Achievable Rates for Multicell Systems with Femtocells and Network MIMO

O. Simeone
CWCSPR, ECE Dept.,
NJIT, Newark, USA

E. Erkip
Dept. of ECE, Polytechnic Inst. of NYU
Brooklyn, NY, USA

S. Shamai (Shitz)
Dept. of EE, Technion
Haifa, Israel

***Abstract*—[1]The uplink of a cellular system where macrocells are overlaid with femtocells is studied. Each femtocell is served by a home base station (HBS) that is connected to the macrocell base station (BS) via a last-mile access link, such as DSL or cable followed by the Internet. Decoding at the BSs takes place via either standard single-cell processing or multicell processing (i.e., network MIMO). Closed and open-access femtocells are considered. Achievable per-cell sum-rates are derived in this setting for a linear cellular network. Overall, the analysis lends evidence to the performance advantages of open-access femtocells and sheds light on the performance trade-offs between single/multi-cell processing and different relaying strategies at the femtocells.**

I. INTRODUCTION

With the recent advances in coding and multiantenna technology, interference is becoming the performance-limiting factor in terms of area and spectral efficiency of cellular systems. To cope with interference, two diametrically opposite strategies are currently being investigated. On one end, *femtocells* reduce the size of a cell to contain only the customer's premises, thus allowing transmission with smaller powers and the possibility to reuse the spectrum more aggressively [1]. On the other end, *network MIMO* or *multicell processing* (MCP) [2][3] creates clusters of macrocells for joint coding/ decoding in order to better manage *inter-cell interference*.

A femtocell consists of a short-range low-cost *home base station (HBS)*, installed within the customer's premises, that serves either only indoor users, in case of *closed-access* femtocells, or possibly also outdoor users that are within the HBS coverage range, in case of *open-access* femtocells. Femtocells in open-access mode provide an asset that the network designer can exploit to manage the interference created by outdoor users towards the femtocell and other macrocells. In this work, we provide an information-theoretic look at the performance trade-offs between open and closed-access femtocells, on the one hand, and the deployment of femtocells and MCP, on the other. Analysis is performed by resorting to a simple cellular model that extends [2] and by deriving achievable rates that are then compared via numerical results.

Notation: We define $\mathcal{C}(\mathbf{A}) = 1/2\log_2\det(\mathbf{I}+\mathbf{A})$ for a positive definite $\mathbf{A}$; Notation $[1,N]$ represents the set of numbers $\{1,...,N\}$.

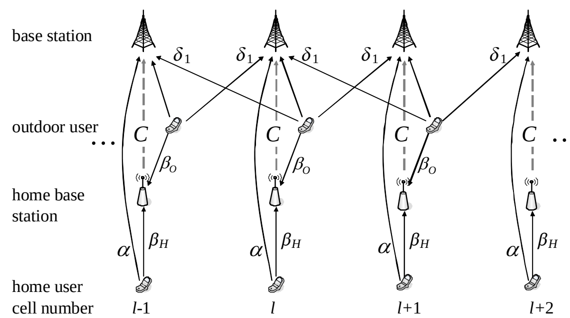

Fig. 1. A linear multicell system where each macrocell is overlaid with a femtocell. Each HBS is connected to the local BS via a last-mile link of capacity C ($L = 1$ in the figure).

II. SYSTEM MODEL

Consider a linear cellular system similar to [2], where M cells are arranged on a line, as for a corridor or a highway, as shown in Fig. 1. Each cell, served by a base station (BS), contains a single femtocell, served by a HBS, and presents the same number of outdoor (i.e., outside the femtocell) and home (i.e., within the femtocell) users. Assuming that the channel gains are the same for different home/outdoor users in the same cell, and focusing the analysis on achievable sum-rates, we can concentrate without loss of generality on a single outdoor and home user per cell, as shown in Fig. 1 [5].

Signals generated within each femtocell are received with relevant power only by the local BS with power gain α and the local HBS with power gain β_H, while outdoor users are received not only by the local BS and HBS (with power gains $\delta_0 = 1$ and β_O, respectively), but also by L adjacent BSs on either side with symmetric power gains $\delta_l,\ l \in [1,L]$. Given the above, the received signals at a given time instant for the BS and home BS in the lth cell can be expressed as, respectively

$$Y_l = \sum_{i=-L}^{L} \sqrt{\delta_i} X_{O,[l+i]} + \sqrt{\alpha} X_{H,l} + N_{Y,l} \qquad (1a)$$

$$\text{and } Z_l = \sqrt{\beta_O} X_{O,l} + \sqrt{\beta_H} X_{H,l} + N_{Z,l}, \qquad (1b)$$

[1]The work of O. Simeone was supported by U.S. NSF grant CCF-0905446. The work of E. Erkip by U.S. NSF grant CCF-0914899. The work of S. Shamai was supported by the European Commision in the framework of the FP7 Network of Excellence in Wireless COMmunications, NEWCOM++.

where $X_{O,l}$ and $X_{H,l}$ are the signals transmitted by the outdoor ("O") and home ("H") user in the lth cell, and $(N_{Y,l}, N_{Z,l})$ are independent Gaussian noise processes with unit-power. Power constraints for outdoor and home users are defined as $P_O, P_{H.}$, respectively. Moreover, to avoid edge effects, in (1), we have assumed that inter-cell interference affects cells in a circulant fashion, so that every cell is impaired by the same number of interferers (we have defined $[l+i]$ as the modulo-M operation and assumed $M \geq 2L+1$).

Finally, the HBS is assumed to be connected to the corresponding BS via a last-mile connection (such as DSL or cable) followed by the Internet, whose overall capacity is C bits/ dim[2]. This link is wired and orthogonal to the wireless channels [1]. The scenario at hand can be seen as an extension of the model in [2] to include femtocells and is related to the models in [6] and references therein for mesh networks.

We consider two alternatives for decoding at the BSs: (*i*) *Single-cell Processing* (SCP): The BS in each cell decodes independently; (*ii*) *Multicell Processing* (MCP): All BSs in the system are connected to a central processor (CP) for joint decoding. The CP collects the signals of all BSs and jointly decodes all the M outdoor and M home messages jointly. Furthermore, for both SCP and MCP, we will study the performance of closed-access (CA) and open-access (OA) femtocells. CA femtocells treat the signal of the outdoor user as interference, whereas OA femtocells may serve as relays towards the BS for the outdoor users. The aim is to identify pairs of home user and outdoor users rates R_H and R_O, respectively, that are achievable in each cell according to the usual definitions.

III. Single-Cell processing (SCP)

In this section, we study achievable rate pairs (R_H,R_O) with SCP and OA or CA femtocells.

A. Closed-Access Femtocells

We start with CA femtocells.

Proposition 1 (CA,SCP): Rates satisfying the following conditions

$$\begin{aligned} R_H &< \min\left\{\mathcal{C}\left(\frac{\beta_H P_H}{1+\beta_O P_O}\right),\ \mathcal{C}\left(\frac{\alpha P_H}{1+\Delta P_O}\right)+C\right\} \\ R_O &< \mathcal{C}\left(\frac{P_O}{1+\Delta P_O}\right) \\ R_O+R_H &< \mathcal{C}\left(\frac{P_O+\alpha P_H}{1+\Delta P_O}\right)+C, \end{aligned}$$

are achievable with SCP and CA femtocells, where with $\Delta = 2\sum_{l=1}^{L}\delta_l$.

Proof (sketch): The HBS decodes the home user's message by treating the outdoor user as noise (of power $\beta_O P_O$). Having decoded, the HBS provides C bits/dim of the decoded message to the BS. The BS performs joint decoding of home and outdoor users' messages by treating inter-cell signals as noise (of power ΔP_O). In this process, thanks to the C bits received from the HBS, the equivalent rate of the home user to be decoded by the BS is reduced to $R_H - C$ (see, e.g., [4]). The proof is completed using standard arguments.

[2]We measures the rates in bits per (real) dimension (dim).

B. Open-Access Femtocells

Turning to OA femtocells, we consider two classes of strategies. In the first, the HBS decodes both home and outdoor users' messages and then shares the last-mile link capacity C for transmission of bits from both messages (Decode-and-Forward, DF). In the second, the HBS simply compresses and forwards (CF) the received signal. It is noted that the latter scheme does not require codebook information at the HBS and thus reduces the signaling overhead.

1) Decode-and-Forward: Proposition 2 (OA-DF,SCP): The convex hull of the union of the rates that satisfy

$$\begin{aligned} R_H &< \min\left\{\mathcal{C}\left(\beta_H P_H\right),\ \mathcal{C}\left(\frac{\alpha P_H}{1+\Delta P_O}\right)+\gamma C\right\} \\ R_O &< \min\{\mathcal{C}\left(\beta_O P_O\right),\ \mathcal{C}\left(\frac{P_O}{1+\Delta P_O}\right)+(1-\gamma)C\} \\ R_O+R_H &< \min\{\mathcal{C}\left(\beta_H P_H+\beta_O P_O\right), \\ &\qquad \mathcal{C}\left(\frac{\alpha P_H+P_O}{1+\Delta P_O}\right)+C\} \end{aligned}$$

for some $0 \leq \gamma \leq 1$ is achievable with SCP and OA femtocells employing DF relaying.

Proof (sketch): The HBS decodes both home and outdoor users' messages and then sends γC bits/dim of the decoded home message and $(1-\gamma)C$ bits/dim of the decoded outdoor message to the BS. The BS performs joint decoding as discussed for Proposition 1, but on codebooks of equivalent rates $R_H - \gamma C$ and $R_O - (1-\gamma)C$.

2) Compress-and-Forward: Proposition 3 (OA-CF,SCP): Rates satisfying the following conditions

$$\begin{aligned} R_H &< \mathcal{C}\left(\frac{\alpha P_H}{1+\Delta P_O}+\frac{\beta_H P_H}{1+\sigma^2}\right) \\ R_O &< \mathcal{C}\left(\frac{P_O}{1+\Delta P_O}+\frac{\beta_O P_O}{1+\sigma^2}\right) \\ R_O+R_H &< \mathcal{C}\left(\mathbf{A}\right) \end{aligned}$$

with

$$\mathbf{A} = \begin{bmatrix} \frac{P_O+\alpha P_H}{1+\Delta P_O} & \frac{\sqrt{\beta_O}P_O+\sqrt{\alpha\beta_H}P_H}{\sqrt{(1+\Delta P_O)(1+\sigma^2)}} \\ \frac{\sqrt{\beta_O}P_O+\sqrt{\alpha\beta_H}P_H}{\sqrt{(1+\Delta P_O)(1+\sigma^2)}} & \frac{\beta_H P_H+\beta_O P_O}{1+\sigma^2} \end{bmatrix}$$

are achievable with SCP and OA femtocells employing CF relaying, where

$$\sigma^2 = \frac{\left[1+\beta_O P_O+\beta_H P_H-\frac{(\sqrt{\beta_O}P_O+\sqrt{\alpha\beta_H}P_H)^2}{P_O+\alpha P_H+\Delta P_O}\right]}{2^{2C}-1}. \quad (2)$$

Proof (sketch): The HBS compresses the received signal to a description $\hat{Z}_l$ of C bits/dim using Wyner-Ziv quantization, exploiting the fact that the BS has side information Y_l. The compression noise (2) is found by imposing $I(Z_l;\hat{Z}_l|Y_l) = C$ following standard arguments (see, e.g., [7]). The lth BS performs joint decoding based on the signals $(Y_l, \hat{Z}_l)$.

IV. Multicell Processing (MCP)

In this section, we address achievable rates in the presence of MCP. We recall that, with MCP, decoding is performed at a CP connected via ideal links to all BSs. For notational convenience, we define the channel matrix $\mathbf{H}$ between outdoor users and the M BSs as the $M \times M$ circulant matrix whose first column is given by

$$[\sqrt{\delta_0}\sqrt{\delta_1}\cdots\sqrt{\delta_{L_C}}\mathbf{0}_{L-(2L_C+1)}\sqrt{\delta_{L_C}}\sqrt{\delta_{L_C-1}}\cdots\sqrt{\delta_1}].$$

We also denote the eigenvalues of $\mathbf{HH}^T$ as $\lambda_l = \left(1+2\sum_{l=1}^{L_C}\sqrt{\delta_l}\cos\left(\frac{2\pi}{L}l\right)\right)^2$, $l \in [0, M-1]$.

A. Closed Access

Proposition 4 (CA,MCP): Rates satisfying the following conditions

$$\begin{aligned} R_H &< \min\left\{\mathcal{C}\left(\frac{\beta_H P_H}{1+\beta_O P_O}\right),\ \mathcal{C}\left(\alpha P_H\right)+C\right\} \\ R_O &< \frac{1}{L}\sum_{l=0}^{L=1}\mathcal{C}\left(\lambda_l P_O\right) \\ R_O+R_H &< \frac{1}{L}\sum_{l=0}^{L=1}\mathcal{C}\left(\lambda_l P_O+\alpha P_H\right)+C \end{aligned}$$

are achievable with MCP and CA femtocells.

Proof (sketch): The HBS operates as for Proposition 1. The CP decodes jointly all the home and outdoor users' messages based on the signals Y_l, $l \in [1, M]$ and the MC bits/dim received from the HBSs. The equivalent rate of the home users to be decoded is $R_H - C$ due to the bits received from the HBS, as, e.g., for Proposition 1.

B. Open Access

Turning to OA femtocells, as for SCP, we study both DF and CF strategies.

1) Decode-and-Forward: Proposition 5 (OA-DF,MCP): The convex hull of the union of the rates that satisfy

$$\begin{aligned} R_H &< \min\left\{\mathcal{C}\left(\beta_H P_H\right),\ \mathcal{C}\left(\alpha P_H\right)+\gamma C\right\} \\ R_O &< \min\left\{\begin{array}{l}\mathcal{C}\left(\beta_O P_O\right), \\ \frac{1}{L}\sum_{l=0}^{L=1}\mathcal{C}\left(\lambda_l P_O\right)+(1-\gamma)C\end{array}\right\} \\ R_O+R_H &< \min\left\{\begin{array}{l}\mathcal{C}\left(\beta_H P_H+\beta_O P_O\right), \\ \frac{1}{L}\sum_{l=0}^{L=1}\mathcal{C}\left(\lambda_l P_O+\alpha P_H\right)+C\end{array}\right\} \end{aligned}$$

for some $0 \le \gamma \le 1$ is achievable with MCP and OA femtocells employing DF relaying.

Proof (sketch): The HBS operates as for Proposition 2 and the CP performs joint decoding as for Proposition 4.

2) Compress-and-Forward: Proposition 6 (OA-CF,MCP): Rates satisfying the following conditions

$$\begin{aligned} R_H &< \mathcal{C}\left(\alpha P_H+\frac{\beta_H P_H}{1+\sigma^2}\right) \\ R_O &< \frac{1}{L}\sum_{l=0}^{L=1}\mathcal{C}\left(\lambda_l P_O+\frac{\beta_O P_O}{1+\sigma^2}\right) \\ R_O+R_H &< \frac{1}{L}\mathcal{C}\left(\mathbf{B}\right) \end{aligned}$$

with (2) and

$$\mathbf{B} = \begin{bmatrix} P_O\mathbf{HH}^T+\alpha P_H\mathbf{I} & \frac{\sqrt{\beta_O}P_O\mathbf{H}+\sqrt{\alpha\beta_H}P_H\mathbf{I}}{\sqrt{1+\sigma^2}} \\ \frac{\sqrt{\beta_O}P_O\mathbf{H}^T+\sqrt{\alpha\beta_H}P_H\mathbf{I}}{\sqrt{1+\sigma^2}} & \left(\frac{\beta_O P_O}{1+\sigma^2}+\frac{\beta_H P_H}{1+\sigma^2}\right)\mathbf{I} \end{bmatrix}$$

are achievable with MCP and OA femtocells employing CF relaying.

Proof (sketch): The HBS operates as for Proposition 3 and the CP decodes jointly all messages based on the signals $(Y_l, \hat{Z}_l)$, $l \in [1, M]$. It is noted that using σ^2 in (2) implies that decompression of $\hat{Z}_l$ is performed at the lth BS. However, with MCP, one could potentially improve the performance by moving decompression from the BSs to the CP, which has better side information (namely, all Y_l with $l \in [1, M]$). We do not pursue this further here.

V. Numerical Results

In this section, we provide some insight into the performance comparison of different scenarios and strategies through numerical results. Throughout, we set parameters $P_O = P_H = 4$, $\beta_H = 20dB$ and $\alpha = -10dB$, which implies that the indoor channel gain between home user and HBS is 30dB better than the channel home user-BS [1], $M = 30$, $L = 1$. We focus on maximum achievable equal rates $R_H = R_O$ for the different considered techniques.

We start by concentrating on the performance comparison between CA and OA femtocells, by varying the outdoor-HBS power gain β_O with fixed $\delta_1 = 0.4$ and $C = 1.5$. Fig. 2 shows that CA femtocells, due to the macro-to-femto interference, are largely outperformed by OA techniques for increasing β_O. More specifically, OA-DF becomes advantageous over CA for sufficiently large β_O, while OA-CF, for the range of β_O shown in the figure, performs always at least as well as CA. As for the comparison between OA-CF and OA-DF, on the one hand, OA-CF has the advantage of enabling joint decoding at the receiver (BS for SCP or CP for MCP), while having the drawback of adding extra noise via compression. On the other hand, OA-DF has the advantage of providing "clean" information bits to the receiver, at the cost of causing a potential performance bottleneck at the home BS for decoding. This trade-off is clear from Fig. 2: Whenever decoding at the HBS does not set the performance bottleneck (i.e., for β_O large enough), OA-DF outperforms OA-CF, while the opposite is true when β_O is small so that decoding of the outdoor users at the home BS limits the performance of OA-DF[3].

We further discuss the comparison between the performance of MCP and SCP in Fig. 3 for $\beta_O = 10$, and varying inter-cell interference power gain δ_1. It can be seen that as the inter-cell interference δ_1 increases, the advantages of MCP become more pronounced for all techniques. It is also noted, similar to the example above, that CF appears to be performing better when deployed with MCP than with SCP. This is further discussed below.

[3]For β_O larger than β_H (not shown in the figure given the minor relevance of this regime), the performance of CF keeps degrading as β_O increases due to the larger quantization noise, down to the performance attainable with $C = 0$.

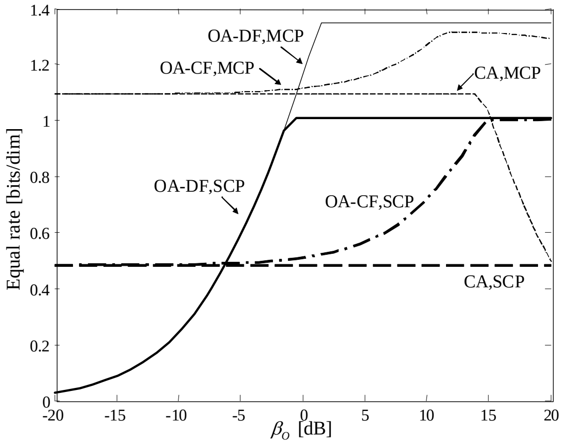

Fig. 2. Equal achievable rate $R_H = R_O$ versus the outdoor-HBS power gain β_O ($\delta_1 = 0.4$, $C = 1.5$, $P_O = P_H = 4$, $\beta_H = 20dB$, $\alpha = -10dB$, $M = 30$, $L = 1$).

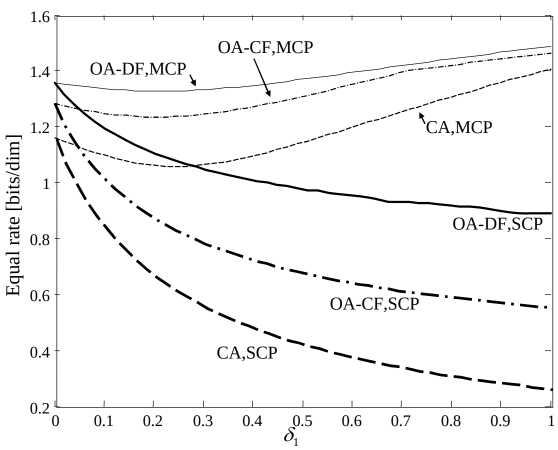

Fig. 3. Equal achievable rate $R_H = R_O$ versus the inter-cell interference power gain δ_1 ($\beta_O = 10$, $C = 1.5$, $P_O = P_H = 4$, $\beta_H = 20dB$, $\alpha = -10dB$, $M = 30$, $L = 1$).

Fig. 4 shows the maximum equal rate of different techniques versus the last-mile link capacity C for $\delta_1 = 0.5$ and $\beta_O = -3dB$. It is seen, following the discussion above, that, if C is small, OA-DF is appropriate since the performance is limited by decoding at the BS. However, as C increases, the equal rate achievable by OA-DF saturates to the maximum equal rate decodable at the HBS (which is the same for both SCP and MCP), while OA-CF does not suffer from such saturation and keeps exploiting larger values of C to improve the quality of the compressed signal provided to the receiver. It is also noted that with MCP the crossing point between the performance of OA-DF and OA-CF occurs for smaller values of C than SCP, due to the greater decoding power at the CP with respect to the single BS.

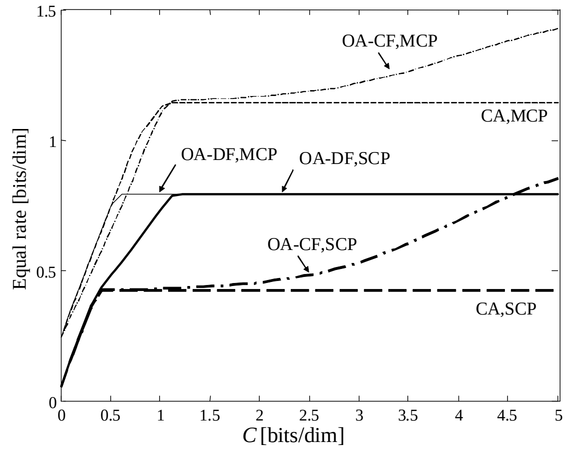

Fig. 4. Equal achievable rate $R_H = R_O$ versus the last-mile HBS-BS link capacity C ($\delta_1 = 0.5$, $P_O = P_H = 4$, $\beta_O = -3dB$, $\beta_H = 20dB$, $\alpha = -10dB$, $M = 30$, $L = 1$).

VI. Concluding Remarks

While network MIMO and femtocells are being mostly developed and studied in separation, this paper has argued for a joint analysis, given the interplay between the two technologies. An important observation is that femtocells, when allowed to work in an open-access mode, have a potentially relevant role for interference management, since they can exploit their dedicated (wired) connection to the BS to reduce radio interference by serving also outdoor users. However, the relaying strategy must be carefully designed according to whether decoding at the BSs implements network MIMO or not, in order not to create performance bottlenecks. This increased interference margin may be dually turned into a corresponding reduction in power emissions, thus moving towards "greener" wireless communications.

References

[1] V. Chandrasekhar, J. G. Andrews and A. Gatherer, “Femtocell networks: a survey,” *IEEE Comm. Magazine*, vol. 46, no. 9, pp. 59-67, Sept. 2008.

[2] A. D. Wyner, “Shannon-theoretic approach to a Gaussian cellular multiple-access channel,” *IEEE Trans. Inform. Theory*, vol. 40, no. 6, pp. 1713-1727, Nov. 1994.

[3] M. K. Karakayali, G.J. Foschini and R.A. Valenzuela, "Network coordination for spectrally efficient communications in cellular systems," *IEEE Wireless Communications*, vol. 13, no. 4, pp. 56-61, Aug. 2006.

[4] Y.-H. Kim, “Coding techniques for primitive relay channels,” in *Proc. Forty-Fifth Annual Allerton Conf. Commun., Contr. Comput.*, Monticello, IL, Sep. 2007.

[5] O. Simeone, E. Erkip and S. Shamai (Shitz), "Robust transmission and interference management for femtocells with unreliable network access," submitted.

[6] O. Somekh, O. Simeone, H. V. Poor, and S. Shamai (Shitz), “Cellular Systems with Full-Duplex Compress-and-Forward Relaying and Cooperative Base-Stations,” in *Proc. IEEE International Symposium Inform. Theory (ISIT 2008)*, Toronto, Canada, July 6-11, 2008.

[7] G. Kramer, *Topics in Multi-User Information Theory*, Foundations and Trends in Communications and Information Theory, vol. 4, no. 4-5, pp. 265-444, 2007.