constraints, transmission from the source and relays must be appropriately scheduled. As discussed in [10][11], four transmission phases can be considered, where each ith phase is allocated a fraction $0 \leq t_i \leq 1$, with $\sum_{i=1}^{4} t_i = 1$, of the overall n channel uses. Specifically, the four phases are depicted in Fig. 2 and given by: (*i*) *Phase 1* ($t_1 n$ channel uses): S transmits, R_1 and R_2 receive (broadcast phase); (*ii*) *Phase 2* ($t_2 n$ channel uses): S and R_1 transmit, R_2 and D receive; (*iii*) *Phase 3* ($t_3 n$ channel uses): S and R_2 transmit, R_1 and D receive; (*iv*) *Phase 4* ($t_4 n$ channel uses): R_1 and R_2 transmit, D receives (multiple access phase). As in [10], in this paper, we consider two specific time schedules, namely Time Pattern I and Time Pattern II. Time Pattern I is composed of two phases, *Phase 1* (broadcast phase) and *Phase 4* (multiple access phase) so that $t_1 + t_4 = 1$ and $t_2 = t_3 = 0$; Time Pattern II is composed of *Phase 2* and *Phase 3*, so that $t_2 + t_3 = 1$ and $t_1 = t_4 = 0$. Note that Time Pattern I and II can be combined to include all four phases. While in general the schedule (t_1, t_2, t_3, t_4) can be optimized, other network constraints may enforce the use of a fixed schedule. To model this, in each time pattern, we set $t_i = 1/2$.

We assume that the interferer employs a fixed (and given) codebook that is not subject to design [1][8]. The codebook of the interferer is assumed to be chosen by the interfering terminal independently to communicate with some other destination which is not modeled explicitly. We assume that the interferer is active during the entire transmission block and that it transmits according to one of the two following interference models. (*i*) *Interference model A*: In phase i, the interferer transmits an independent codeword, of $n/2$ channel uses, carrying an independent message $W_{I,i}$ with rate $R_{I,i}$ so that $W_{I,i}$ is uniformly distributed in the set $\{1, ..., 2^{nR_{I,i}/2}\}$; (*ii*) *Interference model B:* The interferer transmits only one codeword of n channel uses carrying message W_I with rate R_I during the entire block (W_I is uniformly distributed in the set $\{1, ..., 2^{nR_I}\}$). In both cases, we assume that the interferer's codebooks are generated according to a Gaussian distribution with power P_I, which is known by all the nodes. Moreover, the interferer's messages, $W_{I,1}$, $W_{I,2}$ in interference model A and W_I in interference model B, are known to R_2.

Interference model A could arise, for example, if the interferer follows the same fixed network schedule. Moreover, note that it is not a priori clear which one of the two interference models is more benign for communication between S and D. In fact, consider for instance Time Pattern I, where interference affects transmission only during *Phase 4*. Assume, to fix the ideas that the interference rates, satisfy $R_I = R_{I,i} = R$, $i = \{1, 2, 3, 4\}$. With interference model A, the message of the interference in this phase contains $nR/2$ bits, which is less than the number of bits nR carried by the interferer's message under interference model B. This would seem to make it easier to handle interference under model A. However, under model B, the destination can further leverage the signal received during *Phase 1* to aid interference mitigation, while this is not the case with model A. In general, either effect may dominate the other and the relative performance under the two models must be assessed case by case. Similar arguments also hold for Time Pattern II.

Finally, we define the function $C(x) = 1/2 \log_2(1 + x)$.

III. Achievable rates

We propose a number of achievable schemes that are classified with respect to: (*i*) Interference model: A or B; (*ii*) Treating the interference as structured (S) or unstructured (U), following the discussion in Sec. I; (*iii*) Time pattern: I or II. Thus, for instance scheme (A,S,I) will refer to a scheme that operates on interference model A, with a structured approach to treating the interference and with time pattern I.

A. Interference Model A

With interference model A, the codewords sent by the interferer over the different protocol phases encode independent messages.

1) Scheme (A,U,I): We first propose an achievable scheme for Time Pattern I which ignores the structure of the interference, thus treating it as an i.i.d. state. In *Phase 1*, the source S sends information to the two relays over a Gaussian broadcast channel, whereas in *Phase 4* the relays transmit to the destination as over a multiple access channel with common messages and states known at only one encoder.

In general, the source S can provide a common information, of rate R_c, to both relays. This common message can then be sent cooperatively by the relays in *Phase 4*. Moreover, if

$h_2^2 \geq h_1^2$, in *Phase 1*, the relay $\mathtt{R}_2$ can obtain extra information from the source $\mathtt{S}$ in the form of a "private" message of rate R_p that cannot be decoded at relay $\mathtt{R}_1$. In this case, the scenario in *Phase 4* reduces to the channel model studied in [9], where it is shown that relay $\mathtt{R}_2$, that has an extra message and is also aware of the interference (state sequence), should employ GDPC to achieve optimal performance. We recall that with GDPC, relay $\mathtt{R}_2$ is able to precode over the interference to send the extra (private) message and also to partially cancel the interference. If instead $h_1^2 \geq h_2^2$, then it is relay $\mathtt{R}_1$ that is able to collect a private message from the source $\mathtt{S}$, beside the common message. In this second case, relay $\mathtt{R}_2$ cannot transmit an additional message and is limited to transmit the common message and to partially cancel the interference.

The discussion above leads to the following rates.

Proposition 3.1: Let $\rho_p = \sqrt{1-\rho_c^2-\rho_I^2}$. The following rate is achievable for a scheme (A,U,I) if $h_2^2 \geq h_1^2$:

$$R_{(A,U,I)} = \max_{\substack{\alpha,\rho_c,\rho_I: \\ \rho_c^2+\rho_I^2 \leq 1 \\ \alpha,\rho_c\in[0,1] \text{ and } \rho_I\in[-1,0]}} R_p + R_c \tag{2}$$

$$\text{with } R_p \leq \min\left\{\frac{1}{2}C(h_2^2\alpha P_S),\ \frac{1}{2}C(g_2^2\rho_p^2 P_2)\right\} \tag{3}$$

$$R_p + R_c \leq \min\left\{\begin{array}{l} \frac{1}{2}C\left(h_2^2\alpha P_S\right) + \frac{1}{2}C\left(\frac{h_1^2(1-\alpha)P_S}{1+h_1^2\alpha P_S}\right), \\ \frac{1}{2}C\left(\frac{(g_1\sqrt{P_1}+g_2\rho_c\sqrt{P_2})^2}{g_2^2\rho_p^2P_2+(g_I\sqrt{P_I}+g_2\rho_I\sqrt{P_2})^2+1}\right) \\ +\frac{1}{2}C(g_2^2\rho_p^2P_2) \end{array}\right. \tag{4}$$

Sketch of the proof: The proof follows easily from the discussion above, where R_p and R_c are the rates of the private message sent only to relay $\mathtt{R}_1$ (with power αP_S) and R_c is the rate of the common message decoded by both relays in *Phase 1*. The first terms in (3) and (4) correspond to the constraints imposed by decoding the source messages in *Phase 1* [12]. Instead, the second terms in (3) and (4) corresponds to the rate achievable by GDPC [8, eq. (41)]. Notice that $\rho_I \in [-1,0]$, $\rho_c, \rho_p \in [0,1]$ rule the fraction of power that relay $\mathtt{R}_2$ uses for interference cancellation, forwarding the common and private messages, respectively.

Proposition 3.2: The following rate is achievable for a scheme (A,U,I) if $h_1^2 \geq h_2^2$:

$$R_{(A,U,I)} = \max_{\substack{\alpha,\rho_c,\rho_I: \\ \alpha,\rho_c\in[0,1] \text{ and } \rho_I\in[-1,0]}} R_p + R_c \tag{5}$$

$$\text{with } R_p \leq \min\left\{\begin{array}{l} \frac{1}{2}C(h_1^2\alpha P_S), \\ \frac{1}{2}C\left(\frac{g_1^2(1-\rho_c^2)P_1}{1+(g_2\rho_I\sqrt{P_2}+g_I\sqrt{P_I})^2}\right) \end{array}\right. \tag{6}$$

$$R_p + R_c \leq \min\left\{\begin{array}{l} \frac{1}{2}C\left(h_1^2\alpha P_S\right) + \frac{1}{2}C\left(\frac{h_2^2(1-\alpha)P_S}{1+h_2^2\alpha P_S}\right), \\ \frac{1}{2}C\left(\frac{g_1^2(1-\rho_c^2)P_1+(\rho_c g_1\sqrt{P_1}+g_2\sqrt{(1-\rho_I^2)P_2})^2}{1+(g_2\rho_I\sqrt{P_2}+g_I\sqrt{P_I})^2}\right) \end{array}\right. \tag{7}$$

Sketch of the proof: The proof follows again from the discussion above, with the difference that here it is relay $\mathtt{R}_1$ that sends the private message, of rate R_p to the destination $\mathtt{D}$ in *Phase 4*. Therefore, *Phase 4* now reduces to a MAC with common messages, in which we treat interference as noise. The second terms in (6) and (7) are constraints that guarantee correct decoding over such MAC [13]-[15].

2) Scheme (A,S,I): Next, we propose an achievable rate for Time Pattern I that leverages the structure of the interference. In particular, the coding scheme in *Phase 1* is identical to that of Proposition 3.1. However, in *Phase 4*, the decoder attempts joint decoding of both messages from the relays, private and common, and the message of the interferer $W_{I,4}$. In order to ease decoding at the destination, relay $\mathtt{R}_2$, that is aware of $W_{I,4}$ beamforms the codeword of the interference towards the destination.

Proposition 3.3: Let $\rho_p = \sqrt{1-\rho_c^2-\rho_I^2}$. The following rate is achievable for a scheme (A,S,I) if $h_2^2 \geq h_1^2$:

$$R_{(A,S,I)} = \max_{\substack{\alpha,\rho_c,\rho_I: \\ \alpha,\rho_c,\rho_I\in[0,1]}} R_p + R_c \tag{8}$$

$$\text{with } R_p \leq \min\left\{\begin{array}{l} \frac{1}{2}C(h_2^2\alpha P_S), \\ \frac{1}{2}C(g_2^2\rho_p^2P_2), \\ (\frac{1}{2}C(g_2^2\rho_p^2P_2+(g_I\sqrt{P_I}+g_2\rho_I\sqrt{P_2})^2) \\ -\frac{R_{I,4}}{2})^+ \end{array}\right. \tag{9}$$

$$R_p + R_c \leq \min\left\{\begin{array}{l} \frac{1}{2}C\left(h_2^2\alpha P_S\right) + \frac{1}{2}C\left(\frac{h_1^2(1-\alpha)P_S}{1+h_1^2\alpha P_S}\right), \\ \frac{1}{2}C(g_2^2\rho_p^2P_2+(g_1\sqrt{P_1}+g_2\rho_c\sqrt{P_2})^2), \\ (\frac{1}{2}C(g_2^2\rho_p^2P_2+(g_1\sqrt{P_1}+g_2\rho_c\sqrt{P_2})^2 \\ +(g_I\sqrt{P_I}+g_2\rho_I\sqrt{P_2})^2)-\frac{R_{I,4}}{2})^+ \end{array}\right. \tag{10}$$

Sketch of the proof: The coding in *Phase 1* is same as that of Proposition 3.1. In *Phase 4*, $\mathtt{R}_1$, $\mathtt{R}_2$ and the interferer $\mathtt{I}$ form a MAC with common messages where $\mathtt{R}_1$, $\mathtt{R}_2$ and $\mathtt{I}$ have messages (W_c), $(W_c, W_p, W_{I,4})$ and $(W_{I,4})$, respectively. In this MAC, $\mathtt{R}_2$ beamforms the interference such that it is decodable at $\mathtt{D}$ and $\mathtt{D}$ jointly decodes W_p, W_c and $W_{I,4}$ [8]. The rate region of W_c and W_p in *Phase 4* is given in the second and third terms in (9) and (10). This is obtained from the rate region for the MAC with common messages [13]-[15].

Remark 3.1: Proposition 3.3 can easily be extended to the case $h_1^2 \geq h_2^2$ and we do not explicitly report this result here.

3) Scheme (A,U,II): Here, we provide an achievable rate for Time Pattern II where the structure of the interference is ignored. The idea is to send, as in [10], two independent messages of rate R_1 and R_2 along the paths $\mathtt{S}$-$\mathtt{R}_1$-$\mathtt{D}$ and $\mathtt{S}$-$\mathtt{R}_2$-$\mathtt{D}$, respectively. For transmission in *Phase 3*, the informed node, $\mathtt{R}_2$, simply eliminates the interference by DPC. Notice that relay $\mathtt{R}_2$ is aware of the interference that affects the transmission $\mathtt{R}_1$-$\mathtt{D}$ in *Phase 2* as well. Therefore, in *Phase 3*, it can also forward partial information about the interfering sequence (using Wyner-Ziv compression) to the decoder. This way, the decoder can go back to the signal received in *Phase 2* and mitigate such interference. The remaining interference in Phase 2 is treated as noise.

Proposition 3.4: The following rate is achievable for a scheme (A,U,II)

$$R_{(A,U,II)} = \max_{r:\ r\leq\frac{1}{2}C\left(g_2^2P_2\right)} R_1 + R_2 \tag{11}$$

$$\text{where } R_1 \leq \min\left\{\begin{array}{l} \frac{1}{2}C(h_1^2P_S), \\ \frac{1}{2}C\left(\frac{g_1^2P_1}{1+\frac{g_I^2P_I}{1+\left(1+\frac{g_I^2P_I}{1+g_1^2P_1}\right)(2^{4r}-1)}}\right) \end{array}\right. \tag{12}$$

$$R_2 \leq \min\left\{\frac{1}{2}C(h_2^2P_S),\ \frac{1}{2}C\left(g_2^2P_2\right)-r\right\}. \tag{13}$$

Sketch of the proof: Follows from the discussion above. In particular, rate r is devoted for transmission of the compressed interference information relative to Phase 2 from relay $\mathtt{R}_2$ to the destination $\mathtt{D}$. This is done by using Wyner-Ziv coding exploiting the side information available at $\mathtt{D}$ in the form of the signal received over Phase 2. The destination uses this side information to recover interference signal in a lossy manner and then mitigates the interference.

4) Scheme (A,S,II): This scheme is similar to the one studied above. However, here $\mathtt{R}_2$ helps the $\mathtt{R}_1 - \mathtt{D}$ transmission in *Phase 2* by forwarding partial interference information to $\mathtt{D}$ in *Phase 3* in terms of the message $W_{I,2}$ that the interferer encodes in *Phase 2*. This is different from the scheme above in which partial information was provided about the interfering sequence, which was treated as unstructured. In particular, relay $\mathtt{R}_2$ forwards nr bits with $r \leq \frac{R_{I,2}}{2}$ of the overall $\frac{nR_{I,2}}{2}$ bits of message $W_{I,2}$. This reduces the effective rate of the interference to be decoded in *Phase 3* to $\frac{R_{I,2}}{2} - r$. Note that even though this scheme is denoted as structured, $\mathtt{R}_2$ uses the unstructured approach (DPC) to mitigate the interference in *Phase 3*.

Proposition 3.5: The following rate is achievable for a scheme (A,S,II)

$$R_{(A,S,II)} = \max_{r:\ r \leq \min\{\frac{1}{2}C(g_2^2 P_2),\ \frac{R_{I,2}}{2}\}} R_1 + R_2 \tag{14}$$

$$\text{where } R_1 \leq \min \left\{ \begin{array}{l} \frac{1}{2}C(h_1^2 P_S), \\ \frac{1}{2}C(g_1^2 P_1), \\ \left(\frac{1}{2}C(g_1^2 P_1 + g_I^2 P_I) - (\frac{R_{I,2}}{2} - r)\right)^+ \end{array} \right. \tag{15}$$

$$R_2 \leq \min \left\{ \frac{1}{2}C(h_2^2 P_S),\ \frac{1}{2}C\left(g_2^2 P_2\right) - r \right\}. \tag{16}$$

Sketch of the proof: The scheme works similarly to (A,U,II). However, as discussed above, in *Phase 3*, relay $\mathtt{R}_2$ provides nr bits of the message $W_{I,2}$ to $\mathtt{D}$. This reduces the rate of message W_2, conveyed over the path $\mathtt{S}$-$\mathtt{R}_2$-$\mathtt{D}$, to $\frac{1}{2}C\left(g_2^2 P_2\right) - r$, as shown in (16). Moreover, the rate of the interferer that is left to be decoded in *Phase 2* is $(\frac{R_{I,2}}{2} - r)$ as seen in (15).

B. Interference Model B

With the interference model B, the interferer sends the same message over all phases. We derive achievable rates with such model, focusing on structured strategies alone. The reason is that unstructured strategies would perform the same way as for interference model A. In fact, unstructured approaches would not leverage the fact that the same message is sent over all phases and instead treat the interfering sequence as i.i.d.

1) Scheme (B,S,I): We extend (A,S,I) of Proposition 3.3 to Interference Model B.

Proposition 3.6: The rate in Proposition 3.3 is achievable with a scheme (B,S,I) for $h_2^2 \geq h_1^2$ by substituting $\frac{R_{I,4}}{2}$ with $(R_I - \frac{1}{2}C(g_I^2 P_I))^+$.

Sketch of the proof: The scheme works in the same way as (A,S,I). The only difference is that the destination, when jointly decoding W_p, W_c and W_I in *Phase 4*, it uses the information received about the interferer in *Phase 1*, which amounts to mutual information $\frac{1}{2}C(g_I^2 P_I)$. This reduces the effective rates of the interferer to $(R_I - \frac{1}{2}C(g_I^2 P_I))^+$.

Comparing Proposition 3.3 to Proposition 3.6 confirms the discussion at the end of Sec. II. In fact, the relative performance under the two interference models depends on the relationship between the interference rate $\frac{R_{I,4}}{2}$ seen by the destination in model A and the effective rate $(R_I - \frac{1}{2}C(g_I^2 P_I))^+$ seen in model B due to the use of the signal received also during *Phase 1*.

2) Scheme (B,S,II): In this scheme, $\mathtt{R}_2$ follows a fully structured approach and forwards interference as well as information. Since the interfering message is the same over both phases in model B, the decoder can jointly decode the messages sent by the two relays and by the interferer by jointly processing the signals received over *Phases 2* and *3*. Hence, the destination decodes the interference using three different observations: interference signals in *Phase 2* and *3* sent by the interferer and interference signal forwarded by $\mathtt{R}_2$ in *Phase 3*. This is unlike model A, where the destination cannot take advantage of the interference observation in *Phase 3*. Similar to the discussion above for Time Pattern I, depending on the relationship between $R_{I,i}$ in model A and R_I in model B, and the channel parameters, scheme (B,S,II) can perform better or worse than scheme (A,S,II).

Proposition 3.7: The following rate is achievable for a scheme (B,S,II)

$$R_{(B,S,II)} = \max_{\rho:\rho\in[0,1]} R_1 + R_2 \tag{17}$$

$$\text{where } R_1 \leq \min\{\frac{1}{2}C(h_1^2 P_S),\ \frac{1}{2}C(g_1^2 P_1)\} \tag{18}$$

$$R_2 \leq \min \left\{ \begin{array}{l} \frac{1}{2}C(h_2^2 P_S), \\ \frac{1}{2}C((1-\rho^2) g_2^2 P_2), \\ \frac{1}{2}C((1-\rho^2) g_2^2 P_2 + (g_2 \rho \sqrt{P_2} + g_I \sqrt{P_I})^2) \\ + \frac{1}{2}C(g_I^2 P_I) - R_I \end{array} \right. \tag{19}$$

$$\begin{aligned} R_1 + R_2 \leq (&\frac{1}{2}C((1-\rho^2) g_2^2 P_2 + (g_2 \rho \sqrt{P_2} + g_I \sqrt{P_I})^2) \\ &\frac{1}{2}C(g_1^2 P_1 + g_I^2 P_I) - R_I)^+ \end{aligned} \tag{20}$$

Sketch of the proof: As described above, $\mathtt{R}_2$ beamforms with the interference. The decoder performs joint decoding of messages W_1, W_2 and W_I by observing the signals received over both phases. By considering the signals received over the two phases as the output of an appropriate MAC with common message, the constraints (18)-(20) can be obtained.

IV. A Note on Capacity Results

There are relatively few results on the capacity of the diamond relay channel. In [11], a capacity result was derived for the case where the interferer is absent. Specifically, denoting by C_{ij} the inteference-free capacity of the link between terminal i and j, it was shown that, if $C_{SR_1} C_{SR_2} = C_{R_1 D} C_{R_2 D}$, then using only Time Pattern II is optimal. This result is shown by proving that the scheme of Proposition 3.4, in the absence of interference ($P_I = 0$), coincides with the cut-set upper bound. Here we point out that the same result cannot be extended to the scenario at hand where interference is present. In fact, in this case, the cut-set bound may not be achievable even for $C_{SR_1} C_{SR_2} = C_{R_1 D} C_{R_2 D}$. For instance, when considering the cut ($\mathtt{S}$,$\mathtt{R}_1$)-($\mathtt{R}_2$,$\mathtt{D}$), since the interference is known at $\mathtt{R}_2$, the rate across the cut in Phase 3 is upper bounded by $C_{SR_2} + C_{R_1 D}$, but this does not seem to be achievable in the presence of interference unless the interference can be decoded by treating the relay signals as noise (akin to the very strong interference regime in the interference channel). The problem of obtaining capacity results in this scenario, apart from degraded and very strong interference models, is thus still open.

V. Numerical Results

In this section, we numerically evaluate the achievable rates for the all proposed schemes and compare their performance with the following reference schemes. (i) *No interference and Time Pattern I (NI,I)*: We consider an interference-free diamond relay channel [10] [11], that is $X_I = 0$ in (1), and evaluate an achievable rate $R_{(NI,I)}$ for Time Pattern I using Proposition 3.1 with $P_I = 0$ (or Prop. 3.3 with $P_I = 0$ and $R_{I,i} = 0$ for all $i = 1, 2, 3, 4$). (ii) *No interference and Time Pattern II (NI,II)*: For the interference-free diamond relay channel, we evaluate an achievable rate $R_{(NI,II)}$ with Time Pattern II by using Proposition 3.4 with $P_I = 0$.

Remark 5.1: Interference-free rates $R_{(NI,I)}$ and $R_{(NI,II)}$ can also be achieved when the non-causal interference information is available to both relays $\mathtt{R}_2$ and $\mathtt{R}_1$. The achievability of $R_{(NI,II)}$ is simply based on DPC. On the other hand, the achievability for $R_{(NI,I)}$ utilizes multi user version of DPC [16] in Phase 4.

In Fig. 3, the achievable rates for Time Pattern I are illustrated as a function of the interference power P_I when $P_S = 10$, $P_1 = P_2 = 1$, $h_1 = g_1 = h_I = g_I = h_2 = g_2 = 1$ and $R_I = R_{I,i} = 0.5$ for all $i = 1, 2, 3, 4$. Schemes that exploit the interference structure (i.e., (A,S,I) and (B,S,I)) tend to perform better for larger values of the interferer's power P_I since in this regime it becomes easier to decode the interferer's codeword. In fact, for large P_I, it can be seen from Fig. 3, that structured schemes can achieve the upper bound of no interference. Conversely, unstructured schemes (A,U,I/II) tend to perform close to the upper bound for small values of P_I, but have increasing worse performance for larger values of P_I. This is due to the fact that the proposed unstructured schemes inevitably treat at least part of the interference as noise. For time pattern I, the relative performances of the structured schemes depend on the interference power.

In Fig. 4, the achievable rates for Time Pattern II are instead illustrated as a function of the interference rate R_I when $R_{I,i} = R_I$ for all $i = 1, 2, 3, 4$, $P_S = P_I = 10$, $P_1 = P_2 = 1$, $h_1 = g_1 = h_I = g_I = h_2 = g_2 = 1$. Clearly, the performance of unstructured schemes is independent of the rate R_I of the interferer, whereas structured schemes have decreasing performance for increasing R_I, since decoding the interferer's codeword becomes more difficult. In fact, for small values of R_I, structured schemes achieve the no-interference bound. (B,S,II) achieves zero rate for high R_I whereas (A,S,II) is able to maintain a nonzero rate by using DPC over the $\mathtt{R}_2 - \mathtt{D}$ link. Finally, comparing the performance of structured schemes (A,S,II) with (B,S,II), it can be seen that even though the decoder can exploit the fact that the interferer sends the same message over all phases in Interference Model B, the use of DPC in (A,S,II) together with lower effective interference rate to be decoded in model A results in higher rates for scheme (A,S,II).

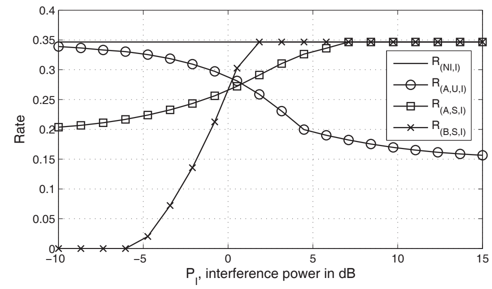

Figure 3. Rates as a function of P_I when $P_S = 10dB$, $P_1 = P_2 = 0dB$, $R_I = R_{I,i} = 0.5$ for all $i = 1, 2, 3, 4$.

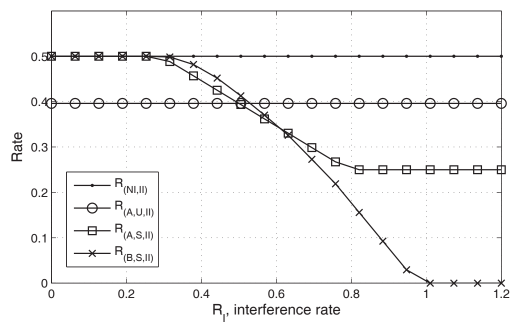

Figure 4. Rates as a function of R_I where $R_{I,i} = R_I$ for all $i = 1, 2, 3, 4$, $P_S = P_I = 10dB$ and $P_1 = P_2 = 0dB$.

VI. Conclusion

This paper studies a diamond relay channel with an external interference where the interference is non-causally available to the one of the relays. Two interference models are proposed, interference model A and B that complement time patterns used for diamond relay channel. Several achievable schemes are proposed for the two interference models and two time patterns based on three interference mitigation techniques: interference precoding, cancelation and forwarding. Performances of the proposed schemes are numerically evaluated and compared. Numerical results reveal that even when one relay is aware of the interfering signal, this knowledge along with the structure of the interference can be crucial in mitigating interference.

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