Interference Channel aided by an Infrastructure Relay

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***Abstract*—A Gaussian Interference Channel with an Infrastructure Relay (ICIR) is investigated. The relay has finite-capacity links to both sources and destinations that are orthogonal to each other and to the underlying interference channel. A general achievable rate region is presented by using the relay both to convey additional information from the sources (*signal relaying*) and to ease interference cancelation (*interference forwarding*). Outer bounds to the capacity region are also derived, and used to determine a number of regimes of interest where either signal relaying only or both signal relaying and interference forwarding are optimal.**

I. Introduction

Consider two terminals of, say, a cellular system, communicating at the same time and over the same bandwidth with the corresponding receivers (base stations). The base stations cannot cooperate for decoding and thus generally suffer from inter-cell interference. Assume now that a fifth node is available in between the two cells that has no data of its own to communicate, but can assist the two ongoing transmissions towards the base stations. This scenario can be modelled as a (two-user) Interference Channel (IC) assisted by a relay, that has been recently dealt with in a number of works [8]-[12]. As shown in these references (briefly reviewed below), the presence of a relay terminal may aid reception at both receivers, not only via (useful) *signal relaying*, as in a standard relay channel, but also, remarkably, via *interference forwarding* (see also [13] for related discussion).

The IC aided by a relay has been first studied in [8], where a Gaussian model is considered and an achievable rate region is obtained via rate splitting into common and private messages at the sources [2], decode-and-forward (DF) at the relay and joint decoding at the destinations [2]. It is shown via numerical results that the sum-rate in a symmetric IC is maximized when the relay only forwards common messages, which need to be decoded at both destinations. The discrete memoryless and Gaussian IC with a relay is further investigated in [9] [10], in which simplified channel models are considered where the relay only receives from one source. DF-based strategies at the relays with joint decoding at the destinations are proposed without rate splitting and shown to exhaust the capacity region under some conditions. These works emphasize the fact that forwarding the interference of even a single source may improve the rates of both users. Related work is also presented in [11] and [12], where the relay is assumed to be aware a priori of the users' messages (cognitive relay) and sophisticated achievable strategies are investigated.

This work is partially supported by NSF grant No. 0520054 and The Wireless Internet Center for Advanced Technology at Polytechnic Institute of NYU.

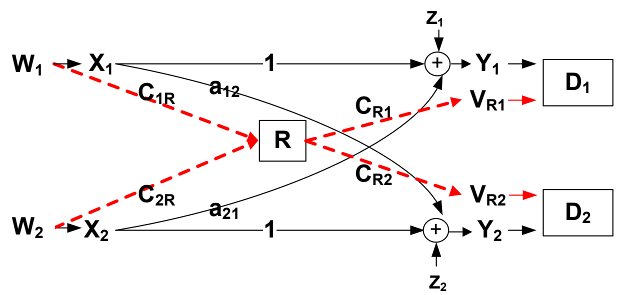

Fig. 1. System Model

Previous work, as summarized above, assumed that the relay operates over the same time and bandwidth of the IC, thus opening up the possibility of deploying a large number of strategies, such as coherent transmission, dirty paper coding, interference cancellation, etc. As such, the analysis has turned out to be quite prohibitive without resorting to simplified models or numerical evaluations. In this paper, we focus on a more fundamental model in which the operation of the relay is limited in a way to clearly emphasize optimality of *signal relaying versus interference forwarding* in different operating regimes. Specifically, we consider the model in Figure I, in which, on top of a standard Gaussian IC, a relay is connected to the sources and to the destination via finite-capacity links. Such links are orthogonal to one another and to the underlying IC. Besides the technical reason discussed above, this model finds justifications in scenarios in which transmission to and from the relay takes place via orthogonal wireless interfaces (e.g., Wi-Fi cards). Since the introduction of the relay here assumes the presence of such additional communication infrastructure, we term this channel as IC with Infrastructure Relay (IR), or in short ICIR. We first derive a general achievable rate based on a four-way rate splitting strategy and then show optimality of either signal relaying only or both signal relaying and interference in a number of scenarios of interest.

The paper is organized as follows. In Sec. II, we provide the system model. A general achievable rate region is given in Sec. III. In Sec. IV, we derive outer bounds for the rate region of the system and show that the outer bounds are tight for some channel conditions. The paper is concluded in Sec. VI.

II. System Model

We focus on the Gaussian ICIR, as shown in Fig. I. Each source S_i, $i = 1, 2$, wishes to send a message index W_i, uniformly drawn from the set $[1, 2^{nR_i}]$, to its destination D_i, with the help of an IR R. The sources S_1 and S_2 communicate simultaneously to their respective destinations D_1 and D_2 via a Gaussian ("wireless") IC. Moreover, each source is also connected via a noiseless finite-capacity link to a relay, which in turn has finite-capacity links to each destination. All the four links are orthogonal to each other and to the Gaussian IC. Moreover, the links from S_1, S_2 to the relays have capacities C_{1R}, C_{2R} in bits/ channel use (of the IC), respectively, and the links from the relay to the destinations D_1, D_2 have capacity of C_{R1}, C_{R2} [bits/channel use], respectively. The signals received at time $t = 1, ..., n$ on the Gaussian channel by D_1 and D_2 are:

$$Y_{1,t} = X_{1,t} + a_{21}X_{2,t} + Z_{1,t} \quad (1a)$$
$$Y_{2,t} = a_{12}X_{1,t} + X_{2,t} + Z_{2,t}, \quad (1b)$$

respectively, where $X_{i,t} \in \mathbb{R}$ represents the (real) input symbols of source S_i, on which we enforce the power constraint $1/n \sum_{t=1}^{n} x_{i,t}^2 \leq P_i$ for each codeword, and $\{Z_{i,t}\}$ is an independent identically distributed (i.i.d.) Gaussian noise process with unit power. Finally, we define as $V_{iR} \in [1, 2^{nC_{iR}}]$ the messages sent by source S_i over the finite-capacity link to the IR R, and $V_{Ri} \in [1, 2^{nC_{Ri}}]$ as the messages sent by the IR to D_i, for $i = 1, 2$.

A $(2^{nR_1}, 2^{nR_2}, n)$ code for the ICIR is defined by the encoding function at the source S_i, $i = 1, 2$:

$$f_i\colon [1, 2^{nR_i}] \rightarrow \mathbb{R}^n \times [1, 2^{nC_{iR}}], \quad (2)$$

which maps a message $W_i \in [1, 2^{nR_i}]$ into a codeword $X^n \in \mathbb{R}^n$ and a message to the IR $V_{iR} \in [1, 2^{nC_{iR}}]$; the encoding function at the IR R

$$f' : [1, ...2^{nC_{1R}}] \times [1, ...2^{nC_{2R}}] \rightarrow [1, ...2^{nC_{R1}}] \times [1, ...2^{nC_{R2}}] \quad (3)$$

which maps the received messages (V_{1R}, V_{2R}) into messages (V_{R1}, V_{R2}); and by the decoding function at the destination D_i, $i = 1, 2$,

$$g_i : \mathbb{R}^n \times [1, ...2^{nC_{Ri}}] \rightarrow [1, ...2^{nR_i}], \quad (4)$$

which maps the received signal over the IC, Y^n, and from the IR, V_{Ri}, into an estimated message $\hat{W}_i$. Probability of error, achievable rate region and capacity region are defined in standard manner as given in, e.g., [5].

III. A General Achievable Region

In this section, we derive a general achievable rate region for the ICIR. It is noted that, due to the different channel structure, it is not possible to directly borrow techniques from previous work on the IC with a relay of [8] [9] [11] [12]. As in [1] [2], in the proposed strategy, we employ rate splitting into private and common messages, where the private message of each source is to be decoded only by the intended destination and the common is to be decoded at both intended and interfered destinations. However, private and common parts are further split into two messages as follows. The private message is sent in part over the IC and in part via the IR directly to the intended destination. As for the common message, both parts are sent over the IC, but one of the two is also sent over the IR to the interfered destination for interference cancellation. More specifically, we have the following four-way split of each message W_i, $W_i = (W_{iR}, W_{ip}, W_{ic'}, W_{ic''})$, $i = 1, 2$, where: (*i*) $W_{iR} \in [1, ...2^{nR_{iR}}]$ is a private message that is transmitted via the IR only, directly to D_i. Notice the since the IR has orthogonal channels to the IC, this message is conveyed interference-free to D_i; (*ii*) $W_{ip} \in [1, ...2^{nR_{ip}}]$ is a private message that is transmitted over the IC, decoded at D_i and treated as noise at D_j, $j \neq i$; (*iii*) $W_{ic'} \in [1, 2^{nR_{ic'}}]$ is a common message that is transmitted over the IC *and* IR. Specifically, the IR conveys $W_{ic'}$ to D_j only, $j \neq i$, to enable interference cancellation; (*iv*) $W_{ic''} \in [1, ...2^{nR_{ic''}}]$ is a common message that is transmitted over IC only and decoded at both destinations.

Overall, it is noted that the IR conveys both messages *independent* of the transmission on the IC (W_{iR}), which bring additional information bits directly to the destinations and can be seen as *signal relaying*, and messages that are *correlated* with the transmission over the IC and enable interference cancellation ($W_{ic'}$), which can be seen as *interference forwarding*. Based on the strategy outline above, we have the following achievable rate region.

Theorem 1: The convex hull of the union of all rates (R_1, R_2) with $R_i = R_{ic'} + R_{ic''} + R_{ip} + R_{iR}$, $i = 1, 2$, that satisfy the inequalities

$$\sum_{j \in \mathcal{S}_1} R_j \leq \frac{1}{2}\log\left(\frac{\sum_{j \in \mathcal{S}_1} k_{j1}^2 P_j}{N_1}\right) \quad (5)$$
$$\sum_{j \in \mathcal{S}_2} R_j \leq \frac{1}{2}\log\left(\frac{\sum_{j \in \mathcal{S}_2} k_{j2}^2 P_j}{N_2}\right) \quad (6)$$
$$R_{1c'} + R_{1R} \leq C_{1R} \quad (7)$$
$$R_{2c'} + R_{2R} \leq C_{2R} \quad (8)$$
$$R_{2c'} + R_{1R} \leq C_{R1} \quad (9)$$
$$R_{1c'} + R_{2R} \leq C_{R2}, \quad (10)$$

provides an achievable rate region for the Gaussian ICIR, where conditions (5)-(6) must hold for all subsets $\mathcal{S}_1 \subseteq \mathcal{T}_1 = \{1c, 1p, 2c''\}$ and $\mathcal{S}_2 \subseteq \mathcal{T}_2 = \{2c, 2p, 1c''\}$, *except* $\mathcal{S}_1 = \{2c''\}$ and $\mathcal{S}_2 = \{1c''\}$; and we define $R_{ic} = R_{ic'} + R_{ic''}$, $N_1 = a_{21}^2 P_{2p} + 1$, $N_2 = a_{12}^2 P_{1p} + 1$, and the parameters $k_{j1} = 1$, $k_{j2} = a_{12}$ if $j \in \{1c, 1p\}$, and $k_{j1} = a_{21}$, $k_{j2} = 1$ if $j \in \{2c, 2p\}$. Moreover, we use the convention $P_{jc''} = P_{jc}$, $j = 1, 2$, and the power allocations must satisfy the power constraints $P_{1c} + P_{1p} \leq P_1$ and $P_{2c} + P_{2p} \leq P_1$.

Proof: *Codeword Generation and Encoding:* The sources divide their messages as $W_1 = (W_{1c}^{'}, W_{1c}^{''}, W_{1p}, W_{1R})$, and $W_2 = (W_{2c}^{'}, W_{2c}^{''}, W_{2p}, W_{2R})$ as explained above. Messages W_{ip} and ($W_{ic'}$,$W_{ic''}$) are encoded into codewords X_{ip}^n and X_{ic}^n with rates R_{ip}, $R_{ic'} + R_{ic''}$ for $i = 1, 2$, respectively, and sent over the IC in n channel uses. Such codewords are generated i.i.d. from Gaussian distributions with zero-mean and powers P_{ip}, P_{ic}, respectively. Overall, we have the transmitted codewords:

$$X_1^n(W_1) = X_{1p}^n(W_{1p}) + X_{1c}^n(W_{1c'}, W_{1c''}) \quad (11a)$$
$$X_2^n(W_2) = X_{2p}^n(W_{2p}) + X_{2c}^n(W_{2c'}, W_{2c''}). \quad (11b)$$

Message W_{iR} is transmitted to D_i via the IR only, through the messages V_{iR} and V_{Ri}. Moreover, to facilitate interference cancellation, source S_i transmits message $W_{ic}^{'}$ to the interfered destination D_j, $j \neq i$, via the IR in the messages V_{iR} and V_{Rj}.

Thus, the messages sent over the links are given by:

$$V_{1R} = (W_{1c'}, W_{1R}),\ V_{2R} = (W_{2c'}, W_{2R}) \quad (12)$$
$$V_{R1} = (W_{2c'}, W_{1R}),\ V_{R2} = (W_{1c'}, W_{2R}). \quad (13)$$

Decoding: The destination D_i immediately recovers V_{Ri} from the incoming noiseless link. The signals received on the IC are given by (1) with (11). Moreover, destination D_1 knows $W_{2c'}$ and thus sees an equivalent codebook $X^n_{2c}(W_{2c'}, W_{2c''})$ with only $2^{nR_{2c''}}$ codewords (and power P_{ic}). Similarly, D_2 sees an equivalent codebook $X^n_{1c}(W_{1c'}, W_{1c''})$ with rate $R_{1c''}$. Decoding of the messages $(W_{1c'}, W_{1c''}, W_{1p}, W_{2c''})$ at destination D_1 (and $(W_{2c'}, W_{2c''}, W_{2p}, W_{1c''})$ at destination D_2) is then performed jointly as over a multiple access channel with three sources of rates $R_{1c} = R_{1c'} + R_{1c''}$, R_{1p} and $R_{2c''}$ (and $R_{2c} = R_{2c'} + R_{2c''}$, R_{2p} and $R_{1c''}$ for D_2), by treating the private messages as noise, thus with equivalent noise power $N_i = a^2_{ji}P_{jp} + 1$ for $i, j = 1, 2$, $i \neq j$. It is finally noted that, as explained in [14], error events corresponding to erroneous decoding of only message $W_{2c''}$ at destination D_1 and $W_{1c''}$ at destination D_2 do not contribute to the probability of error and thus can be neglected. □

IV. Outer Bounds and Capacity Results

In this section, we first present a general outer bound to the capacity region of an ICIR in terms of multi-letter mutual informations (Theorem 2). This bound is then specialized to a number of special cases of interest, allowing the identification of the capacity region of ICIR for such scenarios (Theorems 3-5).

Theorem 2 (General outer bound): For an ICIR, the capacity region $\mathcal{C}_{ICIR}$ is contained within the set of rates (R_1, R_2) satisfying

$$R_1 \leq \frac{1}{n}I(X^n_1; Y^n_1) + \min(C_{R1}, C_{1R} + C_{2R}) \quad (14)$$
$$R_1 \leq \frac{1}{n}I(X^n_1; Y^n_1|X^n_2) + \min(C_{1R}, C_{R1}) \quad (15)$$
$$R_2 \leq \frac{1}{n}I(X^n_2; Y^n_2) + \min(C_{R2}, C_{1R} + C_{2R}) \quad (16)$$
$$R_2 \leq \frac{1}{n}I(X^n_2; Y^n_2|X^n_1) + \min(C_{2R}, C_{R2}), \quad (17)$$

for some multi-letter input distribution $p(x^n_1)p(x^n_2)$ that satisfy the power constraints $1/n \sum^n_{t=1} x^2_{i,t} \leq P_i$, $i = 1, 2$.

Proof: We start with the bound (14), then (16) follows similarly. We have

$$\begin{aligned} nR_1 &= H(W_1) && (18)\\ &= I(W_1; Y^n_1, V_{R1}) + H(W_1|Y^n_1, V_{R1}) && (19)\\ &\leq I(W_1; Y^n_1, V_{R1}) + n\epsilon_n && (20)\\ &= I(W_1; Y^n_1) + I(W_1; V_{R1}|Y^n_1) + n\epsilon_n && (21)\\ &\leq I(X^n_1; Y^n_1) + H(V_{R1}) + n\epsilon_n && (22)\end{aligned}$$

where (20) is from Fano's inequality, (22) is from $W_1 \rightarrow X^n_1 \rightarrow Y^n_1$ and from the fact that conditioning decreases entropy: $I(W_1; V_{R1}|Y^n_1) \leq H(V_{R1}|Y^n_1) \leq H(V_{R1})$. Now, we have $H(V_{R1}) \leq nC_{R1}$ by definition, and $H(V_{R1}) \leq H(V_{1R}, V_{2R}) \leq H(V_{1R}) + H(V_{2R}) = nC_{1R} + nC_{2R}$, since V_{R1} is a function of (V_{1R}, V_{2R}). Therefore, we have $nR_1 \leq I(X^n_1; Y^n_1) + n\min\{C_{R1}, C_{1R} + C_{2R}\}$, which gives (14). Similarly, we obtain, $nR_2 \leq I(X^n_2; Y^n_2) + n\min\{C_{R2}, C_{1R} + C_{2R}\}$, which is the outer bound in (16). For the remaining bounds, consider the following

$$\begin{aligned} nR_1 &= H(W_1) && (23)\\ &= H(W_1|W_2) && (24)\\ &= I(W_1; Y^n_1, V_{R1}|W_2) + H(W_1|Y^n_1, V_{R1}, W_2) && (25)\\ &\leq I(W_1; Y^n_1, V_{R1}|W_2) + n\epsilon_n && (26)\\ &= I(W_1; Y^n_1|W_2) + I(W_1; V_{R1}|Y^n_1, W_2) + n\epsilon_n && (27)\\ &\leq I(X^n_1; Y^n_1|X^n_2) + H(V_{R1}|Y^n_1, W_2)\\ &\quad - H(V_{R1}|Y^n_1, W_1, W_2) + n\epsilon_n && (28)\\ &\leq I(X^n_1; Y^n_1|X^n_2) + H(V_{R1}|W_2) + n\epsilon_n && (29)\\ &= I(X^n_1; Y^n_1|X^n_2) + H(V_{R1}|V_{2R}) + n\epsilon_n && (30)\end{aligned}$$

where (24) is from independence of W_1 and W_2, (26) is from Fano's inequality, $H(W_1|Y^n_1, V_{R1}) \leq n\epsilon_n$ and conditioning decreases entropy, (28) is from $W_i \rightarrow X^n_i \rightarrow Y^n_i$, $i = 1, 2$, (29) is from conditioning decreases entropy, and (30) is due to the fact that V_{2R} is a function of W_2 and V_{R1} is a function of V_{2R}. Moreover, we have,

$$H(V_{R1}|V_{2R}) \leq H(V_{1R}) \leq nC_{1R} \quad (31)$$
$$H(V_{R1}|V_{2R}) \leq H(V_{R1}) \leq nC_{R1}, \quad (32)$$

where (31) is from the fact that V_{R1} is a function of (V_{1R}, V_{2R}). Thus, the upper bound on nR_1 becomes $nR_1 \leq I(X^n_1; Y^n_1|X^n_2) + n\min\{C_{1R}, C_{R1}\}$, giving the bound (15). Bound (17) is proved similarly. □

The next result shows that, for the infrastructure relay if the links to the destinations form the bottleneck, i.e., $C_{R1} \leq C_{1R}$ and $C_{R2} \leq C_{2R}$, transmission of independent (rather than correlated) messages via the IR achieves capacity. In other words, in this regime *signal relaying* only is optimal. The theorem below is expressed in terms of the capacity region $\mathcal{C}_{IC}$ of a regular IC, which is generally unknown in single-letter formulation apart from special cases.

Theorem 3 (Multi-letter capacity region for $C_{R1} \leq C_{1R}$ and $C_{R2} \leq C_{2R}$): For an ICIR with $C_{R1} \leq C_{1R}$ and $C_{R2} \leq C_{2R}$, the capacity region $\mathcal{C}_{ICIR}$ is given by the capacity region $\mathcal{C}_{IC}$ of the IC, enhanced by (C_{R1}, C_{R2}) along the individual rates as

$$\mathcal{C}_{ICIR} = \{(R_1, R_2)\colon (R_1 - R'_1, R_2 - R'_2) \in \mathcal{C}_{IC}\},$$

for $R'_1 \leq C_{R1}$, $R'_2 \leq C_{R2}$. Equivalently, the capacity region $\mathcal{C}_{ICIR}$ is given by the union over the sets of rates (R_1, R_2) that satisfy

$$R_1 \leq \frac{1}{n}I(X^n_1; Y^n_1) + C_{R1} \quad (33)$$
$$R_2 \leq \frac{1}{n}I(X^n_2; Y^n_2) + C_{R2}, \quad (34)$$

for some input distributions $p(x^n_1)p(x^n_2)$ that satisfy the power constraints.

Proof: The converse follows immediately from (14) and (16) with the conditions $C_{Ri} \leq C_{iR}$, $i = 1, 2$. Achievability follows by sending independent messages, say (W_{1R}, W_{2R}) in the notation of Theorem 1, of rates C_{R1} and C_{R2} from sources S_1 and S_2, respectively, via the IR, and then using the Gaussian IC as a regular IC, stripped of the infrastructure links. For the latter channel, it is in fact known that the capacity region is given by the multi-letter expressions $R_i \leq \frac{1}{n}I(X^n_i; Y^n_i)$, $i = 1, 2$.□

Remark 1: Due to Theorem 3, in any scenario where a single-letter capacity region is known for the regular IC, the capacity result immediately carries over to the ICIR with $C_{R1} \leq C_{1R}$ and $C_{R2} \leq C_{2R}$. Therefore, for instance, we can obtain a single-letter capacity region expression for an ICIR in the strong interference regime ($a_{21} \geq 1$ and $a_{12} \geq 1$) [2] [3] or noisy interference regime [16] [17] [18], as long as $C_{R1} \leq C_{1R}$ and $C_{R2} \leq C_{2R}$.

Remark 2: Both Theorem 2 and 3 apply also to a general discrete memoryless ICIR (with the caveat of eliminating the power constraint).

While Theorem 3 provides a general capacity result for the case where the IR-to-destination links set the performance bottleneck, i.e., $C_{R1} \leq C_{1R}$ and $C_{R2} \leq C_{2R}$, we next investigate the capacity region for the complementary scenario in which such condition is not satisfied. We focus specifically on the case characterized by $C_{R1} \leq C_{1R}$ and $C_{2R} \leq C_{R2}$, where the extension to the *dual* scenario $C_{1R} \leq C_{R1}$ and $C_{R2} \leq C_{2R}$ will be straightforward (and not explicitly stated) by appropriately switching indices. Under the assumption at hand, the following rate

$$R_{ex_{12}} = \min\left\{C_{1R} - C_{R1},\ C_{R2} - C_{2R},\right\}$$

plays a key role. This can be interpreted as the *excess rate from* S_1 *to* D_2 *on the IR links*, once user 1 and user 2 have allocated the maximum possible rate on the IR links for *signal relaying*, namely $R_{1R} = \min\{C_{R1}, C_{1R}\} = C_{R1}$ and $R_{2R} = \min\{C_{R2}, C_{2R}\} = C_{2R}$.

Theorem 4: In a Gaussian ICIR with channel conditions $a_{21} \geq 1$ and $R_{ex_{12}} \geq \max\left\{0, \frac{1}{2}\log(1 + P_1 + a_{21}^2 P_2) - \frac{1}{2}\log(1 + a_{12}^2 P_1 + P_2)\right\}$, the following gives the capacity region,

$$R_1 \leq \frac{1}{2}\log\left(1 + P_1\right) + C_{R1} \quad (35)$$

$$R_2 \leq \frac{1}{2}\log\left(1 + P_2\right) + C_{2R} \quad (36)$$

$$R_1 + R_2 \leq \frac{1}{2}\log\left(1 + P_1 + a_{21}^2 P_2\right) + C_{R1} + C_{2R}. \quad (37)$$

Proof: The converse follows from Theorem 2. Namely, the upper bounds on individual rates (35) and (36) are a consequence of (15) and (17), while the upper bound on the sum rate (37) follows by summing (14) and (17) and accounting for the condition $a_{21} \geq 1$ as

$$\begin{aligned} R_1 + R_2 &\leq \frac{1}{n}I(X_1^n; Y_1^n) + \frac{1}{n}I(X_2^n; Y_2^n | X_1^n) \\ &+ \min(C_{R1}, C_{1R} + C_{2R}) + \min(C_{2R}, C_{R2}) \quad (38) \\ &= \frac{1}{n}I(X_1^n; Y_1^n) + \frac{1}{n}I(X_2^n; Y_2^n | X_1^n) \\ &+ C_{R1} + C_{2R} \quad (39) \\ &= \frac{1}{n}h(X_1^n + a_{21}X_2^n + Z_1^n) - \frac{1}{n}h(a_{21}X_2^n + Z_1^n) \\ &+ \frac{1}{n}h(X_2^n + Z_2^n) - h(Z_2^n) + C_{R1} + C_{2R} \quad (40) \\ &\leq \frac{1}{2}\log(1 + P_1 + a_{21}^2 P_2) + C_{R1} + C_{2R} \quad (41) \end{aligned}$$

where (38) is due to the conditions $C_{R1} \leq C_{1R}$ and $C_{2R} \leq C_{R2}$, (41) is from the worst-case noise result [6], i.e., $h(X_2^n + Z_2^n) - h(a_{21}X_2^n + Z_1^n) \leq n\log(1)$ for $a_{21} \geq 1$, and the first entropy is maximized by i.i.d. Gaussian inputs.

For achievability, we use the general result of Theorem 1, where the sources here transmit common messages $(W_{1c''}, W_{2c''})$ over the IC which are decoded at both destinations. In addition, S_1 transmits also the message $W_{1c'}$ to be decoded at D_1. The other rates are set to $R_{2c'} = R_{1p} = R_{2p} = 0$. The IR is used to transmit independent messages W_{1R}, W_{2R} with rates $R_{1R} = C_{R1}$ and $R_{2R} = C_{2R}$, but also message $W_{1c'}$ of rate $R_{1c'}$ to D_2 in order to facilitate interference cancellation. From Theorem 1, and applying Fourier-Motzkin elimination, we obtain the following achievable region

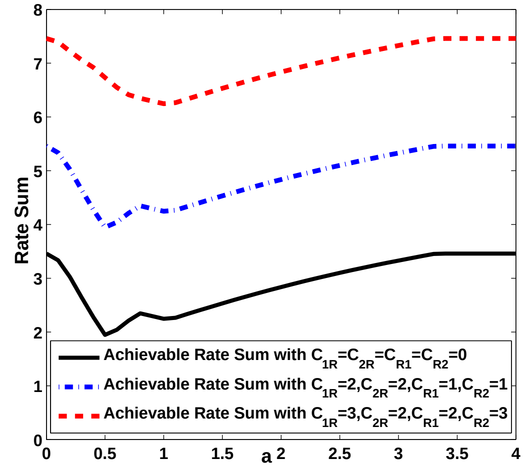

Fig. 2. Maximum achievable sum-rate (from Theorem 1) for various IR link capacities: $C_{1R} = C_{R1} = C_{2R} = C_{R2} = 0$ corresponds to the maximum sum-rate with no relay, $C_{1R} = C_{2R} = 2, C_{R1} = C_{R2} = 1$ satisfies the conditions in Theorem 3, and $C_{1R} = C_{R2} = 3, C_{2R} = C_{R1} = 2$, satisfies the conditions in Theorem 4.

$$R_1 \leq \frac{1}{2}\log(1 + P_1) + C_{R1} \quad (42)$$

$$R_2 \leq \frac{1}{2}\log(1 + P_2) + C_{2R} \quad (43)$$

$$R_1 + R_2 \leq \frac{1}{2}\log(1 + P_1 + a_{21}^2 P_2) + C_{R1} + C_{2R} \quad (44)$$

$$\begin{aligned} R_1 + R_2 &\leq \frac{1}{2}\log(1 + a_{12}^2 P_1 + P_2) + R_{ex_{12}} \\ &+ C_{R1} + C_{2R}, \quad (45) \end{aligned}$$

so that for $R_{ex_{12}} \geq \max\{0, \frac{1}{2}\log(1 + P_1 + a_{21}^2 P_2) - \frac{1}{2}\log(1 + a_{12}^2 P_1 + P_2)\}$, the claim is proved.$\square$

Remark 3: The assumptions in Theorem 4 encompass two different situations. In the first, we have the channel conditions $(1 - a_{12}^2)P_1 + (a_{21}^2 - 1)P_2 \leq 0$, so that the sum-rate bound (44) to receiver D_1 forms the performance bottleneck in terms of sum-rate irrespective of a positive excess rate $R_{ex_{12}}$ (which increases the sum-rate at D_2 as per (45)). Therefore, it can be seen that the capacity region of Theorem 4 is attained without performing interference forwarding, $R_{1c'} = 0$. In the second case, we have $(1 - a_{12}^2)P_1 + (a_{21}^2 - 1)P_2 > 0$, so that, conversely, the sum-rate bound (45) at D_2 may be more restrictive than (44). In this scenario, it can be seen that it is optimal to exploit the excess rate $R_{ex_{12}}$ to perform interference forwarding from S_1 to D_2 with rate equal to $R_{1c'} = \log(1 + P_1 + a_{21}^2 P_2) - \log(1 + a_{12}^2 P_1 + P_2)$.

Theorem 4 assumes that the excess rate satisfies a given lower bound. The next result considers a scenario where the complementary upper bound is assumed.

Theorem 5: In a Gaussian ICIR with the conditions $a_{12} \geq 1$, $0 \leq R_{ex_{12}} = C_{R2} - C_{2R} \leq \frac{1}{2}\log(1 + P_1 + a_{21}^2 P_2) - \frac{1}{2}\log(1 + a_{12}^2 P_1 + P_2)$, the following conditions gives the capacity region

$$R_1 \leq \frac{1}{2}\log(1 + P_1) + C_{R1} \quad (46)$$

$$R_2 \leq \frac{1}{2}\log(1 + P_2) + C_{2R} \quad (47)$$

$$R_1 + R_2 \leq \frac{1}{2}\log\left(1 + a_{12}^2 P_1 + P_2\right) + C_{R1} + C_{R2}. \quad (48)$$

Proof: The converse is again a consequence of Theorem 2. Specifically, the single rate bounds (46) and (47) follow immediately from (15) and (17), while the bound on the sum-rate (48) is obtained from the summation of (15) and (16) and exploiting the condition $a_{12} \geq 1$ as

$$\begin{aligned} R_1 + R_2 &\leq \frac{1}{n}I(X_2^n; Y_2^n) + \frac{1}{n}I(X_1^n; Y_1^n | X_2^n) \\ &+ \min(C_{1R}, C_{R1}) + \min(C_{R2}, C_{1R} + C_{2R}) \quad (49) \\ &= \frac{1}{n}I(X_2^n; Y_2^n) + \frac{1}{n}I(X_1^n; Y_1^n | X_2^n) \\ &+ C_{R1} + C_{R2} \quad (50) \\ &= \frac{1}{n}h(a_{12}X_1^n + X_2^n + Z_2^n) - \frac{1}{n}h(a_{12}X_1^n + Z_2^n) \\ &+ \frac{1}{n}h(X_1^n + Z_1^n) - h(Z_1^n) + C_{R1} + C_{R2} \quad (51) \\ &\leq \frac{1}{2}\log(a_{12}^2 P_1 + P_2 + 1) + C_{R1} + C_{R2}, \quad (52) \end{aligned}$$

where (49) is due to the conditions $C_{R1} \leq C_{1R}$ and $C_{R2} + C_{R1} \leq C_{1R} + C_{2R}$, (52) is from the worst-case noise result [6], i.e., $h(X_1^n + Z_1^n) - h(a_{12}X_1^n + Z_2^n) \leq n\frac{1}{2}\log(1)$ for $a_{12} \geq 1$, and the fact that first entropy is maximized by i.i.d. Gaussian inputs.

For the achievability, consider the achievable rate region given in Theorem 4 (42)-(45). Clearly, when the conditions in Theorem 5 which can also be written as $C_{R2} - C_{2R} \geq C_{1R} - C_{R1}$ and $R_{ex_{12}} \leq \frac{1}{2}\log(1 + P_1 + a_{21}^2 P_2) - \frac{1}{2}\log(1 + a_{12}^2 P_1 + P_2)$ are satisfied, (52) is achievable, hence gives the sum capacity. □

Remark 4: Similarly to the second case described in Remark 3, under the assumptions of Theorem 5, interference forwarding is useful in increasing the capacity region. However, unlike Theorem 4, here the excess rate is not large enough to make the sum-rate constraint (44) at D_1 the performance bottleneck with respect to (45). In fact, under the assumptions of Theorem 5, the sum-rate bound (45) for D_2 is always more restrictive than (44) for D_1 in terms of the sum-rate, given the constraint $R_{ex_{12}} \leq \log(1 + P_1 + a_{21}^2 P_2) - \log(1 + a_{12}^2 P_1 + P_2)$. The capacity region of Theorem 5 is attained by setting $R_{1c'} = R_{ex_{12}} = C_{R2} - C_{2R}$.

Remark 5: The assumptions in Theorem 5 imply the strong interference conditions $a_{12} \geq 1$ and $a_{21} \geq 1$. In contrast, the assumptions of Theorem 4 in general do not imply strong interference. This is, however, the case if we further assume the condition $(1 - a_{12}^2)P_1 + (a_{21}^2 - 1)P_2 \leq 0$ (as in the first case discussed in Remark 3).

V. Numerical Results

In Fig. 2, we show the maximum achievable sum-rate of Theorem 1 for different configurations of the IR link capacities and with $P_1 = P_2 = 10$ and $a_{21} = a_{12} = a$. Power allocations at the sources are optimized numerically. For comparison, we show the case $C_{1R} = C_{2R} = C_{R1} = C_{R2} = 0$. Moreover, we first consider a scenario where IR-to-destination links have smaller capacities than the source-to-IR links, $C_{1R} = C_{2R} = 2, C_{R1} = C_{R2} = 1$, thus falling within the assumptions of Theorem 3. It can be seen that the sum-rate increases by $C_{R1} + C_{R2} = 2$ for all values of a. Moreover, from Theorem 3, it is known that in the *noisy* [17] $\left(a(1 + 10a^2) \leq 0.5,\ \text{i.e, } a \leq 0.28\right)$ and *strong* ($a \geq 1$) [2] [3] interference regimes, the sum-rate shown in the figure is actually the sum-capacity. Finally, we consider a situation with $C_{1R} = C_{R2} = 3, C_{2R} = C_{R1} = 2$, which falls under the conditions of Theorem 4 for $a \geq 1$. As stated in the Theorem, for $a \geq 1$, the sum-rate shown in the sum-capacity and is $C_{R1} + C_{2R} = 4$ bits/channel use larger than the reference case of zero IR capacities.

VI. Conclusion

Relaying in an interference-limited system is investigated by focusing on a model in which orthogonal and finite capacity links exist between terminals and a dedicated relay. It is shown that for different conditions on the underlying Gaussian IC and on the links to/ from the relay, signal relaying only or both signal relying and interference forwarding by the relay is optimal. It is noted that such conclusions do not necessarily hinge on the availability of a single-letter capacity region for the Gaussian IC (see Theorem 3), but the latter is often instrumental to the proofs (see Theorem 4 and 5). Along these lines, extensions of these results to other scenarios where the capacity region the Gaussian IC is known [15]-[17] will be considered in future work. Also interesting is to impose further constraints on the links to and from the relay, such as broadcasting or multiple access interference.

References

[1] A. Carleial, "Interference channels", *IEEE Trans. Inform. Theory*, Vol. IT-24, pp. 60-70, 1978.

[2] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inform. Theory*, Vol. IT-27, pp. 49-60, 1981.

[3] H. Sato, "The capacity of the Gaussian interference channel under strong interference," *IEEE Trans. Inform. Theory*, Vol. IT-27, pp. 786-788, 1981.

[4] M. H. M. Costa, "On the Gaussian interference channel," *IEEE Trans. Inform. Theory*, Vol. IT-31, pp. 607-615, Sep. 1985.

[5] T. Cover and J. Thomas, *Elements of Information Theory*, New York: Wiley, 1991.

[6] T. Liu and P. Viswanath, "An extremal inequality motivated by multi terminal information theoretic problems,", *IEEE Trans. Inform. Theory*, Vol. IT-53, pp. 1839-1851, 2007.

[7] O. Sahin and E. Erkip, "On achievable rates for interference relay channel with interference cancellation," in *Proc. Asilomar Conference on SSC*, 2007.

[8] O. Sahin and E. Erkip, "Achievable rates for the Gaussian interference relay channel,"in *Proc. Globecom Conference*, 2007.

[9] I. Maric, R. Dabora and, A. Goldsmith, "On the capacity of the interference channel with a relay," in *Proc. ISIT*, 2008.

[10] R. Dabora, I. Maric, and A. Goldsmith, "Relay strategies for interference-forwarding", in *Proc. IEEE ITW*, 2008.

[11] O. Sahin and E. Erkip, "Cognitive relaying with one-sided interference", in *Proc. Asilomar Conference on SSC*, 2008.

[12] S. Sridharan, S. Vishwanath, S.A. Jafar, and S. Shamai, "On the capacity of cognitive relay assisted Gaussian interference channel" in *Proc. ISIT*, 2008.

[13] W. Yu and L. Zhou, "Gaussian Z-interference channel with a relay link: achievability region and asymptotic sum capacity," submitted to *Trans. on Inform. Theory*.

[14] H. Chong, M. Motani, H. Garg, and H. El Gamal, "On the Han-Kobayashi region for the interference channel", *IEEE Trans. on Inform. Theory*, Vol. IT-54, pp. 3188-3195, 2008

[15] R.H. Etkin, D. N. C. Tse, and H. Wang,"Gaussian interference channel capacity to within one bit", *IEEE Trans. Inform. Theory*, Vol. IT-54, pp. 5534-5562, 2008

[16] X. Shang, G. Kramer, and B. Chen, "A new outer bound and noisy-interference sum-rate capacity for the Gaussian interference channels," *IEEE Trans. Inform. Theory*, Vol. IT-55, pp. 689-699, 2009

[17] S. Annapureddy and V. Veeravalli, "Sum capacity of the Gaussian interference channel in the low interference regime," in *Proc. ITA Workshop*, 2008.

[18] A. S. Motahari and A. K. Khandani, "Capacity bounds for the Gaussian interference channel", *IEEE Trans. Inform. Theory*, Vol. IT-55, pp. 620-643, 2009.