Relaying Simultaneous Multicast Messages

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***Abstract*—The problem of multicasting multiple messages with the help of a relay, which may also have an independent message of its own to multicast, is considered. As a first step to address this general model, referred to as the compound multiple access channel with a relay (cMACr), the capacity region of the multiple access channel with a "cognitive" relay is characterized, including the cases of partial and rate-limited cognition. Achievable rate regions for the cMACr model are then presented based on decode-and-forward (DF) and compress-and-forward (CF) relaying strategies. Moreover, an outer bound is derived for the special case in which each transmitter has a direct link to one of the receivers while the connection to the other receiver is enabled only through the relay terminal. Numerical results for the Gaussian channel are also provided.**

I. Introduction

Consider two non-cooperating satellites each multicasting radio/TV signals to users on Earth. The coverage area and the quality of the transmission is limited by the strength of the direct links from the satellites to the users. To extend coverage, to increase capacity or to improve robustness, a standard solution is that of introducing relay terminals, which may be other satellite stations or stronger ground stations. The role of the relay terminals is especially critical for users that lack a direct link from any of the satellites.

Cooperative transmission has been extensively studied both for a single user with a dedicated relay terminal [1], [2] and for two cooperating users [3]. In this work, we study cooperation in a model with two source terminals simultaneously multicasting independent information to two receivers with the help of a relay. While the source terminals cannot directly cooperate, the relay can support both transmissions simultaneously to enlarge the multicast capacity region. Moreover, it is assumed that the relay station has also its own message to be multicast.

The model under study is a *compound multiple access channel with a relay* (cMACr) and can be seen as an extension of several channel models, for example, the compound multiple access channel (MAC), the broadcast channel and the relay channel. The main goal of this work is to provide achievable rate regions and an outer bound on the capacity region for this model. We start our analysis by studying a simplified version of the cMACr that consists of a MAC with a "cognitive" relay. In this scenario the cognitive relay is assumed to know both messages non-causally. We provide the capacity region for this model. As an intermediate step between the cognitive relay model and cMACr, we also consider the relay with finite capacity unidirectional links from the transmitters and find its capacity region. In this scenario, parts of the messages are transmitted to the relay over the finite capacity links and the rates of these links determine how much the relay can help each user. This is not the case in the general cMACr model since decoding at the relay might be restrictive, yet we can use these techniques to obtain achievable rate regions.

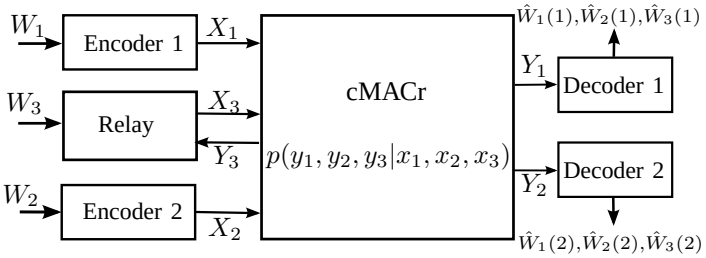

Fig. 1. A compound MAC with a relay (cMACr).

We provide achievable rate regions for cMACr with decode-and-forward (DF) and compress-and-forward (CF) relaying. In the CF scheme, the relay, instead of decoding the messages, quantizes and broadcasts its received signal. This corresponds to the joint source-channel coding problem of broadcasting a common source to two receivers, each with its own correlated side information, in a lossy fashion, studied in [8]. This result indicates that the pure channel coding rate regions for certain multi-user networks can be improved by exploiting related joint source-channel coding techniques. The cMACr model is also studied in [5], where DF and amplify-and-forward (AF) based protocols are analyzed, assuming that no private relay message is available.

The rest of the paper is organized as follows. The system model is introduced in Section II. In Section III we study several cognitive relay models. The compound multiple access channel with a relay is studied in Section IV. In Section V numerical results for the Gaussian channel setup are presented. Section VI concludes the paper.

II. System Model

A compound multiple access channel with relay consists of three input alphabets $\mathcal{X}_1$, $\mathcal{X}_2$ and $\mathcal{X}_3$ of transmitter 1, transmit-

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ter 2 and the relay, respectively, and three output alphabets $\mathcal{Y}_1$, $\mathcal{Y}_2$ and $\mathcal{Y}_3$ of receiver 1, receiver 2 and the relay, respectively. We consider a discrete memoryless time-invariant channel without feedback characterized by $p(y_1, y_2, y_3|x_1, x_2, x_3)$ (see Fig. 1). Transmitter i has message $W_i \in \mathcal{W}_i$, $i = 1, 2$, while the relay terminal also has a message $W_3 \in \mathcal{W}_3$ of its own, all of which need to be transmitted reliably to both receivers.

Definition 2.1: A $(2^{nR_1}, 2^{nR_2}, 2^{nR_3}, n)$ code for the cMACr consists of three sets $\mathcal{W}_i = \{1, \ldots, 2^{nR_i}\}$ for $i = 1, 2, 3$, two encoding functions f_i at the transmitters, $i = 1, 2$, $f_i : \mathcal{W}_i \to \mathcal{X}_i^n$, a set of (causal) encoding functions g_j at the relay, $j = 1, \ldots, n$, $g_j : \mathcal{W}_3 \times \mathcal{Y}_3^{j-1} \to \mathcal{X}_3$, and two decoding functions h_i at the receivers, $i = 1, 2$, $h_i : \mathcal{Y}_i^n \to \mathcal{W}_1 \times \mathcal{W}_2 \times \mathcal{W}_3$.

We assume that the relay terminal is capable of full-duplex operation, i.e., it can receive and transmit at the same time instant. The average error probability is defined as

$$P_e^n \triangleq \frac{1}{2^{n(R_1+R_2+R_3)}} \sum_{\mathbf{W}} \Pr\left[\bigcup_{i=1,2} \{\hat{\mathbf{W}}(i) \neq \mathbf{W}\}\right],$$

where we defined $\mathbf{W} \triangleq (W_1, W_2, W_3)$ and $\hat{\mathbf{W}}(i) \triangleq (\hat{W}_1(i), \hat{W}_2(i), \hat{W}_3(i))$.

Definition 2.2: A rate triplet (R_1, R_2, R_3) is said to be *achievable* for the cMACr if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, 2^{nR_3}, n)$ codes with $P_e^n \to 0$ as $n \to \infty$.

Definition 2.3: The *capacity region* $\mathcal{C}$ for the cMACr is the closure of the set of all achievable rate triplets.

III. MAC with a Cognitive Relay

As stated in Section I, before addressing the general cMACr model we first study the MAC with a cognitive relay scenario in which the messages W_1 and W_2 are assumed to be non-causally available at the relay terminal (in a "cognitive" fashion [6]) and there is only one receiver ($\mathcal{Y}_2 = \mathcal{Y}_3 = \emptyset$ and $\mathcal{Y} = \mathcal{Y}_1$). The next proposition provides the capacity region for this model.

Proposition 3.1: For the MAC with a cognitive relay, the capacity region is the closure of the set of all non-negative (R_1, R_2, R_3) satisfying

$$R_3 \leq I(X_3; Y|X_1, X_2, U_1, U_2, Q), \quad (1a)$$
$$R_1 + R_3 \leq I(X_1, X_3; Y|X_2, U_2, Q), \quad (1b)$$
$$R_2 + R_3 \leq I(X_2, X_3; Y|X_1, U_1, Q), \text{ and} \quad (1c)$$
$$R_1 + R_2 + R_3 \leq I(X_1, X_2, X_3; Y|Q) \quad (1d)$$

for some joint distribution of the form

$$p(q)p(x_1, u_1|q)p(x_2, u_2|q)p(x_3|u_1, u_2, q)p(y|x_1, x_2, x_3) \quad (2)$$

for some auxiliary random variables U_1, U_2 and Q.

Proof: The capacity region of a MAC with three users and any combination of "common messages" (i.e., messages known "cognitively" to more than one user) is given in [7]. ■

We next consider the cases of partial and limited-rate cognition.

Proposition 3.2: The capacity region of the MAC with a partially cognitive relay (informed only of W_1) is the closure of the set of all non-negative (R_1, R_2, R_3) satisfying

$$R_2 \leq I(X_2; Y|X_1, X_3, Q), \quad (3a)$$
$$R_3 \leq I(X_3; Y|X_1, X_2, Q), \quad (3b)$$
$$R_1 + R_3 \leq I(X_1, X_3; Y|X_2, Q), \quad (3c)$$
$$R_2 + R_3 \leq I(X_2, X_3; Y|X_1, Q), \text{ and} \quad (3d)$$
$$R_1 + R_2 + R_3 \leq I(X_1, X_2, X_3; Y|Q). \quad (3e)$$

for an input distribution of the form $p(q)p(x_2|q)p(x_1, x_3|q)$.

Proof: Proof can be found in [9]. ■

Remark 3.1: The capacity region in Proposition 3.2 follows, like Proposition 3.1, from the general result in [7]. However, the formulation given in (3) is more convenient than the one obtained from [7], since, in the case of partial cognition, the capacity region characterization does not require auxiliary random variables in addition to the time-sharing random variable Q. This is because, unlike the scenario covered by Proposition 3.1, in which the relay's codeword can depend on both W_1 and W_2, and the auxiliary random variables quantify the amount of dependence on each message, for Proposition 3.2, the relay cooperates with only one source, and no auxiliary random variable is needed.

The MAC with a cognitive relay model can be further generalized to a scenario with *limited-capacity cognition*, in which the sources are connected to the relay via finite-capacity orthogonal links, rather than having a priori knowledge of the terminals' messages. In particular, assume that terminal i can communicate with the relay, prior to transmission, via a link of capacity C_i for $i = 1, 2$. The following proposition establishes the capacity region of this model.

Proposition 3.3: The capacity region of the MAC with a cognitive relay connected to the source terminals via (unidirectional) links of capacities C_1 and C_2 is given by

$$\begin{aligned}
R_1 &\leq I(X_1; Y|X_2, X_3, U_1, U_2, Q) + C_1,\\
R_2 &\leq I(X_2; Y|X_1, X_3, U_1, U_2, Q) + C_2,\\
R_3 &\leq I(X_3; Y|X_1, X_2, U_1, U_2, Q),\\
R_1 + R_2 &\leq I(X_1, X_2; Y|X_3, U_1, U_2, Q) + C_1 + C_2,\\
R_1 + R_3 &\leq \min\left\{\begin{array}{l} I(X_1, X_3; Y|X_2, U_1, U_2, Q) + C_1,\\ I(X_1, X_3; Y|X_2, U_2, Q)\end{array}\right.\\
R_2 + R_3 &\leq \min\left\{\begin{array}{l} I(X_2, X_3; Y|X_1, U_1, U_2, Q) + C_2\\ I(X_2, X_3; Y|X_1, U_1, Q)\end{array}\right.\\
R_1 + R_2 + R_3 &\leq \min\{I(X_1, X_2, X_3; Y|U_1, Q) + C_1,\\
&\qquad I(X_1, X_2, X_3; Y|U_2, Q) + C_2, I(X_1, X_2, X_3; Y|Q),\\
&\qquad I(X_1, X_2, X_3; Y|U_1, U_2, Q) + C_1 + C_2\}
\end{aligned}$$

for auxiliary random variables U_1, U_2 and Q with joint distribution of the form (2).

Proof: Proof can be found in [9]. ■

Remark 3.2: Based on the results of this section, we can now take a further step towards the analysis of the cMACr of Fig. 1 by considering the *compound MAC with a cognitive relay*. This channel is given as in Fig. 1 with

the only difference that the relay here is informed "for free" of the messages W_1 and W_2 and that the signal received at the relay is non-informative, e.g., $\mathcal{Y}_2 = \emptyset$. The capacity of such a channel follows easily from Proposition 3.1 by taking the union over the distribution $p(q)p(x_1,u_1|q)p(x_2,u_2|q)p(x_3|u_1,u_2,q)p(y_1,y_2|x_1,x_2,x_3)$ of the intersection of the two rate regions (1) evaluated for the two outputs Y_1 and Y_2. Notice that this capacity region depends on the channel inputs only through the marginal distributions $p(y_1|x_1,x_2,x_3)$ and $p(y_2|x_1,x_2,x_3)$.

IV. Inner and Outer bounds on the Capacity Region of the Compound MAC with a Relay

In this section, we focus on the general cMACr model illustrated in Fig. 1. Single-letter characterization of the capacity region for this model is open even for various special cases. Our goal here is to provide inner and outer bounds.

The following inner bound is obtained by the DF scheme. The relay fully decodes messages of both users so that we have a MAC from the transmitters to the relay terminal. Once the relay has decoded the messages, the transmission to the receivers takes place similarly to the MAC with a cognitive relay model of Section III.

Proposition 4.1: For the cMACr as seen in Fig. 1, any rate triplet (R_1,R_2,R_3) with $R_j \geq 0$, $j=1,2,3$, satisfying

$$R_1 \leq I(X_1;Y_3|U_1,X_2,X_3,Q), \tag{5a}$$
$$R_2 \leq I(X_2;Y_3|U_2,X_1,X_3,Q), \tag{5b}$$
$$R_1+R_2 \leq I(X_1,X_2;Y_3|U_1,U_2,X_3,Q), \tag{5c}$$
$$R_3 \leq \min\{I(X_3;Y_1|X_1,X_2,U_1,U_2,Q), I(X_3;Y_2|X_1,X_2,U_1,U_2,Q)\}, \tag{5d}$$
$$R_1+R_3 \leq \min\{I(X_1,X_3;Y_1|X_2,U_2,Q), I(X_1,X_3;Y_2|X_2,U_2,Q)\}, \tag{5e}$$
$$R_2+R_3 \leq \min\{I(X_2,X_3;Y_1|X_1,U_1,Q), I(X_2,X_3;Y_2|X_1,U_1,Q)\} \text{ and} \tag{5f}$$
$$R_1+R_2+R_3 \leq \min\{I(X_1,X_2,X_3;Y_1|Q), I(X_1,X_2,X_3;Y_2|Q)\} \tag{5g}$$

for auxiliary random variables U_1,U_2 and Q with a joint distribution of the form $p(q)p(x_1,u_1|q)p(x_2,u_2|q)p(x_3|u_1,u_2,q)$ $p(y_1,y_2,y_3|x_1,x_2,x_3)$ is achievable by DF.

Proof: The proof follows by combining the block-Markov transmission strategy with DF at the relay studied in [2], the joint encoding to handle the private relay message and backward decoding at the receivers. Notice that conditions (5a)-(5c) ensure correct decoding at the relay, whereas (5d)-(5g) follow similarly to Proposition 3.1 and Remark 3.2 ensuring correct decoding at both receivers. ■

Next, we consider applying the CF strategy at the relay terminal. In CF relaying introduced in [1], the relay does not decode the source message, but facilitates decoding at the destination by transmitting a quantized version of its received signal. In quantizing its received signal, the relay takes into consideration the correlated received signal at the destination terminal and applies Wyner-Ziv source compression (see [1] for details). In the cMACr scenario, unlike the single-user relay channel, we have two distinct destinations, each with different side information correlated with the relay received signal. This situation is similar to the problem of lossy broadcasting of a common source to two receivers with different side information sequences considered in [8] (and solved in some special cases), and applied to the two-way relay channel setup in [4]. Here, for simplicity, we consider broadcasting only a single quantized version of the relay received signal to both receivers. The following proposition states the corresponding achievable rate region.

Proposition 4.2: For the cMACr of Fig. 1, any rate triplet (R_1,R_2,R_3) with $R_j \geq 0$, $j=1,2,3$, satisfying

$$R_1 \leq \min\{I(X_1;Y_1,\hat{Y}_3|X_2,X_3,Q), I(X_1;Y_2,\hat{Y}_3|X_2,X_3,Q)\},$$
$$R_2 \leq \min\{I(X_2;Y_2,\hat{Y}_3|X_1,X_3,Q), I(X_2;Y_1,\hat{Y}_3|X_1,X_3,Q)\},$$

and

$$R_1+R_2 \leq \min\{I(X_1,X_2;Y_1,\hat{Y}_3|X_3,Q), I(X_1,X_2;Y_2,\hat{Y}_3|X_3,Q)\}$$

such that

$$R_3+I(Y_3;\hat{Y}_3|X_3,Y_1,Q) \leq I(X_3;Y_1|Q) \text{ and}$$
$$R_3+I(Y_3;\hat{Y}_3|X_3,Y_2,Q) \leq I(X_3;Y_2|Q)$$

for random variables $\hat{Y}_3$ and Q with a joint distribution $p(q,x_1,x_2,x_3,y_1,y_2,y_3,\hat{y}_3) = p(q)p(x_1|q)p(x_2|q)p(x_3|q)$ $p(\hat{y}_3|y_3,x_3,q)p(y_1,y_2,y_3|x_1,x_2,x_3)$ is achievable with $\hat{Y}_3$ having bounded cardinality.

Proof: The proof can be found in [9]. ■

Remark 4.1: The achievable rate region given in Proposition 4.2 can potentially be improved. Instead of broadcasting a single quantized version of its received signal, the relay can transmit two descriptions so that the receiver with an overall better quality in terms of its channel from the relay and the side information received from its transmitter, receives a better description, and hence higher rates (see [8] and [4] for details). Another possible extension which we will not pursue here is to use the partial DF scheme together with the above CF scheme [1], [4].

We are now interested in studying the special case in which each source terminal can reach only one of the destination terminals directly. Assume, for example, that there is no direct connection between source terminal 1 and destination terminal 2, and similarly between source terminal 2 and destination terminal 1. In practice, this setup might model either a larger distance between the disconnected terminals, or some physical constraint in between the terminals blocking the connection. In such a scenario, the relay is essential in providing coverage to multicast data to both receivers. We model this scenario by the (symbol-by-symbol) Markov chain conditions:

$$Y_1-(X_1,X_3)-X_2 \text{ and } Y_2-(X_2,X_3)-X_1. \tag{6}$$

The following proposition, given without proof, provides an outer bound for the capacity region in such a scenario.

Proposition 4.3: Assuming that the Markov chain conditions (6) hold for any channel input distribution, a rate triplet (R_1, R_2, R_3) with $R_j \geq 0$, $j = 1, 2, 3$, is achievable only if

$$\begin{aligned}
R_1 &\leq I(X_1; Y_3|U_1, X_2, X_3, Q), \\
R_2 &\leq I(X_2; Y_3|U_2, X_1, X_3, Q), \\
R_3 &\leq \min\{I(X_3; Y_1|X_1, X_2, U_1, U_2, Q), \\
&\qquad I(X_3; Y_2|X_1, X_2, U_1, U_2, Q)\}, \\
R_1 + R_3 &\leq \min\{I(X_1, X_3; Y_1|U_2, Q), \\
&\qquad I(X_3; Y_2|X_2, U_2, Q)\}, \\
R_2 + R_3 &\leq \min\{I(X_3; Y_1|X_1, U_1, Q), \\
&\qquad I(X_2, X_3; Y_2|U_1, Q)\} \\
R_1 + R_2 + R_3 &\leq \min\{I(X_1, X_3; Y_1|Q), I(X_2, X_3; Y_2|Q)\}
\end{aligned}$$

for some auxiliary random variables U_1, U_2 and Q satisfying the joint distribution $p(q)p(x_1, u_1|q)p(x_2, u_2|q)$ $p(x_3|u_1, u_2, q)p(y_1, y_2, y_3|x_1, x_2, x_3)$.

By imposing the condition (6) on the DF achievable rate region of Proposition 4.1, it can be easily seen that the only difference between the outer bound and the achievable region with DF (5) is that the latter contains the additional constraint (5c), which generally reduces the rate region. The constraint (5c) accounts for the fact that the DF scheme prescribes both messages W_1 and W_2 to be decoded at the relay terminal. The following remark provides two examples in which the DF scheme achieves the outer bound in Proposition 4.3 and thus the capacity region. In both cases, the multiple access interference at the relay terminal is eliminated from the problem setup so that the condition (5c) does not limit the performance of DF.

Remark 4.2: In addition to the Markov conditions in (6), consider orthogonal channels from the two users to the relay terminal, that is, we have $Y_3 \triangleq (Y_{31}, Y_{32})$, where Y_{3k} depends only on inputs X_k and X_3 for $k = 1, 2$; that is, we assume $X_1 - (X_2, X_3) - Y_{32}$ and $X_2 - (X_1, X_3) - Y_{31}$ form Markov chains for any input distribution. Then, it is easy to see that the sum-rate constraint at the relay terminal is redundant and hence the outer bound in Proposition 4.3 and the achievable rate region with DF in Proposition 4.1 match, yielding the full capacity region for this scenario. As another example where DF is optimal, we consider a *relay multicast channel* setup, in which a single relay helps transmitter 1 to multicast its message W_1 to both receivers, i.e., $R_2 = R_3 = 0$ and $X_2 = \emptyset$. For such a setup, under the assumption that $X_1 - X_3 - Y_2$ forms a Markov chain, the achievable rate with DF relaying in Proposition 4.1 and the above outer bound match. Specifically, the capacity C for this *multicast relay channel* is given by

$$C = \max_{p(x_1, x_3)} \min\{I(X_1; Y_3|X_3),\ I(X_1, X_3; Y_1),\ I(X_3; Y_2)\}.$$

Remark 4.3: This work is limited to random coding techniques to provide achievable rate regions. However, the structure of the network under the Markov assumptions in (6) can be exploited using structured codes rather than the random codes. This enables the relay to decode only the modulo sum of the messages relaxing the sum-rate constraint at the relay. We explore this in more detail in [9] and show that structured coding schemes achieve the capacity in certain scenarios.

V. Gaussian Compound MAC with a Relay

A Gaussian cMACr satisfying the Markov conditions (6) is given by

$$Y_1 = X_1 + \eta X_3 + Z_1 \tag{7a}$$

$$Y_2 = X_2 + \eta X_3 + Z_2 \tag{7b}$$

$$Y_3 = \gamma(X_1 + X_2) + Z_3, \tag{7c}$$

where $\gamma \geq 0$ is the channel gain from the users to the relay and $\eta \geq 0$ is the channel gain from the relay to both receiver 1 and receiver 2. The noise components Z_i, $i = 1, 2, 3$ are i.i.d. zero-mean unit variance Gaussian random variables. We enforce the average power constraints $\frac{1}{n}\sum_{i=1}^{n} E[X_{ji}^2] \leq P_j$ for $j = 1, 2, 3$. We define $C(x) = \frac{1}{2}\log(1 + x)$ for $x \in \mathbb{R}^+$. Considering for simplicity the case $R_3 = 0$, we have the following result.

Proposition 5.1: The following rate region is achievable for the Gaussian cMACr characterized by (7) with DF:

$$R_1 \leq \min\left\{ \begin{array}{l} C\left(\gamma^2 P_1 \left(1 - \frac{\alpha_1\alpha_3'}{1-\alpha_2\alpha_3''}\right)\right), \\ C\left(P_1 + \eta^2 P_3(1 - \alpha_3'')\right) \end{array} \right\}, \tag{8a}$$

$$R_2 \leq \min\left\{ \begin{array}{l} C\left(\gamma^2 P_2 \left(1 - \frac{\alpha_2\alpha_3''}{1-\alpha_1\alpha_3'}\right)\right), \\ C\left(P_2 + \eta^2 P_3(1 - \alpha_3')\right) \end{array} \right\} \tag{8b}$$

and

$$\begin{aligned}
R_1 + R_2 \leq \min\Big\{ & C\left(P_1 + \eta^2 P_3 + 2\eta\sqrt{\alpha_1\alpha_3' P_1 P_3}\right), \\
& C\left(P_2 + \eta^2 P_3 + 2\eta\sqrt{\alpha_1\alpha_3'' P_2 P_3}\right), \\
& C\left(\gamma^2(P_1 + P_2)\left(1 - \frac{(\sqrt{\alpha_1\alpha_3' P_1} + \sqrt{\alpha_2\alpha_3'' P_2})^2}{P_1 + P_2}\right)\right)\Big\},
\end{aligned} \tag{8c}$$

with the union taken over the parameters $0 \leq \alpha_1, \alpha_2, \alpha_3', \alpha_3'' \leq 1$ and $\alpha_3' + \alpha_3'' \leq 1$. An outer bound to the capacity region is given by (8) without the last sum-rate constraint in (8c).

Proof: The proof can be found in [9] ∎

It is noted that the parameters α_3' and α_3'' represent the fractions of power that the relay uses to cooperate with transmitter 1 and 2, respectively.

Next, we characterize the achievable rate region for the Gaussian setup with the CF strategy of Proposition 4.2. Here, we assume a Gaussian quantization codebook without claiming optimality.

Proposition 5.2: The following rate region is achievable for the Gaussian cMACr characterized by (7):

$$R_1 \leq C\left(\frac{\gamma^2\alpha_1 P_1}{1 + N_q}\right) \tag{9a}$$

$$R_2 \leq C\left(\frac{\gamma^2\alpha_2 P_2}{1 + N_q}\right) \text{ and} \tag{9b}$$

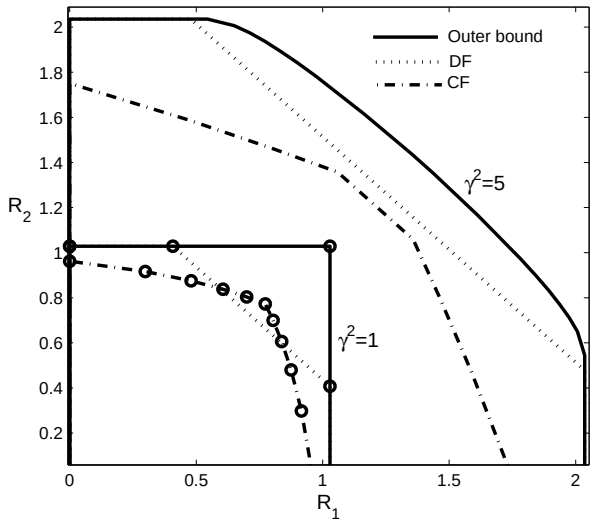

Fig. 2. Achievable rate region and outer bound for $P_1 = P_2 = P_3 = 5\ dB$, $\eta^2 = 10$ and different values of the channel gain from the terminals to the relay, namely $\gamma^2 = 1, 5$.

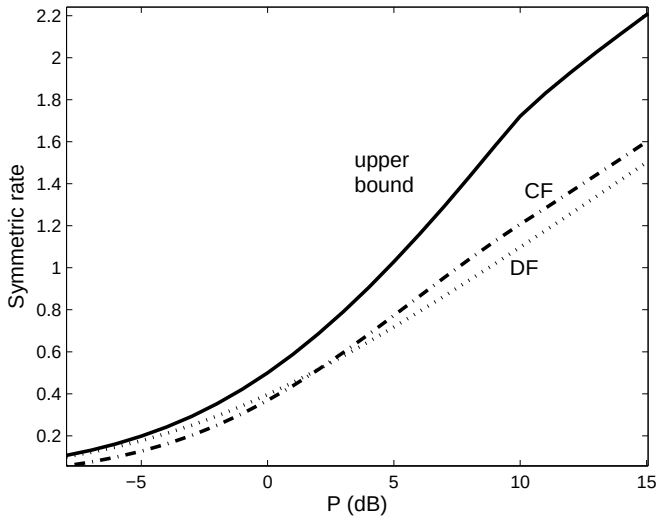

Fig. 3. Symmetric rate achievable with DF and CF strategies compared with the upper bound for $\gamma^2 = 1$ and $\eta^2 = 10$ versus $P_1 = P_2 = P_3 = P$.

$$R_1 + R_2 \leq C(\bar{P}) + C\left(\frac{\gamma^2(\alpha_1 P_1 + \alpha_2 P_2)}{1 + N_q}\right) \qquad (9c)$$

where

$$N_q = \frac{1 + \gamma^2(\alpha_1 P_1 \alpha_2 P_2 + \alpha_1 P_1 + \alpha_2 P_2) + \bar{P}}{\eta^2 P_3},$$

for all $0 \leq \alpha_i \leq 1$, $i = 1, 2$, where we defined $\bar{P} = \min\{\alpha_1 P_1, \alpha_2 P_2\}$.

1) Numerical examples: Consider a cMACr with powers $P_1 = P_2 = P_3 = 5\ dB$ and channel gain $\eta^2 = 10$ from the relay to the two terminals. Fig. 2 shows the achievable rate region and the outer bound for different values of the channel gain from the terminals to the relay, namely $\gamma^2 = 1$ and $\gamma^2 = 5$. It can be seen that, if the channel to the relay is weak, then CF improves upon DF at certain parts of the rate region. However, as γ^2 increases, DF gets very close to the outer bound dominating the CF rate region, since the sum rate constraint in DF scheme becomes less restricting.

In Fig. 3, the symmetric rate achievable with DF and CF is compared with the upper bound for $\gamma^2 = 1$ and $\eta^2 = 10$ versus $P_1 = P_2 = P_3 = P$. We see that while DF achieves a higher rate than CF and performs very close to the upper bound at low power values, with increasing power CF surpasses the DF rate for this channel setup. There is a constant bit gap between the upper bound and the achievable rates with DF and CF at higher power values.

VI. Conclusions

We have considered a compound MAC with a relay terminal. In this model, the relay simultaneously assists both transmitters while multicasting its own information at the same time. We have first characterized the capacity region for a MAC with a cognitive relay and related models of partially cognitive relay and cognition through finite capacity links. We then have used the coding technique that achieves the capacity for these models to provide an achievable rate region with DF relaying in the case of a general cMACr. We have also considered a CF relaying scheme, in which the relay broadcasts a compressed version of its received signal considering the received signals at the receivers as side information. Here we have used a novel joint source-channel coding scheme to improve the achievable rate region of the underlying multi-user channel coding problem. Strategies based on structured codes and physical layer network coding are studied in [9].

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