Energy-Efficient Sensing and Communication of Parallel Gaussian Sources

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***Abstract*—Energy efficiency is a key requirement in the design of wireless sensor networks. While most theoretical studies only account for the energy requirements of communication, the sensing process, which includes measurements and compression, can also consume comparable energy. In this paper, the problem of sensing and communicating parallel sources is studied by accounting for the cost of both communication and sensing. In the first formulation of the problem, the sensor has a separate energy budget for sensing and a rate budget for communication, while, in the second, it has a single energy budget for both tasks. Furthermore, in the second problem, each source has its own associated channel. Assuming that sources with larger variances have lower sensing costs, the optimal allocation of sensing energy and rate that minimizes the overall distortion is derived for the first problem. Moreover, structural results on the solution of the second problem are derived under the assumption that the sources with larger variances are transmitted on channels with lower noise.**

I. Introduction

Sensor networks consisting of battery-limited nodes need to be operated in an energy-efficient manner in order to attain a satisfactory lifetime. Energy consumption of a sensor node usually consists of both computational and communication energy [1], which come primarily from sensing and communication. The sensing component consumes energy in the process of digitizing given information sources through a cascade of acquisition, sampling, quantization and compression tasks, while the communication component spends power for the transmit circuitry and for the power amplifier. It is known that the overall energy spent for compression is generally comparable to that used for communication and that a joint design of compression and transmission is critical to improve the energy efficiency [2] [3]. We refer to the energy cost associated with measurements and compression of information sources as "sensing cost".

The problem of allocating energy across sensing and communication components of sensors in a wireless sensor network, was studied in [4], where an on-line algorithm that is able to choose between a finite number of possible compression algorithms with different energy costs for a multi-hop set-up was proposed. In this paper, instead, we consider an integrated sensor device consisting of multiple sensor interfaces [5] that can simultaneously measure multiple information sources, which are modeled as parallel Gaussian sources. Being part of the same device, the sensor interfaces share the same overall resource budget. Moreover, since the sensor interfaces have distinct hardwares and sensitivities, we assume the sensing costs of different sources are generally different. Finally, for tractability, we model the sensing cost of a given source as being constant per source sample. This is analogous to the model used in the literature to account for the transmitter processing cost of the communication component of a wireless device [6]. In [6], it was shown that, when the transmitter processing energy cost is not negligible, it is no longer optimal to transmit continuously, but instead, bursty transmission is optimal in terms of the achievable rate.

A. Contributions

With sensing costs present, we aim at optimizing the resource (energy or rate) allocation so as to minimize the overall mean squared error distortion of all the sources. We consider two types of resource constraints. In the first, the sensor has a given energy budget used for sensing and a separate rate constraint for communication to the destination (separate sensing/communication). In the second, the sensor has an overall energy budget which is to be spent for both sensing and communication (joint sensing/communication). Moreover, in the joint sensing/communication scenario, the sensed sources are assumed to be transmitted over orthogonal additive white Gaussian channels with different noise variances. This set-up can model a scenario in which different sensor interfaces of the integrated device are used at different times and, to avoid delay and buffer overflow, the measurements are transmitted over a time-varying channel to the destination as they are measured.

For the separate sensing/communication problem, we obtain a closed-form solution for the case where the sources with larger variances have lower sensing costs. This corresponds to a situation when sources with lower variances might require more energy-consuming sensor interfaces with higher sensitivity for sensing. For the joint sensing/communication problem, assuming that sources with larger variances not only have lower sensing costs, but also are transmitted over channels with lower noise variances, we obtain structural results on the optimal solution. Moreover, a closed-form solution is obtained

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for the case where the energy budget is sufficiently large.

This paper is organized as follows. In Section II, we formulate the problems of interest. Section III derives the analytical optimal solution to the separate sensing/communication problem when the source variance and the sensing cost are ordered. In Section IV, the structure of the optimal solution to the joint sensing/communication problem is analyzed for the ordered case. Finally, we make some concluding remarks in Section V.

II. Problem Formulation

We consider a system in which a sensor measures Q independent parallel Gaussian sources and communicates them to a single destination. The ith source consists of n independent and identically distributed (i.i.d.) samples with variance σ_i^2, $i = 1, ..., Q$. In this paper, we assume measuring each sample of the ith source entails a given sensing cost $\epsilon_{S,i}$ joules per source sample, which takes into account the energy spent for acquisition, sampling, quantization and compression. Note that, more generally, the energy costs associated with quantization and compression may depend on the compression rate and the target distortion level [7], which is not pursued here for simplicity. We are interested in minimizing the overall average distortion D of the reproduction of the sources at the destination. We consider two related problems. In the first (separate sensing/communication), we assume that the sensor has two resource budgets, an energy budget for sensing and a rate budget for communication. In the second (joint sensing/communication), instead, we consider energy allocation between the tasks of sensing and communications.

A. Separate Sensing/Communication of Parallel Sources

For the separate sensing/communication problem, we assume the sensor has an energy budget E to be used exclusively for sensing of the Q sources, and a total rate R that can be allocated for communication. Both E and R are normalized by n so that E is the energy budget per source sample and similarly for R. When E and R are limited, it might not be optimal, or possible, to sense all the samples from all the sources. We assume instead that the sensor node measures a fraction $\theta_{S,i}$, with $0 \leq \theta_{S,i} \leq 1$, of samples from of the ith source, and then sends a compressed version of the sensed samples of the ith source with rate R_i ($R_i \geq 0$). Given the above, the mean square error (MSE) of the reconstruction at the destination for the ith source can be obtained as $D_i = \sigma_i^2 f(\theta_{S,i}, R_i)$ [8], where

$$f(\theta_{S,i}, R_i) = \begin{cases} 1 & \text{if } \theta_{S,i} = 0 \\ (1-\theta_{S,i}) + \theta_{S,i} 2^{-\frac{2R_i}{\theta_{S,i}}} & \text{otherwise} \end{cases}. \quad (1)$$

We define the sampling fraction vector and rate allocation vector as $\boldsymbol{\theta}_S = [\theta_{S,1} \ ... \ \theta_{S,Q}]^T$ and $\mathbf{R} = [R_1 \ ... \ R_Q]^T$, respectively. The problem of minimizing the total MSE is finally given by

$$\min_{\boldsymbol{\theta}_S, \mathbf{R}} D(\boldsymbol{\theta}_S, \mathbf{R}) = \sum_{i=1}^{Q} \sigma_i^2 f(\theta_{S,i}, R_i), \quad (2)$$

subject to

$$\sum_{i=1}^{Q} \theta_{S,i} \epsilon_{S,i} \leq E, \quad (3a)$$

$$\text{and} \quad \sum_{i=1}^{Q} R_i \leq R, \quad (3b)$$

where (3a) and (3b) reflect the sensing energy and rate budget constraints respectively.

B. Joint Sensing/Communication of Parallel Sources

For the joint sensing/communication problem, the communication link is modeled as a collection of Q orthogonal channels. We assume that the compressed version of the sensed samples from the ith source ($1 \leq i \leq Q$) are transmitted over the ith channel, which is an independent complex Gaussian noise channel with noise variance N_i. Each channel consists of $n\tau$ channel uses, where τ is the channel-source bandwidth ratio for each source-channel pair. It is also assumed that the sensor has a joint energy constraint B on the sensing and communication components. Similar to E and R in Section II-A, the energy B is normalized by n as well. The sensor measures a fraction $\theta_{S,i}$, with $0 \leq \theta_{S,i} \leq 1$, of the samples of the ith source, and transmits the corresponding compressed samples with power P_i ($P_i \geq 0$) over the ith channel. The MSE of the reproduction of the ith source at the destination can be obtained as $D_i = \sigma_i^2 h(\theta_{S,i}, P_i)$, where

$$h(\theta_{S,i}, P_i) = \begin{cases} 1 & \text{if } \theta_{S,i} = 0 \\ (1-\theta_{S,i}) + \theta_{S,i}(1+\frac{P_i}{N_i})^{-\frac{2\tau}{\theta_{S,i}}} & \text{otherwise} \end{cases}, \quad (4)$$

since the compression rate for each sensed sample of the ith source is given by $\frac{\tau}{\theta_{S,i}} \log_2 \left(1 + \frac{P_i}{N_i}\right)$.

We define the power allocation vector as $\mathbf{P} = [P_1 \ ... \ P_Q]^T$. The problem of minimizing the overall MSE is then given by

$$\min_{\boldsymbol{\theta}_S, \mathbf{P}} D(\boldsymbol{\theta}_S, \mathbf{P}) = \sum_{i=1}^{Q} \sigma_i^2 h(\theta_{S,i}, P_i), \quad (5)$$

subject to

$$\sum_{i=1}^{Q} \theta_{S,i} \epsilon_{S,i} + \tau P_i \leq B, \quad (6)$$

where (6) reflects the overall energy budget constraint.

III. Separate sensing/communication: The ordered variance/cost case

In this section, we consider the separate sensing and communication problem described in Section II-A. To facilitate the analysis, we divide the Q Gaussian sources into K classes with class k ($1 \leq k \leq K$) containing q_k sources with the same variance σ_k^2. Without loss of generality, the variances are in descending order, i.e., $\sigma_1^2 > \sigma_2^2 > ... > \sigma_K^2$. Since each class can contain an arbitrary number q_k of sources, we have strict inequalities among the variances. It is also assumed that sources in class k have the same sensing cost $\epsilon_{S,k}$ and that sources with larger variances have lower sensing costs, i.e., $\epsilon_{S,1} \leq ... \leq \epsilon_{S,K}$. Such an order would be valid if more energy-consuming sensor interfaces with higher sensitivities are required to measure sources with lower variances. Note that, while for the general case, the problem in (2)-(3) can be shown to be convex, there is no closed-form solution. For details, we refer to the reader to [9]. Focusing on the ordered case allows us to obtain an analytical expression for the optimal solution and gain insights into the problem.

For convenience, we divide the range of the energy E into a sequence of intervals $\mathcal{E}_1 = (e_0, e_1]$, $\mathcal{E}_2 = (e_1, e_2]$,..., $\mathcal{E}_K = (e_{K-1}, e_K)$, where $e_0 = 0$, $e_K = +\infty$ and $e_m = \sum_{i=1}^{m} q_i \epsilon_{S,i}$ for $1 \leq m \leq K-1$, and divide the range of rate R into a sequence of intervals $\mathcal{R}_1 = (r_0, r_1]$, $\mathcal{R}_2 = (r_1, r_2]$,..., $\mathcal{R}_K = (r_{K-1}, r_K)$, where $r_0 = 0$, $r_K = +\infty$ and

$$r_l = \frac{1}{2}\sum_{j=1}^{l} q_j \log_2\left(\frac{\sigma_j^2}{\sigma_{l+1}^2}\right), \quad 1 \leq l \leq K-1. \qquad (7)$$

By the convexity of function $D(\boldsymbol{\theta}_S, \mathbf{R})$, it is easy to see that we can assume the same fraction $\theta_{S,k}$ and rate R_k are assigned to each source in the kth class[1]. We have the following result.

Proposition 1: For $K \geq 2$, assuming $\sigma_1^2 > ... > \sigma_K^2$ and $\epsilon_{S,1} \leq ... \leq \epsilon_{S,K}$, the optimal solution for the separate sensing and communication problem in Section II-A is obtained as follows. Given $E \in \mathcal{E}_m$ for some $1 \leq m \leq K$,

1) If $R \in \mathcal{R}_l$ for some integer l with $1 \leq l \leq m-1$, then it is optimal to fully sample the first l classes of sources, i.e., $\theta_{S,k}^* = 1$ for $1 \leq k \leq l$, and to allocate rates as

$$R_k^* = \frac{1}{\sum_{j=1}^{l} q_j}\left(R + \frac{1}{2}\sum_{j=1, j\neq k}^{l} q_j \log_2\left(\frac{\sigma_k^2}{\sigma_j^2}\right)\right), \qquad (8)$$

where $1 \leq k \leq l$. Moreover, there is no need to sense the remaining $K-l$ classes of sources, i.e., $\theta_{S,k}^* = 0$ and $R_k^* = 0$, for $l+1 \leq k \leq K$.

2) If instead $R > r_{m-1}$ (or $R \in \bigcup_{l \geq m} \mathcal{R}_l$), then it is optimal to sample the first $m-1$ classes of sources fully, i.e., $\theta_{S,k}^* = 1$ for $1 \leq k \leq m-1$, and the mth class for a fraction $\theta_{S,m}^* = \min((E - e_{m-1})/(q_m \epsilon_{S,m}), 1)$, and to allocate rates as

$$R_k^* = \frac{\theta_{S,k}^*}{\sum_{j=1}^{m} q_j \theta_{S,j}^*}\left(R + \frac{1}{2}\sum_{j=1, j\neq k}^{m} q_j \theta_{S,j}^* \log_2\left(\frac{\sigma_k^2}{\sigma_j^2}\right)\right), \qquad (9)$$

where $1 \leq k \leq m$. Moreover, there is no need to sense the remaining $K-m$ classes of sources, i.e., $\theta_{S,k}^* = 0$ and $R_k^* = 0$ for $m+1 \leq k \leq K$.

Proof: The proof is based on solving the KKT conditions [11] but special care must be taken since the objective function in (2) is not continuously differentiable in the entire feasible set. Details of the proof are provided in [9]. ■

[1] In fact, fixing all other parameters, function $D(\boldsymbol{\theta}_S, \mathbf{P})$ is Schur-convex with respect to the fractions of samples and the rates assigned to the sources in a class. Therefore, an equal fraction and rate allocation is optimal (see, e.g., [10]).

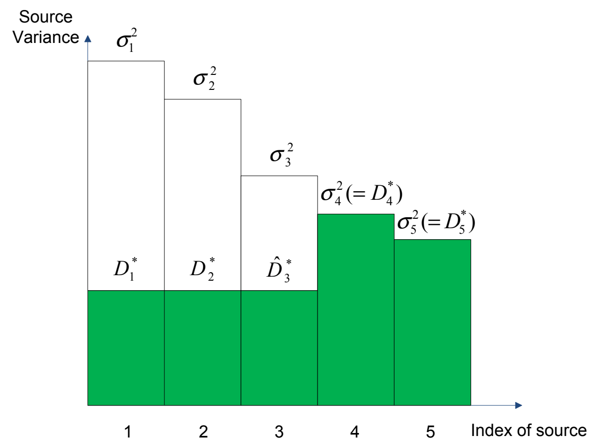

Fig. 1. Illustration of the optimal solution for case 2) of Proposition 1, where $K = 5$, $q_k = 1$ for $k = 1, ..., 5$ and E and R are chosen to satisfy $e_2 < E < e_3$ and $R > r_4$.

Before we discuss the solution given in Proposition 1, we revisit the classical reverse water-filling approach, which solves the separate sensing and communication problem in (2) in the case of zero sensing costs, i.e., $\epsilon_{S,k} = 0$ for all $1 \leq k \leq K$. With zero sensing costs, it is optimal to sample all of the sources fully, that is, $\theta_{S,k}^* = 1$ for all $1 \leq k \leq K$. Moreover, if $R \in \mathcal{R}_l$ with $1 \leq l \leq K$, only the first l source classes are assigned positive rates and the rate allocation is given by (8). It is noted that this rate allocation leads to the same distortion level $\sigma_k^2 2^{-2R_k^*}$ for all the sources that are assigned a non-zero rate. Proposition 1 states that, when the sensing costs are taken into account, the optimal solution in the ordered case entails sensing sources with the highest variances and then optimally allocating rates among the sensed sources using either the reverse water-filling procedure or a variation of it.

Specifically, in case 1) of Proposition 1, that is, if $E \in \mathcal{E}_m$ with $1 \leq m \leq K$ and $R \in \mathcal{R}_l$ with $1 \leq l \leq m-1$, the first l classes of sources are fully sensed and compression rates are assigned according to the classic reverse water-filling solution. Note that in this case, even though there is enough energy to sample more than l classes of sources, given the rate constraint, the optimal rate allocation only assigns positive rate to the first l classes. Instead, in case 2) of Proposition 1, i.e., if $E \in \mathcal{E}_m$ and $R > r_{m-1}$, it is optimal to fully sample the first $m-1$ classes of sources, while the sources in the mth class are sampled only partially using the remaining energy. For the mth class, the optimal sampling fraction is equal to $\theta_{S,m}^* = \min((E - e_{m-1})/(q_m \epsilon_{S,m}), 1)$ and the optimal rate is assigned according to (9) such that the normalized distortion for the sampled fraction of any source in the mth class $\hat{D}_m^* = \sigma_m^2 2^{-2R_m^*/\theta_{S,m}^*}$ is equal to the distortion of any source in the first $m-1$ classes. Moreover, from (2), the distortion attained for each source in class m is given by $\theta_{S,m}^* \hat{D}_m^* + (1-\theta_{S,m}^*)\sigma_m^2$, where $\theta_{S,m}^* \hat{D}_m^*$ is the total distortion of the sampled fraction of the source, while $\sigma_m^2(1-\theta_{S,m}^*)$ corresponds to the total distortion of the non-sampled fraction.

We pictorially illustrate the solution for case 2) of Proposition 1 in Fig. 1, where we assume $K=5$ and $q_k=1$, $k=1,...,5$. In this example, the energy E and the rate R are assumed to satisfy $e_2 < E < e_3$ and $R > r_4$. Thus, it is optimal to have source 1 and source 2 both fully sampled and have source 3 only partially sampled for a fraction $\theta_{S,3}^* = (E-e_2)/\epsilon_{S,3}$. The first two sources and the sampled fraction of source 3 are all described with the same distortion, i.e., $D_1^* = D_2^* = \hat{D}_3^*$, where we recall that $\hat{D}_3^*$ is the average distortion only for the sampled fraction of source 3. The overall distortion for source 3 is $\theta_{S,3}^*\hat{D}_3^* + (1-\theta_{S,3}^*)\sigma_3^2$. Since source 4 and source 5 are not sampled at all, they are assigned zero rates and thus the corresponding distortions are equal to their variances. Recall that in the zero sensing cost case, all the five sources would be fully sampled and since $R > r_4$, all of them would be described with the same distortion, i.e., $D_1^* = D_2^* = ... = D_5^*$.

IV. Joint Sensing and Communication: The Ordered Variance/Cost/Noise Case

In Section III, we have investigated the optimal solution to the separate sensing and communication problem in (2)-(3) when source variances and sensing costs are ordered. In this section, we analyze the joint sensing and communication problem in (5)-(6) when the source variances, the sensing costs and the channel noise variances are ordered. Similar to Section III, we divide the Q parallel source-channel pairs into K classes, with class k having q_k source-channel pairs, where $1 \le k \le K$. It is assumed that the sources in class k have the same variance σ_k^2 and the channels in class k have the same noise variance N_k. Following Section III, we assume the source variances satisfy $\sigma_1^2 > \sigma_2^2 > ... > \sigma_K^2$ and the sensing costs satisfy $\epsilon_{S,1} \le ... \le \epsilon_{S,K}$. It is also assumed that the channel noise variances satisfy $N_1 \le ... \le N_K$. While for the general case, the problem in (5)-(6) can be shown to be convex (see [9]), there is no closed form solution. However, for the ordered case described above, finding an analytical solution in closed form is possible under certain conditions.

Similar to Section III, it can be readily shown that it is optimal to allocate the same sampling fraction $\theta_{S,k}$ and the same transmit power P_k to all source-channel pairs in class k. For convenience, we divide the range of B to a sequence of intervals: $\mathcal{B}_1 = (b_0, b_1]$, $\mathcal{B}_2 = (b_1, b_2]$,..., $\mathcal{B}_K = (b_{K-1}, b_K)$, where $b_0 = 0$, $b_K = +\infty$, and

$$b_i = \tau \sum_{j=1}^{i} q_j N_j \left(\left(\frac{\sigma_j^2 N_{i+1}}{\sigma_{i+1}^2 N_j} \right)^{\frac{1}{2\tau+1}} - 1 \right), \quad 1 \le i \le K-1. \tag{10}$$

We now first summarize the solution of (5)-(6) in the special case of zero sensing costs, i.e., when $\epsilon_{S,k} = 0$ for all $1 \le k \le K$. In this case, we can sample all the sources fully, i.e, we set $\theta_{S,k} = 1$ for all $1 \le k \le K$, without loss of optimality.

Lemma 1: For $K \ge 2$, assuming $\sigma_1^2 > ... > \sigma_K^2$, $N_1 \le ... \le N_K$ and $\epsilon_{S,1} = ... = \epsilon_{S,K} = 0$, if $B \in \mathcal{B}_m$ for some $1 \le m \le K$, then it is optimal to assign positive transmit powers only to the first m classes of source-channel pairs as

$$P_k^* = \frac{B + \tau \sum_{j=1, j\neq k}^{m} q_j N_j \left(1 - \left(\frac{\sigma_j^2 N_k}{\sigma_k^2 N_j} \right)^{\frac{1}{2\tau+1}} \right)}{\tau \sum_{j=1}^{m} q_j \left(\frac{\sigma_j^2 N_j^{2\tau}}{\sigma_k^2 N_k^{2\tau}} \right)^{\frac{1}{2\tau+1}}}, \tag{11}$$

where $1 \le k \le m$, and to assign zero power to the remaining classes, i.e., $P_k^* = 0$, for $m+1 \le k \le K$.

Proof: With $\theta_{S,1} = ... = \theta_{S,K} = 1$, the optimization of powers $\mathbf{P}$ in (5) is convex and can be easily performed using the standard Lagrangian approach [12]. ∎

It is interesting to note that, unlike the reverse water-filling solution, all the source-channel pairs that are allocated positive powers (or positive rates for reverse water-filling) are not assigned the same distortion level in the joint sensing and communication problem considered here. Instead, the distortion level $D_k^* = \sigma_k^2(1 + P_k^*/N_k)^{-2\tau}$ for any class k that is assigned a positive power is proportional to $(\sigma_k^2 N_k^{2\tau})^{\frac{1}{2\tau+1}}$ (See [9] for details). This shows that only in the special case $\sigma_k^2 N_k^{2\tau} = 1$ for all $1 \le k \le m$, all the source-channel pairs with positive powers have the same distortion.

Now we proceed to analyze the problem in (5)-(6) for the ordered case when there are nonzero sensing costs.

Proposition 2: For $K \ge 2$, assuming $\sigma_1^2 > ... > \sigma_K^2$, $0 < \epsilon_{S,1} \le ... \le \epsilon_{S,K}$ and $N_1 \le ... \le N_K$, it is optimal to sense and transmit only the first m source classes, for some m with $1 \le m \le K$ depending on the energy budget B. Moreover, for the sensed m classes, the sampling fractions satisfy $0 < \theta_{S,m}^* \le ... \le \theta_{S,1}^* \le 1$, with $\theta_{S,i}^* = \theta_{S,j}^*$ $(1 \le i < j \le m)$ only when both are 1.

Proof: The structural results on the optimal solution are obtained using the KKT conditions. As in the proof of Proposition 1, special care must be taken since the objective function in (5) is not continuously differentiable in the entire feasible set. Details of the proof are provided in [9]. ∎

Proposition 2 suggests that the sources with larger variances are sampled for a fraction greater than or equal to that of the sources with smaller variances. While this is an important property of the optimal solution to the joint sensing/communication problem in the presence of nonzero sensing costs, it appears difficult to obtain an analytical characterization of the optimal solution even for the ordered case. In the following, we characterize the optimal solution in closed form for the special case when the energy budget is sufficiently large so that all sources can be fully sensed. We also compute the minimum energy budget that guarantees this. To this end, let us define the set $\bar{\mathcal{B}}$ as $\bar{\mathcal{B}} = \big[\bar{b}, +\infty\big)$, where $\bar{b}$ is the solution to the equation

$$\frac{\sigma_K^2}{\epsilon_{S,K}} \left(1 - \left(1 + \frac{\bar{P}_K}{N_K} \right)^{-2\tau} \left[1 + 2\tau \ln \left(1 + \frac{\bar{P}_K}{N_K} \right) \right] \right) = \left(\frac{\tau \sum_{j=1}^{K} q_j (2\sigma_j^2 N_j^{2\tau})^{\frac{1}{2\tau+1}}}{\bar{b} - \sum_{j=1}^{K} q_j(\epsilon_{S,j} - \tau N_j)} \right)^{2\tau+1}, \tag{12}$$

where

$$\bar{P}_K = \frac{\bar{b} - b_{K-1} - \sum_{j=1}^{K} q_j \epsilon_{S,j}}{\tau \sum_{j=1}^{K} q_j \left(\frac{\sigma_j^2 N_j^{2\tau}}{\sigma_K^2 N_K^{2\tau}} \right)^{\frac{1}{2\tau+1}}}. \quad (13)$$

Note that with $\bar{b} \geq b_{K-1} + \sum_{j=1}^{K} q_j \epsilon_{S,j}$, the solution to (12) is unique, since over this range, the left side of (12) is a strictly increasing function of $\bar{b}$, while the right side is a strictly decreasing function of $\bar{b}$.

Proposition 3: For $K \geq 2$, assuming $\sigma_1^2 > ... > \sigma_K^2$, $0 < \epsilon_{S,1} \leq ... \leq \epsilon_{S,K}$ and $N_1 \leq ... \leq N_K$, if $B \in \bar{\mathcal{B}}$, it is optimal to fully sample all the K classes of sources, i.e., to set $\theta_{S,k}^* = 1$ for all $1 \leq k \leq K$ and to assign transmit powers as

$$P_k^* = \frac{B - \sum_{j=1}^{K} q_j \epsilon_{S,j} + \tau \sum_{j=1, j\neq k}^{K} q_j N_j \left[1 - \left(\frac{\sigma_j^2 N_k}{\sigma_k^2 N_j} \right)^{\frac{1}{2\tau+1}} \right]}{\tau \sum_{j=1}^{K} q_j \left(\frac{\sigma_j^2 N_j^{2\tau}}{\sigma_k^2 N_k^{2\tau}} \right)^{\frac{1}{2\tau+1}}}, \quad (14)$$

where $1 \leq k \leq K$.

Proof: Details of the proof are provided in [9]. ■

Proposition 3 states that, if the energy budget is larger than the threshold $\bar{b}$, then it is optimal to fully sample all the sources and to allocate power as for the case with no sensing costs (see (11)) but with energy budget discounted by the energy needed for sensing (i.e., with energy $B - \sum_{j=1}^{K} q_j \epsilon_{S,j}$). It is interesting to note that the threshold $\bar{b}$ is, in fact, strictly larger than $b_{K-1} + \sum_{j=1}^{K} q_j \epsilon_{S,j}$. We recall that b_{K-1} is the energy threshold above which it is optimal to assign positive powers to all K classes of source-channel pairs in the zero sensing cost case, while $\sum_{j=1}^{K} q_j \epsilon_{S,j}$ is the total sensing energy needed to sense all the sources.

Fig. 2 shows the optimal sampling fractions for the joint sensing/communication problem as a function of energy budget B when parameters are chosen as $q_1 = q_2 = 1$, $\sigma_1^2 = 1.25$ and $\sigma_2^2 = 1$, $\epsilon_{S,1} = \epsilon_{S,2} = 1$, $N_1 = N_2 = 4$. The results are obtained via numerical methods [11]. It can be seen from Fig. 2, for any B, θ_1^* is greater than or equal to θ_2^*, which is consistent with the optimal structure derived in Proposition 2. Moreover, when $2 < B < 3$, both sources are partially sampled, which is not encountered in the optimal solution of the separate sensing and communication problem of Section III. As B grows beyond 6, both classes are fully sampled. This threshold corresponds to threshold $\bar{b}$ in (12) with $K = 2$ and is strictly larger than $b_1 + q_1\epsilon_{S,1} + q_2\epsilon_{S,2} = 2.3$. It can be observed from Fig. 2 that, if $2.3 < B < 6$, the optimal solution entails partial sampling of at least source 2 which has the lower variance. In this case, fully sampling both sources is strictly suboptimal.

V. Conclusions

In this paper, we studied an energy-constrained sensor system that has a constant sensing energy cost per source sample and we investigated the impact of the sensing energy cost on the end-to-end distortion of parallel Gaussian sources. We formulated a distortion minimization problem with either separate constraints on the sensing energy budget and on the communication rates, or a joint constraint on the energy budget for both sensing and transmission. For both problems, we studied the special case in which sources with larger variances have lower sensing costs. We showed that, for the separate sensing/communication problem, the optimal strategy is to sense the sources starting from the one with the largest variance and to allocate the communication rate using reverse water-filling, or a variant of it, on the sensed sources. Moreover, for the joint sensing/communication problem, it is generally optimal to sense, possibly partially, only a subset of the sources with the largest variances and to allocate the transmit powers among their respective channels.

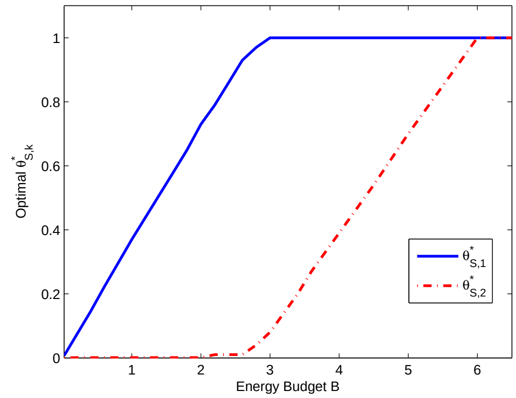

Fig. 2. Optimal sampling fractions $\boldsymbol{\theta}_S$ for $q_1 = q_2 = 1$, $\sigma_1^2 = 1.25$, $\sigma_2^2 = 1$, $\epsilon_{S,1} = \epsilon_{S,2} = 1$ and $N_1 = N_2 = 4$.

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