An Information-Theoretic View of Spectrum Leasing via Secondary Cooperation

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***Abstract*— Spectrum leasing from a primary user to a set of secondary users may be implemented by requiring the secondary nodes to pay back the primary for the leased spectrum via cooperation (relaying). In this paper, this principle is studied from an information-theoretic standpoint by focusing on a scenario with one primary node and multiple secondary nodes, which may act as relays for the primary, communicating to a common receiver. The scenario is modelled as a multirelay channel where each relay (secondary user) has a private message for the destination. Achievable rate regions are derived for discrete memoryless and Gaussian models by considering Decode-and-Forward (DF), with both standard and parity-forwarding techniques, and Compress-and-Forward (CF), along with superposition coding at the secondary nodes. Numerical results for the Gaussian channel confirm that spectrum leasing via secondary cooperation is a promising framework to enable secondary spectrum access.**

I. Introduction

Spectrum leasing is a spectrum management technique that enables regulated coexistence between licensed (primary) and unlicensed (secondary) users in "cognitive radio" networks. With spectrum leasing, unlike the commons model of cognitive radio, primary users actively lease spectrum for secondary access in exchange for some remuneration (see, e.g., [1]). As proposed in [2], spectrum leasing may be effectively implemented by allowing secondary users to pay back the primary for the leased spectrum via cooperation, i.e., by relaying primary packets. With this solution, rather than attempting to fill so-called primary "white spaces", as for the commons model, secondary nodes can capitalize on good channel conditions from primary transmitters and to primary receivers to successfully relay primary packets and, through this, gain access to the primary-owned spectrum. Generally, the secondary nodes are interested in acquiring spectrum access through this mechanism, but only if a minimum quality-of-service (e.g., rate) constraint is guaranteed on their traffic.

While [2] focuses on simple transmission strategies that orthogonalize primary and secondary transmission (e.g., TDMA), in this paper, the idea of spectrum leasing via cooperation is more thoroughly studied from an information-theoretic standpoint. We concentrate on a scenario with one primary node and multiple secondary nodes communicating to a common receiver. The spectrum leasing problem (run at the primary nodes) consists of maximizing the primary rate over a set of strategies that allow for secondary cooperation, but only under the constraint that minimum secondary private rates (i.e., quality-of-service constraints) are guaranteed. Specifically, the scenario of interest is modelled as a multirelay channel where each relay (secondary user) has a private message for the destination (see Fig. 1). To address the spectrum leasing problem, achievable rate regions are proposed for discrete memoryless and Gaussian models via Decode-and-Forward (DF) and Compress-and-Forward (CF). The model of Fig. 1 and the proposed achievable schemes extend the analysis for the multirelay channel (without private relay messages) of [3]-[4] and the single-relay channel with private messages of [5]. Related works are [6], which studies the two-way relay channel with "piggybacking" of a private relay message, papers [7]-[8], which address a two-receiver broadcast channel where the relay is also the recipient of a private message, and [9], where "pairwise" cooperation is analyzed.

II. System Model

In this section, we describe the discrete memoryless multirelay channel with private messages (MCPM) of Fig. 1. While the proposed strategies are designed so that they can be easily extended to an arbitrary number K of secondary nodes (or relays), we focus for simplicity on the case of two secondary nodes, i.e., $K = 2$. Nodes indexed as $0, 1, 2$, and 3 identify the primary transmitter, secondary node 1, secondary node 2, and the common destination, respectively. Notice that we will refer to node 0 as either primary or source, and to nodes 1 and 2 as either secondary users or relays, to emphasize that the considered model applies more generally than only to the spectrum leasing scenario.

The channel of Fig. 1 is memoryless and used without feedback, and is defined by the conditional probability distribution $p(y_1, y_2, y_3|x_0, x_1, x_2)$, where x_0, x_1 and x_2 represent the inputs of the primary source, secondary node 1, and secondary node 2, respectively, which are chosen from the corresponding finite input alphabets $\mathcal{X}_0, \mathcal{X}_1$, and $\mathcal{X}_2$, whereas, y_1, y_2 and y_3 represent the outputs of secondary node 1, secondary node 2, and the destination, respectively, which

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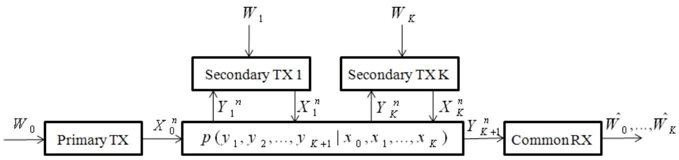

Fig. 1. Spectrum leasing via cooperation modelled as a multirelay channel where relays (secondary users) have private messages (MCPM) for the common destination: K relays are willing to collaborate with the primary transmitter on the condition that minimum individual secondary rates are guaranteed.

belong to the corresponding finite output alphabets $\mathcal{Y}_1, \mathcal{Y}_2$, and $\mathcal{Y}_3$.

A (n, R_0, R_1, R_2) code for the MCPM is defined by: (i) Three message sets $\mathcal{W}_0 = \{1, 2, ..., 2^{nR_0}\}$, $\mathcal{W}_1 = \{1, 2, ..., 2^{nR_1}\}$, and $\mathcal{W}_2 = \{1, 2, ..., 2^{nR_2}\}$; (ii) An encoding function at the primary node $f_0^{(n)}$: $\mathcal{W}_0 \rightarrow \mathcal{X}_0^n$ that maps the primary message into a codeword $x_0^n = f_0^{(n)}(w_0)$; (iii) $2n$ relay functions $f_{j,i}^{(n)}$: $\mathcal{Y}_j^{i-1} \times \mathcal{W}_j \rightarrow \mathcal{X}_j$, for relay index $j = 1, 2$ and $i = 1, ..., n$, that map previously received samples and private message into the symbol sent at each time i as $x_{1,i} = f_{1,i}^{(n)}(y_{1,1}, ..., y_{1,i-1}, w_1)$ and $x_{2,i} = f_{2,i}^{(n)}(y_{2,1}, ..., y_{2,i-1}, w_2)$; (iv) a decoding function at the destination d_3: $\mathcal{Y}_3^n \rightarrow \mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2$ as $(\hat{W}_0, \hat{W}_1, \hat{W}_2) = d(y_3^n)$.

Once an achievable rate region $\mathcal{R}$ of rates (R_0, R_1, R_2) has been obtained for a given transmission strategy, the *spectrum leasing problem* for the primary user (node 0) consists in selecting rates such that

$$\max R_0 \text{ s.t. } \begin{cases} (R_0, R_1, R_2) \in \mathcal{R} \\ R_j \geq R_{j,\min} \text{ for } j = 1, 2 \end{cases}, \quad (1)$$

so that the primary rate R_0 is maximized and the minimum quality-of-service constraints (rates) $R_{j,\min}$ requested by the cooperating secondary (relay) nodes are guaranteed. The definition can be immediately extended to more than two secondary nodes. Notice that the primary could also select only a subset of secondary nodes, say $\mathcal{S}$ (where possibly $\mathcal{S} = \varnothing$) for relaying, in which case problem (1) is easily adapted by considering the corresponding achievable rate region where only the selected relays are active and including only the rate constraints $R_j \geq R_{j,\min}$ for $j \in \mathcal{S}$. The final spectrum leasing decision will be obtained by choosing the subset $\mathcal{S}$ that maximizes the primary rate R_0. In the following, we focus on the derivation of achievable rate regions $\mathcal{R}$ and go back to problem (1) in Sec. V.

III. Achievable Rates

In this section, achievable rate regions for MCPM are derived using DF and CF.

A. Decode-and-Forward (DF)

Within the basic DF approach, we consider a number of possible strategies to be employed at source and relays. In particular, we consider at first an extension of the "*multihop*" technique of [10] and then of the *parity-forwarding* (PF) approach of [4] from the multirelay channel (without relay messages) to the MCPM. As discussed below, the proposed techniques also extend some of the results of [5] from the single-relay channel to the MCPM.

1) DF-MultiHop (DF-MH): In the MH strategy of [10] "downstream" nodes decode the source message (sent in a previous block) based on the signals received from the "upstream" nodes using sliding-window decoding. The destination may either use backward or sliding-window decoding (see, e.g., discussion in [12]). Here we propose an extension of this technique that allows transmission of the private secondary messages. In the proposed method, we perform successive interference cancellation at the destination, by decoding the primary message first (essentially treating the secondary private messages as noise) and then decoding the secondary message after having cancelled the primary signal.

Proposition 1: (DF-MH Achievable Rate Region) The convex hull of the union of all sets of rates (R_0, R_1, R_2) that satisfy

$$R_0 < \min\{I(X_0; Y_1 | X_1, U_1, U_2), \quad (2a)$$
$$I(X_0, U_1; Y_2 | X_2, U_2), \quad (2b)$$
$$I(X_0, U_1, U_2; Y_3)\} \quad (2c)$$

and

$$R_1 < I(X_1; Y_3 | X_0, X_2, U_1, U_2) \quad (3a)$$
$$R_2 < I(X_2; Y_3 | X_0, X_1, U_1, U_2) \quad (3b)$$
$$R_1 + R_2 < I(X_1, X_2; Y_3 | X_0, U_1, U_2) \quad (3c)$$

for some joint distribution

$$p(u_1, u_2) p(x_0 | u_1, u_2) p(x_1 | u_1, u_2) p(x_2 | u_2)$$
$$p(y_1, y_2, y_3 | x_0, x_1, x_2) \quad (4)$$

is achievable for the MCPM via DF-MH.

Proof: See Appendix A. ■

Remark 1: The two auxiliary random variables U_1 and U_2 represent the contribution of relay 1 and 2, respectively, to the primary transmission. Private relay messages of rates R_1 and R_2 are sent superimposed on such cooperative signals.

Remark 2: In the absence of secondary private messages ($R_j = 0$ for $j \in \{1, 2\}$), the achievable rate region reduces to the one derived in [10, Theorem 3.1] for the two-level relay channel (by setting $X_1 = U_1$ and $X_2 = U_2$). Also, with null primary rate ($R_0 = 0$), the result in Proposition 1 coincides with the MAC capacity region by setting the auxiliary random variables U_j to a constant. Finally, a special case of the achievable rates in Theorem 1 of [5] (with $R_{12} = 0$ and $V = X_1$ in the notation of [5]) can be recovered by letting $X_1 = U_1$ =constant (i.e., shutting down the first relay) and neglecting condition (2a).

2) DF-Parity Forwarding (PF): With the PF scheme proposed in [4], the first relay does not cooperate by forwarding the entire primary message sent in the previous block, as in DF-MH of [10], but only parity bits (this is also referred to as irregular encoding [12]). This design choice eases decoding requirements at the second relay that can now decode only the parity bits forwarded by the first relay. In particular, in the absence of relay private messages, reference [4] shows that the PF strategy improves over the MH approach of [10] if the channel from source to relay 2 is poor, and it achieves capacity for doubly degraded channels as defined in [4].

Proposition 2: (DF-PF Achievable Rate Region) The convex hull of the union of all sets of rates (R_0, R_1, R_2) that satisfy

$$\begin{aligned} R_0 \quad < \quad & \min\{I(X_0; Y_1|X_1, U_1, U_2), && (5a)\\ & I(X_0; Y_3|U_1, U_2) + I(U_1; Y_2|X_2, U_2), && (5b)\\ & I(X_0, U_1, U_2; Y_3)\} && (5c)\end{aligned}$$

and (3) for some joint distribution (4) is achievable for the MCPM via DF-PF.

Proof: See Appendix A. ■

Remark 3: The difference between the achievable rate regions of DF-MH and DF-PF is only in the inequalities (2b) and (5b). In fact, inequality (2b) accounts for the fact that DF-MH requires relay 2 to fully decode the source message, whereas (5b) reflects the fact that relay 2 only decodes the parity information transmitted by relay 1 (of rate $I(U_1; Y_2|X_2, U_2)$).

B. Compress-and-Forward (CF)

In this section, the basic idea is that each relay compresses its received signals and performs random binning before transmitting the quantization index (binning exploits the correlation with the received signal of the destination, which serves as side information), see, e.g., [12]. The bin index is sent superimposed on the codeword that carries the relay private message. The destination first decodes the secondary private messages, then decompresses the compressed relay observations and finally decodes the primary message.

Proposition 3: (CF Achievable Rate Region) The convex hull of the union of all sets of rates (R_0, R_1, R_2) that satisfy

$$R_0 < I(X_0; \hat{Y}_1, \hat{Y}_2, Y_3|U_1, U_2, X_1, X_2) \qquad (6)$$

and

$$\begin{aligned} R_1 \quad &< \quad I(U_1; Y_3|U_2) && (7a)\\ R_2 \quad &< \quad I(U_2; Y_3|U_1) && (7b)\\ R_1 + R_2 \quad &< \quad I(U_1, U_2; Y_3) && (7c)\end{aligned}$$

for some joint distribution that factorizes as

$$\begin{aligned} & p(x_0)p(u_1)p(u_2)p(x_1|u_1)p(x_2|u_2)p(\hat{y}_1|y_1, x_1, u_1)\\ & p(\hat{y}_2|y_2, x_2, u_2)p(y_1, y_2, y_3|x_0, x_1, x_2) \qquad (8)\end{aligned}$$

and satisfies the inequalities

$$I(\hat{Y}_1; Y_1|X_1) \leq I(X_1, \hat{Y}_1; Y_3|U_1, U_2, X_2) \qquad (9)$$

$$I(\hat{Y}_2; Y_2|X_2) \leq I(X_2, \hat{Y}_2; Y_3|U_1, U_2, X_1) \qquad (10)$$

$$I(\hat{Y}_1; Y_1|X_1) + I(\hat{Y}_2; Y_2|X_2) \leq I(X_1, X_2, \hat{Y}_1, \hat{Y}_2; Y_3|U_1, U_2) \qquad (11)$$

is achievable for the MCPM via CF.

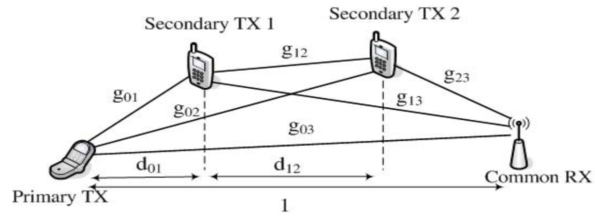

Fig. 2. The Gaussian MCPM. Channel gains g_{tr} depend on the distances d_{tr} and the path loss exponent υ.

Proof: See Appendix B. ■

Remark 4: The CF strategy does not require the secondary nodes to be aware of the primary codebooks, which may reduce the signalling burden between primary and secondary nodes in the spectrum leasing application. Moreover, unlike Propositions 1 and 2, here, random variables U_1 and U_2 represent the codebooks used to convey the private messages of secondary node 1 and secondary node 2, respectively (and not the cooperative signals).

Remark 5: The above achievable rates reduce to a special case of Theorem 2 of [5] (with $R_{12} = 0$, $U_1 = X_1$ in the notation of [5]) by removing the second relay (i.e., setting X_2, U_2, and $\hat{Y}_2$ to constants).

Remark 6: For all schemes developed in this paper, the destination may either use backward decoding or sliding window obtaining the same performance.

IV. Gaussian Model

In this section, we consider the Gaussian MCPM, which is defined by the following received signals for the secondary nodes and the destination:

$$\begin{aligned} Y_1 \quad &= \quad g_{01}X_0 + Z_1\\ Y_2 \quad &= \quad g_{02}X_0 + g_{12}X_1 + Z_2\\ Y_3 \quad &= \quad g_{03}X_0 + g_{13}X_1 + g_{23}X_2 + Z_3,\end{aligned}$$

where Z_1, Z_2 and Z_3 are independent, zero-mean, Gaussian noise with variances N_1, N_2 and N_3, respectively and g_{tr}^2 represent the channel power gains, as discussed below. Consider the geometry of Fig. 2 in which d_{tr} is the distance between transmitting node t and receiving node r. Assume that the distance between the source and the common destination is normalized ($d_{03} = 1$). This is equivalent to the simple collinear geometry in which all nodes lay on a straight line. The corresponding channel gains are then defined as $g_{tr}^2 = \frac{1}{d_{tr}^{\nu}}$ $\forall\ t \in \{0, 1, 2\}$, $r \in \{1, 2, 3\}$ and $t < r$, where ν is the path loss exponent. We impose the following power constraints $\frac{1}{n}\sum_{i=1}^{n} E[X_{ti}^2] \leq P_t$ for $t = 0, 1, 2$. In the following, we adapt the transmission strategies studied in the previous sections to the Gaussian model at hand.

A. Decode-and-Forward (DF)

We define $C(x) = \frac{1}{2}\log_2(1 + x)$, and $\bar{x} = 1 - x$.

Proposition 4: (DF-MH Achievable Rate Region) The convex hull of the union of all sets of rates (R_0, R_1, R_2) that

$$R_0 < \min\left\{ C\left(\frac{g_{01}^2\overline{\beta_1+\beta_2}P_0}{N_1}\right),\right. \quad (12a)$$

$$C\left(\frac{g_{02}^2\overline{\beta_2}P_0+g_{12}^2\overline{\gamma_1}P_1+2g_{02}g_{12}\sqrt{\beta_1\overline{\beta_2}\overline{\gamma_1}P_0P_1}}{g_{12}^2\gamma_1P_1+N_2}\right), \quad (12b)$$

$$\left. C\left(\frac{g_{03}^2P_0+g_{13}^2\overline{\gamma_1}P_1+g_{23}^2\overline{\gamma_2}P_2+2g_{03}g_{13}\sqrt{(\beta_1+\beta_2)\overline{\gamma_1}P_0P_1}+2g_{03}g_{23}\sqrt{\beta_2\overline{\gamma_2}P_0P_2}+2g_{13}g_{23}\sqrt{\beta_2\overline{\gamma_1}\overline{\gamma_2}P_1P_2}}{g_{13}^2\gamma_1P_1+g_{23}^2\gamma_2P_2+N_3}\right)\right\} \quad (12c)$$

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satisfy (12) and

$$R_1 < C\left(\frac{g_{13}^2\gamma_1P_1}{N_3}\right) \quad (13a)$$

$$R_2 < C\left(\frac{g_{23}^2\gamma_2P_2}{N_3}\right) \quad (13b)$$

$$R_1+R_2 < C\left(\frac{g_{13}^2\gamma_1P_1+g_{23}^2\gamma_2P_2}{N_3}\right) \quad (13c)$$

for some value of the parameters $0 \leq \beta_j \leq 1$ and $0 \leq \gamma_j \leq 1$, $j = 1, 2$, is achievable via DF-MH.

Remark 7: As seen in the proof below, parameters β_1 and β_2 control the power allocated by the source (node 0) to the source-to-destination message first transmitted in the previous blocks $b-1$ and $b-2$ respectively (according to the block-Markov strategy). Parameters γ_1 and γ_2 instead rule the power portions allocated by relay 1 and relay 2 respectively to convey private messages.

Proof: Follows from Proposition 1 by fixing the following auxiliary variables: $U_2 \sim N(0, \beta_2 P_1)$, $U_1' \sim N(0, \beta_1 P_1)$, $U_1 = U_2 + U_1'$, $X_0' \sim N(0, \overline{\beta_1+\beta_2}P_0)$, $X_1' \sim N(0, \gamma_1 P_1)$, $X_2' \sim N(0, \gamma_2 P_2)$, $X_0 = U_1 + X_0'$, $X_1 = \sqrt{\frac{\overline{\gamma_1}P_1}{(\beta_1+\beta_2)P_0}}U_1 + X_1'$ and $X_2 = \sqrt{\frac{\overline{\gamma_2}P_3}{\beta_2 P_1}}U_2 + X_2'$. ■

Proposition 5: (DF-PF Achievable Rate Region) The convex hull of the union of all sets of rates (R_0, R_1, R_2) that satisfy

$$R_0 < \min\left\{(12a), C\left(\frac{g_{03}^2\overline{\beta_1+\beta_2}P_0}{g_{13}^2\gamma_1P_1+g_{23}^2\gamma_2P_2+N_3}\right) + C\left(\frac{2g_{02}^2\beta_1P_1}{g_{02}^2\overline{\beta_1+\beta_2}P_0+g_{12}^2\gamma_1P_1+N_2}\right), (12c)\right\} \quad (14)$$

and (13) for some value of the parameters $0 \leq \beta_j \leq 1$ and $0 \leq \gamma_j \leq 1$, $j = 1, 2$, is achievable for the MCPM via DF-PF.

Proof: Follows from Proposition 2 by fixing the same auxiliary variables used in DF-MH. ■

Remark 8: A special case of the achievable rate in Corollary 1 of [5] (with $R_{12} = 0$ and $\alpha = 0$ in the notation of [5]) can be recovered from the Proposition above by setting all channel gains equal to 1, $P_1 = 0$, $\beta_1 = 0$ and neglecting condition (12a).

B. Compress-and-Forward (CF)

Proposition 6: (CF Achievable Rate Region) The convex hull of the union of all sets of rates (R_0, R_1, R_2) that satisfy (15)

$$R_1 < C\left(\frac{g_{13}^2\gamma_1P_1}{g_{03}^2P_0+g_{13}^2\bar{\gamma}_1P_1+g_{23}^2\bar{\gamma}_2P_2+N_3}\right) \quad (16a)$$

$$R_2 < C\left(\frac{g_{23}^2\gamma_2P_2}{g_{03}^2P_0+g_{13}^2\bar{\gamma}_1P_1+g_{23}^2\bar{\gamma}_2P_2+N_3}\right) \quad (16b)$$

$$R_1+R_2 < C\left(\frac{g_{13}^2\gamma_1P_1+g_{23}^2\gamma_2P_2}{g_{03}^2P_0+g_{13}^2\bar{\gamma}_1P_1+g_{23}^2\bar{\gamma}_2P_2+N_3}\right), \quad (16c)$$

$$N_{c_1} \geq \frac{g_{01}^2P_0N_3 + N_1\left(g_{03}^2P_0+N_3\right)}{g_{13}^2\bar{\gamma}_1P_1} \quad (17a)$$

$$N_{c_2} \geq \frac{g_{02}^2P_0N_3 + \left(g_{12}^2P_1+N_2\right)\left(g_{03}^2P_0+N_3\right)}{g_{23}^2\bar{\gamma}_2P_2}, \quad (17b)$$

and (17c) for some parameters $0 \leq \gamma_j \leq 1$, $j = 1, 2$, is achievable via CF.

Remark 9: Parameters γ_j rule the power portions allocated by relay 1 and relay 2 respectively to convey their respective private messages.

Proof: Follows from Proposition 3 by fixing the following auxiliary variables $X_0 \sim \mathcal{N}(0, P_0)$, $U_1 \sim \mathcal{N}(0, \gamma_1 P_1)$, $U_2 \sim \mathcal{N}(0, \gamma_2 P_2)$, $X_1' \sim \mathcal{N}(0, \bar{\gamma}_1 P_1)$, $X_2' \sim \mathcal{N}(0, \bar{\gamma}_2 P_2)$, $X_1 = U_1 + X_1'$, $X_2 = U_2 + X_2'$, $\hat{Y}_1 = Y_1 + Z_{c1}$ and $\hat{Y}_2 = Y_2 + Z_{c_2}$, where $Z_{c_1} \sim \mathcal{N}(0, N_{c_1})$ and $Z_{c_2} \sim \mathcal{N}(0, N_{c_2})$ are the independent compression noise of secondary node 1 and secondary node 2, respectively. ■

Remark 10: A special case of Corollary 2 in [5] (with $R_{12} = 0$ $P_{U_1} = P_0$ and $P_{U_2} = 0$ in the notation of [5]) can be recovered from the Proposition above by nulling g_{02} and P_2 and setting all other gains equal to 1.

V. Numerical Results

In light of the spectrum leasing problem (1), we start by assessing feasible secondary rate requirements $R_{j,\min}$ for given target primary rates R_0, by showing cross section plots of the achievable regions with DF-MH and DF-PF in the R_1, R_2 plane for different primary rates R_0 and fixed source-to-secondary-node distances $d_{01} = 0.3$, and $d_{02} = 0.4$ as shown in Fig. 3. In general, as the primary rate R_0 decreases, the set of feasible private rates R_1 and R_2 becomes larger, but the increase in secondary sum-rate becomes less relevant as the primary rate decreases. Moreover, DF-PF outperforms

$$R_0 < C\left(\frac{g_{01}^2 P_0 N_3 (N_2+N_{c_2}) + g_{03}^2 P_0 (N_1+N_{c_1})(N_2+N_{c_2}) - g_{01}^2 g_{02}^2 P_0^2 N_3}{(N_1+N_{c_1})(N_2+N_{c_2}) N_3}\right) \quad (15)$$

$$N_{c_1} N_{c_2} \geq \left(\frac{(g_{03}^2 P_0+g_{13}^2\bar{\gamma}_1 P_1+N_3)(g_{03}^2 P_0+g_{23}^2\bar{\gamma}_2 P_2+N_3)}{(g_{03}^2 P_0+g_{13}^2\bar{\gamma}_1 P_1+g_{23}^2\bar{\gamma}_2 P_2+N_3)^2}\right) \cdot \left(\frac{\left(g_{01}^2 P_0 N_3 + (N_1+N_{c_1})(g_{03}^2 P_0+N_3)\right)\left(g_{02}^2 P_0 N_3 + (g_{12}^2 P_1+N_2+N_{c_2})(g_{03}^2 P_0+N_3)\right)}{(g_{03}^2 P_0+N_3)^2}\right) \quad (17c)$$

DF-MH for sufficiently low primary rates. Finally, notice that nulling the primary rate retrieves the MAC capacity region.

Fig. 4 shows the maximum primary rate R_0 achievable via the proposed DF schemes for an equal secondary rate requirement $R_{1,\min} = R_{2,\min}$ (i.e., solution of problem (1)). We represent the relays positions as pairs of distances (d_{01}, d_{02}) (see Fig. 2). For comparison between spectrum leasing and a non-cooperative scenario, we also show the rate achievable in case none of the secondary nodes is selected, i.e., $P_1 = P_2 = 0$. Starting with DF-MH, Fig. 4 shows that when both secondary nodes are closer to the destination, i.e. $(0.8, 0.9)$, spectrum leasing gains are possible even for large secondary rates. Moving one or both of the secondary nodes closer to the source, i.e. $(0.1, 0.6)$ or $(0.1, 0.15)$, generally decreases the maximum supported secondary rates, but increases the potential primary rate gains. Comparing with DF-PF, we see that the latter may outperform DF-MH when the two relays are sufficiently close to one another (e.g., for $(0.1, 0.15)$). This is due the fact that with PF secondary node 2 decodes only the signal received from secondary node 1.

Finally, we investigate the possibility of selecting a subset of one of the relays for spectrum leasing, rather than selecting either both or none as done above (recall also discussion in Sec. II). To this end, Fig. 5 shows the maximum primary rate R_0 achievable via DF-MH and CF for the case where the relays are at positions $(0.8, 0.9)$, when the source leases spectrum to either the second relay (at position $d_{02} = 0.9$) or both relays. In general, selecting only one relay allows primary rate gains from spectrum leasing even for large secondary rate requirements $R_{j,\min}$. Conversely, for smaller secondary rate requirements $R_{j,\min}$ it is more advantageous to select both relays when using DF-MH, whereby coherent power gains are accrued, but not necessarily for CF, where the two compression indices are sent as independent codewords to the destination. Finally, when selecting only one relay, it can be seen that the proposed CF scheme attains some gain over DF-MH when the secondary node is closer to the destination.

VI. Concluding Remarks

In this work, we have provided an information-theoretic view on the spectrum leasing approach, by deriving achievable rates for a multirelay channel with private relay message, whereby relay nodes are interpreted as secondary users. The derived achievable rates extend a number of previous works. Moreover, they enable the study of the rate gains achievable by a primary node through spectrum leasing for given secondary rate requirements. Numerical results have shown that these gains can be significant, especially if the primary node is able to optimize the employed cooperation strategy based on the geometry of available secondary users.

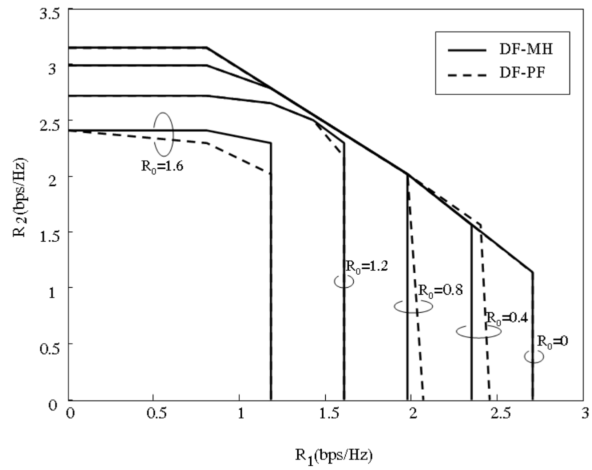

Fig. 3. Feasible secondary rate requirements R_1 and R_2 for different fixed primary rates R_0 via DF-MH and DF-PF ($d_{01} = 0.3$, $d_{02} = 0.4$, $P_0 = P_1 = P_2 = 1$, $N_1 = N_2 = N_3 = 0.1$ and $\nu = 4$).

Appendix A

Proof of Propositions 1 and 2

The messages W_0, W_1 and W_2 are split equally into $B-1$ blocks with rates R_0, R_1 and R_2, respectively. $w_{t,b}$ is the message sent at block b by node t, where $b = 1, 2, ..., B-1$. We focus for the proof on DF-MH. The necessary changes for DF-PF can be easily accommodated following [4].

Random Codebook Construction: We generate the following codebooks independently. Generate $u_2^n(w_0'')$; For each $u_2^n(w_0'')$, generate $u_1^n(w_0', w_0'')$; For each $u_2^n(w_0'')$ and $u_1^n(w_0', w_0'')$, generate $x_0^n(w_0, w_0', w_0'')$; For each $u_2^n(w_0'')$, generate $x_2^n(w_2, w_0'')$ sequences; For each $u_2^n(w_0'')$ and $u_1^n(w_0', w_0'')$, generate $x_1^n(w_1, w_0', w_0'')$, where $w_t \in \left[1, 2^{nR_t}\right]$.

Block-Markov Encoding: At block b, the source transmits $x_0^n(w_{0,b}, w_{0,b-1}, w_{0,b-2})$, where $w_{0,b-1}$and $w_{0,b-2}$ were denoted above as $w_0'$and w_0'', respectively; The secondary 1, having estimated $w_{0,b-1}$ and $w_{0,b-2}$, transmits $x_1^n(w_{1,b}, w_{0,b-1}, w_{0,b-2})$; The secondary 2, having estimated $w_{0,b-2}$, transmits $x_2^n(w_{2,b}, w_{0,b-2})$.

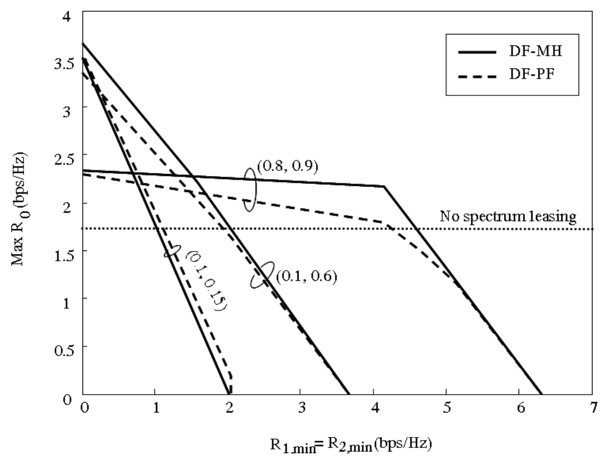

Fig. 4. Maximum primary rate R_0 achievable by spectrum leasing via cooperation using DF-MH and DF-PF for fixed and equal secondary rate requirements $R_{j,\min}$ and different distance pairs (d_{01}, d_{02}) ($P_0 = P_1 = P_2 = 1$, $N_1 = N_2 = N_3 = 0.1$, $\nu = 4$).

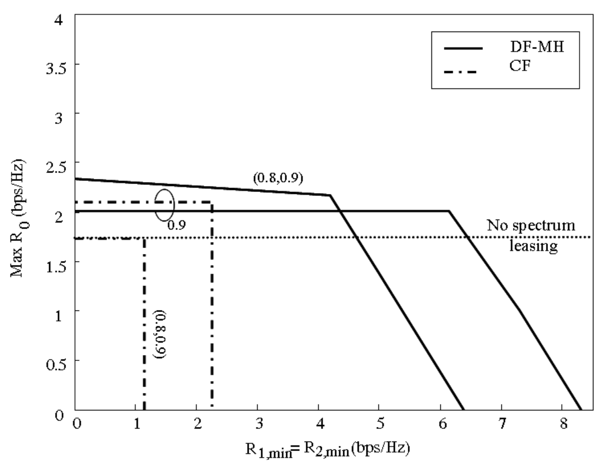

Fig. 5. Spectrum leasing via cooperation using DF-MH and CF for fixed and equal secondary rate requirements $R_{j,\min}$ and selection of either both relays or only the second ($d_{01} = 0.8$, $d_{02} = 0.9$, $P_0 = P_1 = P_2 = 1$, $N_1 = N_2 = N_3 = 0.1$, $\nu = 4$).

Decoding: At block b, secondary 1 searches for $\hat{w}_{0,b}$, such that $(u_1^n(w_{0,b-1}, w_{0,b-2}),\ u_2^n(w_{0,b-2}),\ x_0^n(\hat{w}_{0,b}, w_{0,b-1}, w_{0,b-2}), x_1^n(w_{1,b}, w_{0,b-1}, w_{0,b-2}), y_1^n(b)) \in T_\epsilon^n(P_{U_1,U_2,X_0,X_1,Y_1})$. Secondary 2 uses sliding window over two blocks to search for unique $\hat{w}_{0,b-1}$, such that $(u_1^n(\hat{w}_{0,b-1}, w_{0,b-2}),\ u_2^n(w_{0,b-2}),\ x_2^n(w_{2,b}, w_{0,b-2}),\ y_2^n(b)) \in T_\epsilon^n(P_{U_1U_2X_2Y_2})$ and $(u_1^n(w_{0,b-2}, w_{0,b-3}), u_2^n(w_{0,b-3}), x_0^n(\hat{w}_{0,b-1}, w_{0,b-2}, w_{0,b-3}),\ x_2^n(w_{2,b-1}, w_{0,b-3}),\ y_2^n(b-1)) \in T_\epsilon^n(P_{U_1U_2X_0X_2Y_2})$. Similarly, the destination uses sliding window over three previous blocks to estimate $\hat{w}_{0,b-2}$. Finally, the destination looks for unique $\hat{w}_{1,b}$ and $\hat{w}_{2,b}$ that are jointly typical with the received signal. It can be shown following standard steps that the above steps can be made reliable if (2) and (3) hold.

Appendix B
Proof of Proposition 3

Codebook Construction: The source generates $x_0^n(w_0)$ sequences; Generate $u_1^n(w_1)$ sequences; For each $u_1^n(w_1)$ generate $x_1^n(w_1, z')$ sequences; For each $u_1^n(w_1)$ and $x_1^n(w_1, z')$ generate $\hat{y}_1^n(w_1, z', z)$ sequences, where z and $z' \in [1, 2^{n\hat{R}_1}]$; Generate $u_2^n\ (w_2)$ sequences; For each $u_2^n(w_2)$ generate $x_2^n(w_2, q')$ sequences; For each $u_2^n(w_2)$ and $x_2^n(w_2, q')$ generate $\hat{y}_2^n(w_2, q', q)$ sequences, where q and $q' \in [1, 2^{n\hat{R}_2}]$.

Encoding: The source transmits $x_0^n(w_{0,b})$; Secondary 1 compresses the primary signal then transmits $x_1^n(w_{1,b}, z_{b-1})$; Secondary 2 compresses the primary signal then transmits $x_2^n(w_{2,b}, q_{b-1})$, where z_{b-1} and q_{b-1} are the indices of the compressed signals that were denoted above as z' and q'.

Decoding: Secondary 1 tries to find the index z_b of the quantized output $\hat{y}_1^n(w_{1,b}, z_{b-1}, z_b)$, such that $(\hat{y}_1^n(w_{1,b}, z_{b-1}, \hat{z}_b), y_1^n(b), x_1^n(w_{1,b}, z_{b-1})) \in T_\epsilon^n(P_{\hat{Y}_1,Y_1,X_1})$. This is possible with high probability if $\hat{R}_1 > I(\hat{Y}_1; Y_1|X_1)$. Analogously, for secondary 2 we have the condition $\hat{R}_2 > I(\hat{Y}_2; Y_2|X_2)$. Here we consider backward decoding at the destination. To decode z_{B-1} and q_{B-1}, the destination tries to find a pair $(\hat{z}_{B-1}, \hat{q}_{B-1})$ such that $(\hat{y}_1^n(1, \hat{z}_{B-1}, 1), \hat{y}_2^n(1, \hat{q}_{B-1}, 1), y_3^n(B), x_1^n(1, \hat{z}_{B-1}),\ x_2^n(1, \hat{q}_{B-1}), u_1^n(1),\ u_2^n(1)) \in T_\epsilon^n(P_{\hat{Y}_1,\hat{Y}_2,Y_3,X_1,X_2,U_1U_2})$. The decoding can be made reliable if $\hat{R}_1 < I(X_1, \hat{Y}_1; Y_3|U_1, U_2, X_2), \hat{R}_2 < I(X_2, \hat{Y}_2; Y_3|U_1, U_2, X_1)$, and $\hat{R}_1 + \hat{R}_2 < I(X_1, X_2, \hat{Y}_1, \hat{Y}_2; Y_3|U_1, U_2)$. At block $b < B$, the destination first decodes $w_{1,b}$ and $w_{2,b}$ then $w_{0,b}$ by looking for the corresponding typical sequences. This can be made possible if (7) and (6) hold respectively. The proof is concluded by Fourier-Motzkin elimination.

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