Dynamic Framed-ALOHA for Energy-Constrained Wireless Sensor Networks with Energy Harvesting

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Abstract—[1]The Dynamic Framed-ALOHA (DFA) protocol is studied for wireless sensor networks with energy limitations and *energy-harvesting* capability. The performance of DFA in this scenario is evaluated in terms of the *time efficiency* (or throughput), which is routinely used to evaluate medium access protocols, and by introducing a new metric, referred to as *detection efficiency*, which is tailored to scenarios with energy constraints. Specifically, detection efficiency measures the ability of a multiple access protocol to collect data from nodes without depleting their energy reserves. Analysis is first performed by assuming that DFA is operated with a perfect backlog (i.e., number of sensors left to be interrogated) knowledge. Then, a low-complexity backlog estimation algorithm is presented, which is shown by numerical results to perform close to the ideal case of perfect backlog knowledge.

I. INTRODUCTION

Recent advances in energy scavenging systems and low-power electronics enable the design of wireless sensor networks (WSNs) that collect energy from the surrounding environment. Such self-sustained and maintenance-free WSNs find applications ranging from distributed sensing to portable devices [1]. This new paradigm of energy-limited but potentially "everlasting" WSN calls for novel algorithmic solutions to standard networking and communication problems, such as medium access control [2].

In this work, we address the issue of medium access control in a WSN operated with a single *fusion center* (FC) [3] that collects data from a number of sensors. Sensors are energy-constrained but provided with energy-harvesting capabilities. As the medium access strategy of choice, we consider the *Dynamic Framed-ALOHA* (DFA) protocol [4], since it provides a good trade-off between complexity of implementation [5] and flexibility in adapting to highly variable number of transmitting sensors [4]. The goal of this paper is to analyze the performance of the DFA protocol in terms of the trade-off between two metrics: (*i*) *Time efficiency* (also referred to as throughput), which is routinely used to measure the number of sensor measurements collected in a given time (see, e.g., [4]); and (*ii*) *detection efficiency*, which is a new metric introduced here with the aim of quantifying the ability of a multiple access protocol to collect data from sensors without depleting their energy reserves. It is noted that such trade-off is unique to energy-constrained WSNs, and its analysis is

Fig. 1.   WSN with a single Fusion Center (FC) gathering data from M sensors, which are provided with energy storage and harvesting capabilities (here, $M = 2$).

one of the main contributions of this work. It will be shown, for instance, that the choice of the optimal number of time-slots per frame in DFA is drastically changed if one considers energy limitations and detection efficiency with respect to standard results without energy constraints [4]. Analysis is performed by first assuming perfect knowledge of the *backlog* (i.e., number of active sensors) and then proposing a low-complexity backlog estimator that is shown via numerical results to perform close to the ideal case of perfect backlog knowledge.

II. SYSTEM MODEL

We consider a WSN with a single FC that coordinates network operations, and M wireless sensors $S_1, S_2, ..., S_M$, provided with energy storage devices (ESD) and energy harvesting capabilities, as shown in Fig. 1. The FC retrieves a new set of measurements from sensors at each *inventory round* (IR). In each nth IR, every sensor S_m has a single measure to report to the FC. The FC starts a new IR every T_c [s] by broadcasting an initial *query* command (Q), providing network synchronization and instructions to sensors on how to access the (shared) channel. All sensors mandatorily participate to all IRs for which they have enough energy stored in their ESDs.

Sensor transmissions during an IR are planned according to the DFA protocol, as illustrated in Fig. 2. Accordingly, within each nth IR, time is organized in multiple *frames*. Each kth frame starts with a query (similar to the one that starts the IR), indicating the number $L_k(n)$ of *time-slots* for the current (kth) frame (of the nth IR) available for sensor transmissions. The size of the frame $L_k(n)$ is chosen by the FC according to the (estimated) number of sensors $B_k(n)$ in the backlog. The backlog $B_k(n)$ at the kth frame of the nth IR is composed by all sensors that have enough energy stored in their ESDs to allow transmission in the kth frame (to be explained below) and have not yet successfully transmitted in the current nth IR.

We set $L_k(n) = \lceil \rho \hat{B}_k(n) \rceil$, where $\hat{B}_k(n)$ is the FC estimate of $B_k(n)$, ρ is a design parameter and $\lceil \bullet \rceil$ is the nearest upper integer operator.

According to DFA, each sensor S_m in the backlogged set randomly chooses a time-slot of the current frame for transmission. When a single sensor selects a slot, no collisions occur and the transmitting sensor's measure is correctly detected by the FC; otherwise, no measures are correctly received [4][2]. The FC keeps announcing new frames until the backlog is empty, that is, until no more sensors are left to be read. An example is shown in Fig. 2. The duration of an IR is thus a random variable denoted by $T_{IR}(n)$ (for the nth IR), which is assumed to be $T_{IR}(n) < T_c$, so that successive IRs do not overlap (see Sec. VI for more details).

The energy stored in the ESD of the mth sensor S_m, during the kth frame of the nth IR, is denoted by $E_{m,k}(n) \in [0, E_{\max}]$ [J], where $E_{\max}$ is the ESD's capacity. The initial energy $E_{m,1}(1)$ at the beginning of the first IR ($n = 1$, $k = 1$) is a random variable with known probability density function (pdf) $f_E(\cdot)$. Each time a sensor transmits in a certain frame, it consumes a given amount of energy E_{fr} regardless the frame size. We thus assume that a sensor consumes a non-negligible amount of energy only when the radio is on (to transmit or receive). Therefore, the energy per frame E_{fr} accounts for reception of the Query message and ACK and for transmission in the selected slot (see Fig. 2). Given the above, the backlog $B_k(n)$ is given by all the sensors S_m whose measurements have not been detected in the first $k-1$ frames of the nth IR and that still have enough energy in their ESD for transmission, i.e., $E_{m,k}(n) \geq E_{fr}$ (*non-depleted* sensors).

Each sensor S_m is also equipped with an energy harvesting unit (EHU) that scavenges energy from the environment and stores it in the ESD. We consider that between the nth and $(n+1)$th IRs, the EHU of the mth sensor provides some energy $E_{H,m}(n) = P_{H,m}(n)T_c$. The harvested power $P_{H,m}(n)$ is modelled as a random variable, independent over sensor and IR indices, with known pdf $f_H(\cdot)$ that depends on the energy source. The dynamics of the energy harvesting process is generally slower than an IR. For instance, a sensor usually consumes a E_{fr} of several tens up to few hundreds of μJ when participating to a frame [6], while electromagnetic and small piezoelectric energy harvesters can provide few μJ per second [1]. Moreover, a standard IR lasts a few hundreds of ms for large number of sensors (see [7] for RFID systems). For this reason, we assume that the energy scavenged during an IR is either negligible or is not immediately available to be used by a sensor during the current IR (i.e., a sensor can use only the energy initially available in its ESD at the beginning of the current IR).

A. Performance Metrics

In this section, we review the concept of time efficiency (throughput) and introduce detection efficiency. Where it does
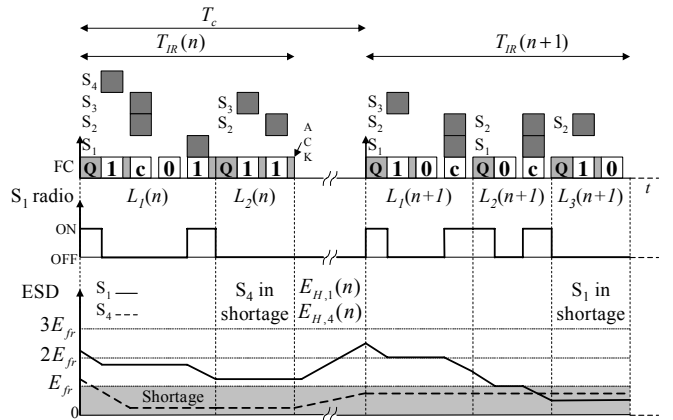
[2]We do not consider capture effects [5].

Fig. 2.   Illustration of the evolution of sensor ESDs along frames and IRs with $M = 4$. In the nth IR, all sensors have enough energy and the FC is able to detect them all, while in the $(n + 1)$th IR, S_4 is in shortage before the beginning of the IR, while S_1 gets depleted after having transmitted in frame 2. The backlog for each frame is indicated over each Q slot.

not cause confusion, we drop the IR index to avoid complicating the notation. First, note that the probability that j sensors out of the B_k in the backlog set simultaneously select the same time-slot in a frame of size $L_k = \lceil \rho B_k \rceil$, is given by [8]

$$p_j(B_k, \rho) = \binom{B_k}{j} \left(\frac{1}{L_k} \right)^j \left(1 - \frac{1}{L_k} \right)^{B_k - j}. \tag{1}$$

Of special importance is $p_1(B_k, \rho)$, which is the probability that a slot is occupied by only one sensor (*detection probability*). For large B_k, we can resort to the standard Poisson approximation of the binomial distribution, and obtain

$$p_1(B_k, \rho) \simeq \frac{1}{\rho} e^{-\frac{1}{\rho}} \triangleq p_d(\rho). \tag{2}$$

This approximation, routinely used in related analyses (see, e.g., [8]), is crucial for this paper since it allows us to consider the detection probability to be independent of the backlog B_k, since (2) is a function of the parameter ρ only.

1) Single-IR Time Efficiency: Based on the above, time efficiency of a single IR is defined as follows. Let $x_{k,i}(n) \in \{0, 1, c\}$ be the random outcome of the ith time-slot within the kth frame of the nth IR. Notation $x_{k,i}(n) = 0, 1, c$ indicate, respectively, empty slot (which happens with probability $p_0(B_k, \rho)$), detection (which happens with probability $p_1(B_k, \rho)$) and collision (with probability $p_c(B_k, \rho) = 1 - p_0(B_k, \rho) - p_1(B_k, \rho)$). The number of sensors detected during frame k of size $L_k(n)$ is the sum of all detections

$$D_k(n) = \sum_{i=1}^{L_k(n)} 1(x_{k,i}(n) = 1), \tag{3}$$

where $1(\bullet)$ is the indicator function.

Time efficiency of the nth IR measures the average number of sensors detected per time-slot during the IR, and is defined as (see also [4])

$$\eta_t(n, \rho) = \frac{E[D_{tot}(n)]}{E[L_{tot}(n)]}, \tag{4}$$

where $D_{tot}(n) = \sum_k D_k(n)$ is the total number of sensors detected in the nth IR and $L_{tot}(n) = \sum_k L_k(n)$ is the total number of time-slots issued in the nth IR. The expected values in (4) are taken with respect to the random selection of the time-slots (1) and the sensor energy distribution at the beginning of the IR. The latter is $f_E(\cdot)$ for the first IR and depends on the previous IRs and the energy harvesting process for successive IRs (see Sec. IV).

2) Single-IR detection Efficiency: The detection efficiency of a single IR measures the ability of a multiple access protocol to collect data from energy-constrained sensors in that IR. It is defined as the ratio between the number of sensors detected during an IR and the number of sensors in the backlog at the beginning of that IR. Mathematically, we define the *detection efficiency* as:

$$\eta_d(n, \rho) = \frac{E[D_{tot}(n)]}{E[B_1(n)]}, \qquad (5)$$

where $B_1(n)$ is the initial backlog for the nth IR (i.e., of the first frame $k = 1$). The expected values in (5) are taken as in (4). Notice that in (5) we considered $B_1(n)$ as initial backlog instead of M, since any given multiple access protocol can at most retrieve data from all non initially-depleted sensors (i.e., $B_1(n)$).

3) Steady-State (SS) Efficiencies: The time and detection efficiencies are defined above for a single IR. It may be of interest to analyze such measures for a "steady-state" IR, that is, for an IR with large n. This way, the effect of the initial distribution $f_E(\cdot)$ potentially vanishes and one can assess the asymptotic effectiveness of energy harvesting. Towards this end, we define the *steady-state* (*SS*) *time efficiency* as $\eta_{t,SS}(\rho) = \lim_{n \to \infty} \eta_t(n, \rho)$ and the *SS detection efficiency* as

$$\eta_{d,SS}(\rho) = \frac{1}{M} \lim_{n \to \infty} E[D_{tot}(n)], \qquad (6)$$

assuming that such limits exist. Notice that $\eta_{d,SS}(\rho)$ is not defined merely as the limit of $\eta_d(n, \rho)$ for $n \to \infty$, but is normalized on the total number of sensors M. This choice allows to better account for the effectiveness of the harvesting process. In fact, for instance, the limit of $\eta_d(n, \rho)$ could be close to one even in situations in which only very few sensors are actually active out of the overall population, while the WSN is not performing well since $B_1(n) \ll M$. Roughly speaking, steady-state detection efficiency accounts for the fraction of sensors that end up with a depleted ESD without having been able to convey their measurements in a steady-state IR.

III. Single-IR Performance with Known Backlog

In this section we study the performance of the DFA protocol, in terms of time (4) and detection (5) efficiencies for a single IR. We assume that the FC perfectly knows the size of the backlog B_k each time it announces a new frame of size $L_k = \lceil \rho B_k \rceil$ (neglecting the IR index). We denote the initial distribution (arbitrary) of the ESD $E_{m,1}$ of each sensor S_m as $f_E(\cdot)$ and assume that the $E_{m,1}$ variables are independent over m (this is rigorously true for the first IR only). We also

recall that energy harvesting is negligible within an IR. The size of the backlog of frame k, according to the DFA protocol described in Sec. II, can be written as:

$$B_k = B_{k-1} - D_{k-1} - U_{k-1} = C_{k-1} - U_{k-1}, \qquad (7)$$

where $C_{k-1} = B_{k-1} - D_{k-1}$ represents the number of sensors that collided in some slots of frame $k - 1$, while U_{k-1} represents the number of sensors that got depleted after having transmitted in frame $k - 1$ out of the collided set of C_{k-1} sensors. For the first frame $k = 1$ we set the initial conditions in (7) as $B_0 = M$, $D_0 = 0$, while U_0 is the number of sensors which are depleted at the beginning of the current IR. To evaluate $E[D_{tot}] = \sum_k E[D_k]$, we calculate

$$
\begin{aligned}
E[D_k] &= \sum_{B_k} E[D_k|B_k] \Pr[B_k] \\
&= \sum_{B_k} \sum_{i=1}^{L_k} E[1(x_{k,i} = 1)|B_k] \Pr[B_k] \\
&= \sum_{B_k} \sum_{i=1}^{L_k} p_1(B_k, \rho) \Pr[B_k] \\
&\simeq \rho p_d(\rho) \sum_{B_k} B_k \Pr[B_k] = \rho p_d(\rho) E[B_k]
\end{aligned} \qquad (8)
$$

where (8) follows from the approximation (2). Now, $E[B_k]$ follows from (7) as

$$E[B_k] = E[C_{k-1}] - E[U_{k-1}], \qquad (9)$$

where

$$E[C_{k-1}] = E[B_{k-1}] - E[D_{k-1}] \simeq E[B_{k-1}][1 - \rho p_d(\rho)], \qquad (10)$$

whereas $E[U_{k-1}]$ represents the average number of sensors that got depleted, out of the C_{k-1} that collided. $E[U_{k-1}]$ can be obtained according to following relation

$$E[U_k] = E[C_k] \Pr[E_{m,1} < kE_{fr}|E_{m,1} \geq (k-1)E_{fr}], \qquad (11)$$

where the probability above can be calculated based on the pdf $f_E(\cdot)$ of the initial energy $E_{m,1}$ (at the beginning of the considered IR). The probability at hand accounts for the event of having still energy left for the new (kth frame) after having transmitted (unsuccessfully) in all previous $k - 1$ frames. By substituting (10) and (11) into (9), after a little algebra we get:

$$E[B_k] \simeq M[1 - \rho p_d(\rho)]^{k-1} G_E(kE_{fr}), \qquad (12)$$

where $G_E(kE_{fr})$ is the complementary cumulative density function (ccdf) of $E_{m,1}$ ($G_E(y) = \int_y^{+\infty} f_E(x)dx$). The above follows from the relationship $\prod_{i=1}^k \Pr[E_{m,1} \geq kE_{fr}|E_{m,1} \geq (k-1)E_{fr}] = G_E(kE_{fr})$.

The preliminary analysis above allows to derive time and detection efficiencies. By substituting (12) into (8), time efficiency (4) becomes:

$$\eta_t(\rho) \simeq \frac{\sum_k E[D_k]}{\sum_k E[L_k]} = \frac{\rho p_d(\rho) \sum_k E[B_k]}{\rho \sum_k E[B_k]} = p_d(\rho), \qquad (13)$$

and similarly the detection efficiency (5) becomes:

$$\eta_d(\rho) \simeq \frac{\rho p_d(\rho)}{G_E(E_{fr})} \sum_{k=1}^{\infty} [1 - \rho p_d(\rho)]^{k-1} G_E(kE_{fr}). \qquad (14)$$

Notice that time efficiency (13) does not depend, as a consequence of approximation (2), on the ESD distribution $f_E(\cdot)$

but only on ρ, while the detection efficiency (14) depends on both.

A remark on approximation (2), used in the analysis above, is in order. One may argue that as frame sizes decrease along the IR, the accuracy of approximation (2) decreases accordingly, thus affecting the validity of the derived results. While the premise of this observation is in general true, we claim that, when starting the DFA from a sufficiently large initial backlog, small frames along an IR only add small contributions to the total metrics (D_{tot} and L_{tot}), thus not affecting noticeably the accuracy of (13) and (14). This is confirmed by our extensive numerical results (see Sec. VI).

Remark 1 (Optimal ρ): The optimal ρ (i.e., ratio between the frame size and backlog) that maximizes the time efficiency is well known to be $\rho = 1$ (see, e.g. [4]). This corresponds to set the size of a frame equal to the number of sensors in the backlog. However, when considering also detection efficiency, this choice may not be optimal anymore. In fact, the value of ρ that maximizes the detection efficiency is easily seen to be ρ arbitrarily large, since no collisions would occur. It follows that the trade-off between the two metrics can be traced by appropriate selections of parameter ρ (see Sec. VI).

IV. Steady-State Performance with Known Backlog and Energy Harvesting

The previous section dealt with the evaluation of time and detection efficiencies for a single IR. The goal of this section is to adapt the analysis to the assessment of steady-state time and detection efficiencies defined in Sec. II-A3. We still assume, as in the previous section, that the FC knows the backlog size. According to the definitions given in Sec. II-A3, in order to study the steady-state performance, one needs to evaluate the steady-state (if any) distribution of the energy $E_{m,1}(n)$ available in the ESD of any mth sensor at the beginning of the nth IR for large n, say $\pi_E(\cdot)$. Notice that this depends on the random slot selection process, and on the harvesting power pdf $f_H(\cdot)$, but not, for well-behaved scenarios, on the initial distribution $f_E(\cdot)$ (i.e., for $n = 1$). Once the steady-state distribution $\pi_E(\cdot)$, and thus its corresponding ccdf G_π, are found, one can immediately use formulas derived in the previous section to find SS time and detection efficiencies. Recall that IR time efficiency (13), as per approximation (2), does not depend on the ESD pdf and thus SS time efficiency is still given by (13). Whereas, similarly to (14), the SS detection efficiency (6) results

$$\eta_{d,SS}(\rho) \simeq \rho p_d(\rho) \sum_{k=1}^{\infty} \left[1 - \rho p_d(\rho)\right]^{k-1} G_\pi\left(kE_{fr}\right). \quad (15)$$

Deriving the exact steady-state distribution $\pi_E(\cdot)$ is not an easy task given the interaction among the evolutions of the ESDs of all sensors. This is due to the fact that the remaining energy in the ESDs determines the backlog, which in turn affects the probability of correct detection. However, under the large-backlog approximation (2), such dependence is broken since the detection probability does not depend on the current backlog. This allows us to derive $\pi_E(\cdot)$ by studying the
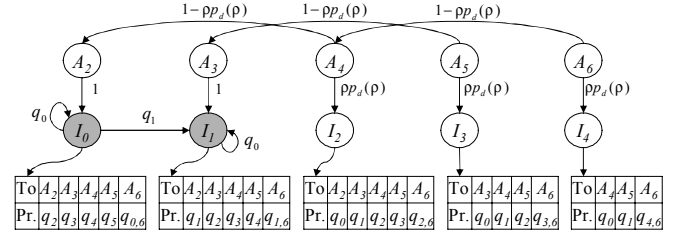

Fig. 3. Example of Markov chain to describe the evolution of the ESD of a test sensor ($N_\Delta = 6$, $r_\Delta = E_{fr}/\Delta E = 2$). In the tables upper and lower rows indicate destination states and the corresponding transition probabilities. Grey shaded states indicate energy shortage.

evolution of the ESD of a single sensor. We do so by building a Markov chain model for the evolution of energy $E_{m,k}(n)$ for a sensor S_m along the IR index n and frame index k. It is emphasized that this model is approximate in that it neglects interactions among the ESDs of different sensors thanks to the approximation (2). This model is developed in the next section. Uninterested readers can proceed to Sec. V, where a low-complexity scheme for backlog estimation is proposed.

A. Discrete Markov Chain (DMC) Model

The energy $E_{m,k}(n) \in [0, E_{\max}]$ stored in the ESD of the mth sensor before the beginning of the nth IR is represented by a discrete set of $N_\Delta + 1$ energy levels $S_E = \{0, \Delta E, 2\Delta E, ..., N_\Delta \Delta E\}$ with $N_\Delta = E_{\max}/\Delta E$. Parameter ΔE, referred to as *energy unit*, determines the accuracy of the discrete model in accounting for the harvested energy, which is a continuous variable. We also define the ratio $r_\Delta = E_{fr}/\Delta E$ (integer), which represents the number of energy units ΔE drawn from the ESD by a sensor when accessing a frame for transmission.

During a generic frame, the sensor at hand can be: *Active*, if participating in the current IR and not having been detected yet; or *Idle*, when it is either in energy shortage or it has already been detected in the current IR (or both). We denote the states of activity and idling as $A_j \in \{A_{r_\Delta}, ..., A_{N_\Delta}\}$ and $I_j \in \{I_0, ..., I_{N_\Delta - r_\Delta}\}$, respectively, where subscript j indicates that $j\Delta E$ J are stored in the ESD. Notice that activity states $\{A_j\}$ have at least r_Δ energy units (sensors are ready to transmit), while states $\{I_j\}$ can have at most $N_\Delta - r_\Delta$ energy units (for the best case in which a successful transmission has taken place when the ESD was full).

Recalling that a sensor transmits when in states $\{A_j\}$ and harvests energy when in states $\{I_j\}$, state transitions in the considered discrete Markov chain (DMC) are determined as follows (see Fig. 3 for an example): i) The sensor transmits successfully consuming r_Δ energy units: Transition from state A_j to state I_{j-r_Δ}. The probability of this event is $\rho p_d(\rho)$; ii) the sensor transmits unsuccessfully consuming r_Δ energy units: Transition from state A_j to either state A_{j-r_Δ}, if $(j - r_\Delta) \geq r_\Delta$, or to I_{j-r_Δ}, if $(j - r_\Delta) < r_\Delta$ (i.e., the sensor gets depleted). The probability of this event is $1 - \rho p_d(\rho)$; iii) harvesting of i energy units: Transition from state I_j to either state I_{j+i} if $(j + i) < r_\Delta$ (energy shortage) or to state

A_{j+i} if $r_\Delta \leq j + i \leq N_\Delta$ (enough energy to participate to the next IR). The probability of transitioning to either I_{j+i} or A_{j+i}, for $(j+i) < N_\Delta$, is given by $q_i = \Pr[i\Delta E \leq E_{H,m} < (i+1)\Delta E]$, while transition from I_j to A_{N_Δ} happens with probability $q_{j,N_\Delta} = 1 - \sum_{l=j}^{N_\Delta-1} q_l$, since the ESD is finite.

Based on the discussion above, one can build the transition matrix $\mathbf{P}$ for the Markov chain, having ordered the states, e.g., as $\{I_0, ..., I_{N_\Delta-r_\Delta}, A_{r_\Delta}, ..., A_{N_\Delta}\}$. Assuming that the DMC at hand is ergodic (this is in practice always true a part from degenerate cases such as probability of detection and harvesting approaching to zero), it admits a unique steady-state distribution, say $\boldsymbol{\phi} = [\phi_{I_0}, ..., \phi_{I_{N_\Delta-r_\Delta}}, \phi_{A_{r_\Delta}}, ..., \phi_{A_{N_\Delta}}]$, regardless the initial state distribution [9]. From this distribution, we are interested in calculating the pdf $\pi_E(\cdot)$ of the energy available immediately before the beginning of the nth IR for $n \to \infty$ (or, more precisely, a discrete approximation of $\pi_E(\cdot)$ with granularity ΔE). This can be done in two steps. First, evaluate the steady-state distribution of the DMC at the end of the current IR, and before the harvesting process, denoted by $\boldsymbol{\phi}_{H^-} = [\phi_{I_0|H^-}, ..., \phi_{I_{N_\Delta-r_\Delta}|H^-}, \mathbf{0}]$ with entries $\phi_{I_j|H^-} = \phi_{I_j}/\sum_{i=0}^{N_\Delta-r_\Delta} \phi_{I_j}, \forall j \in [0, N_\Delta - r_\Delta]$ and $\phi_{A_j|H^-} = 0, \forall j \in [r_\Delta, N_\Delta]$, since at the end of the IR the sensors is necessarily in states $\{I_0, ..., I_{N_\Delta-r_\Delta}\}$. The second step is to consider the state transitions due to harvesting. This can be done by applying the state transition matrix $\mathbf{P}$ to the vector $\boldsymbol{\phi}_{H^-}$ of probabilities before the harvesting as $\boldsymbol{\phi}_{H^+} = \boldsymbol{\phi}_{H^-}\mathbf{P}$. Vector $\boldsymbol{\phi}_{H^+}$ thus contains the steady-state probabilities of DMC states right after harvesting.

We can now map the DMC states to the energy level set S_E, by noticing that states A_j and I_j, for $j \in [r_\Delta, N_\Delta - r_\Delta]$, represent the same energy level in set S_E. Therefore, the steady-state ESD distribution, denoted by $\pi_E(j)$, for $j \in [0, N_\Delta]$, over the discrete set S_E, can be obtained as

$$\pi_E(j) = \begin{cases} \phi_{I_j|H^+} & \text{for } j \in [0, r_\Delta - 1] \\ \phi_{I_j|H^+} + \phi_{A_j|H^+} & \text{for } j \in [r_\Delta, N_\Delta - r_\Delta] \\ \phi_{A_j|H^+} & \text{for } j \in [N_\Delta - r_\Delta + 1, N_\Delta] \end{cases}$$
(16)

V. DFA WITH BACKLOG ESTIMATION

In this section we consider the DFA protocol when the backlog $B_k(n)$ is not assumed to be known a priori, but estimated along frames and IRs. Unlike previous work on the subject, in our scenario we need to account for the harvesting process when estimating the current backlog. While Maximum Likelihood [10] or Bayesian [11] estimation approaches are possible, their complexity become intractable for a large number of sensors [10]. Therefore, to keep complexity low, we propose a simple estimation algorithm and compare its performance to the perfect backlog knowledge case.

A. Backlog Estimation Algorithm

We propose a two-steps backlog estimation algorithm that works as follows: i) The FC estimates the initial backlog $B_1(n)$ of the nth IR based on the ccdf $G_{E(n)}$ of the energy stored in sensor ESD $E_{m,1}(n)$ (see Remark 2 below); ii)

the backlog estimates for the next frames are then obtained according to the channel outcomes and the residual ESD energy after the end of each frame.

More specifically, the initial estimate is $\hat{B}_1(n) = MG_{E(n)}(E_{fr})$. Assuming that the FC announced a frame of size $L_k(n) = \lceil \rho\hat{B}_k(n) \rceil$ in frame k, the backlog for frame $k+1$ is then obtained as follows: 1) The FC counts the number of time-slots $N_{c,k}(n)$ with a collision in frame k. Since the FC can only observe a collision (and not how many sensors collided in a slot), it estimates the total number of collided sensors as $\hat{C}_k(n) = \beta(\rho)N_{c,k}(n)$, where $\beta(\rho)$ accounts for the expected number of sensors per collided time-slot and is defined below. 2) Once the number of sensors left to be decoded (i.e., collided) $\hat{C}_k(n)$ is estimated, the FC estimates the number of sensors, within this set, that do not have enough energy for the next frame as $\hat{U}_k(n) = \hat{C}_k(n)P_s(k+1;n)$, where we have defined $P_s(k+1;n) = \Pr[E_{m,1}(n) < (k+1)E_{fr}|E_{m,1}(n) \geq kE_{fr}]$; 3) the backlog estimation for frame $k+1$ is thus (see (7)) $\hat{B}_{k+1}(n) = \hat{C}_k(n) - \hat{U}_k(n) = \beta(\rho)N_{c,k}(n)[1 - P_s(k+1;n)]$. To summarize, the proposed backlog estimation scheme is given by:

$$\hat{B}_k(n) = \begin{cases} M\Pr[E_{m,1}(n) \geq E_{fr}] & \text{if } k = 1 \\ \beta(\rho)N_{c,k-1}(n)[1 - P_s(k;n)] & \text{if } k > 1 \end{cases}.$$
(17)

We select parameter $\beta(\rho)$ by following the approach in [4], where it was proposed to set $\beta \simeq 2.39$ for $\rho = 1$, which corresponds to the average number of sensors per collided time-slot when the backlog is large. Notice that the choice $\rho = 1$ is optimal if one is concerned with time efficiency only (recall Remark 1). Since here we are interested in more general choices for ρ, we simply extend the result in [4] by calculating $\beta(\rho)$ for a generic ρ. Assuming a large backlog B, we have $E[\text{collided sensors}] = B[1 - p_1(B,\rho)] \simeq B[1 - p_d(\rho)]$ and $E[\text{collided time-slots}] = B[1 - p_1(B,\rho) - p_0(B,\rho)] \simeq B[1 - 2p_d(\rho)]$ so that $\beta(\rho) = \frac{E[\text{collided sensors}]}{E[\text{collided time-slots}]} \simeq \frac{\rho - e^{-\frac{1}{\rho}}}{\rho - 2e^{-\frac{1}{\rho}}}$.

Remark 2 (pdf of $E_{m,1}(n)$): A discrete approximation of the pdf $f_{E(n)}$ of the ESD energy $E_{m,1}(n)$ at the beginning of the nth IR can be obtained by resorting to the Markov model presented in Sec. IV-A. This can be done by exploiting the n-step transition property of DMCs [9] starting from the initial ESD energy pdf f_E (i.e., for $n = 1$). The so obtained pdf is further approximated since the DMC model assumes known backlog (see. Sec. IV-A), while here it is estimated. Moreover, this estimate does not use the observation obtained in the previous IRs, but our simulations show that it is accurate enough. This is due to the fact that the average behavior well represents each IR instance for sufficiently large number of sensors (see next section).

VI. NUMERICAL RESULTS

In this section we present some numerical simulation to validate the analytical results derived in this paper, and to get insight into the system design. DFA performance in terms of steady-state time and detection efficiencies versus ρ are shown
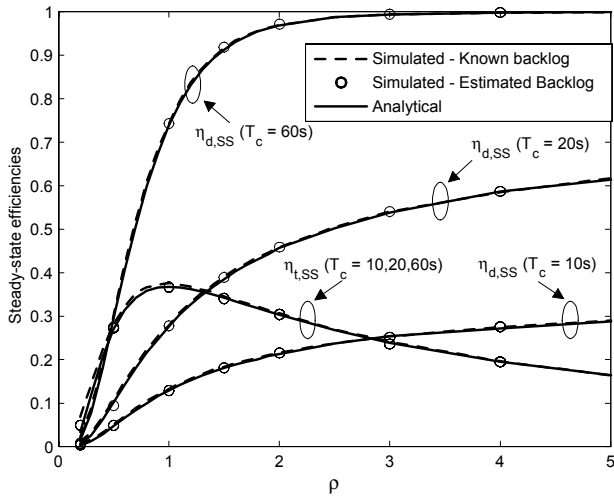
Fig. 4. Time and detection efficiencies versus the frame allocation parameter ρ, for both analytical and simulated results with known and estimated backlog ($M = 200$, $E_{fr} = 100\,\mu\text{J}$, $E_{\max} = 1\,\text{mJ}$, $\Delta E = 5\,\mu\text{J}$, $E[P_H] = 4\,\mu\text{W}$).

Fig. 5. Trade-off between steady-state detection and time efficiencies ($M = 200$, $E_{fr} = 100\,\mu\text{J}$, $E_{\max} = 1\,\text{mJ}$, $\Delta E = 5\,\mu\text{J}$, $E[P_H] = 4\,\mu\text{W}$).

in Fig. 4 for $M = 200$, $E_{fr} = 100\,\mu\text{J}$, $E_{\max} = 1\,\text{mJ}$, $\Delta E = 5\,\mu\text{J}$ and exponential pdf for the harvested power $f_H(\cdot)$ with average harvested power $E[P_H] = 4\,\mu\text{W}$ and $T_c = \{10, 20, 60\}s$. Notice that the initial distribution f_E is immaterial for steady-state performance. We compare the analytical results obtained in Sec. III and IV with simulated scenarios with both known and estimated backlog. The numerical results validate the analysis and approximations made above. Moreover, it is seen that the time efficiency is maximized for $\rho = 1$, while detection efficiency increases with ρ (recall Remark 1). Finally, the advantages of harvesting for larger IR interval T_c are apparent.

The trade-off between steady-state time and detection efficiencies, with same parameters as above, is shown in Fig. 5. The curves are obtained by maximizing the weighted sum $\alpha \eta_{t,SS}(\rho) / \max\{\eta_{t,SS}(\rho)\} + (1 - \alpha)\eta_{d,SS}(\rho)$ with respect to ρ. It can be seen that with larger IR intervals T_c, and thus more harvesting, one can increase the time efficiency (throughput) to a certain extent at almost no cost in terms of detection efficiency. However, for any finite harvesting rate, there is a clear trade-off between the two efficiencies.

Lastly, we can verify the assumption of non-overlapping IRs (i.e., $T_{IR}(n) < T_c$). Consider the typical duration of a time-slot of $T_{TS} = 1.5\,\text{ms}$ (see, e.g., [7]). The average duration of an IR, considering the worst case scenario where all M sensors are active, is given by $E[T_{IR}] = MT_{TS}/\eta_t \simeq MT_{TS}\rho e^{\frac{1}{\rho}}$. According to Figs. 4 and 5, it is reasonable to set $1 \leq \rho \lesssim 5$, since choosing $\rho \leq 1$ is wasteful for both time and detection efficiency. By setting $\rho = 5$, we get $E[T_{IR}] \simeq 1.83\,\text{s}$, which is much smaller than the considered values of T_c.

VII. CONCLUDING REMARKS

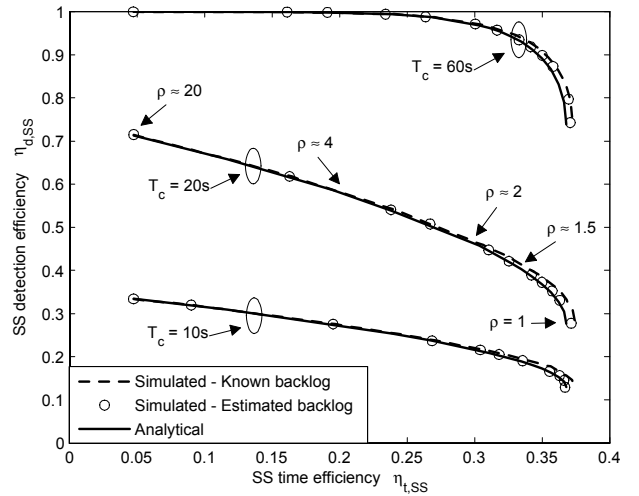The Dynamic Framed-ALOHA (DFA) protocol is studied for energy-constrained wireless sensor networks (WSNs)

with energy harvesting capabilities. Performance of DFA are evaluated in terms of the trade-off between the conventional time efficiency (throughput), and a new proposed metric, referred to as detection efficiency, which assesses the ability of a multiple access protocol to collect measurements from sensors without depleting their energy reserves. The proposed performance evaluation shown that conventional assumptions on the DFA protocol, such as the choice of the frame size, need to be revisited to account for the energy-constrained nature of wireless sensors with energy harvesting capabilities.

REFERENCES

[1] J.A. Paradiso, T. Starner, "Energy Scavenging for Mobile and Wireless Electronics," *IEEE Pervasive Computing*, vol. 4, no. 1, pp. 18-27, Jan.-Mar. 2005.

[2] V. Raghunathan, P.H. Chou, "Design and Power Management of Energy Harvesting Embedded Systems," in *Proc. ISLPED 2006*, Tegernsee, Germany, Oct. 2006.

[3] M. Rabbat, R. Nowak, "Distributed optimization in sensor networks," in *Proc. IPSN 2004*, Berkeley, CA, USA, Apr. 2004.

[4] F. C. Schoute, "Dynamic frame length ALOHA," *IEEE Trans. Comm.*, vol. 31, no. 4, pp. 565-568, Apr. 1983.

[5] J. E. Wieselthier, A. Ephremides, and L. A. Michaels, "An exact analysis and performance evaluation of framed ALOHA with capture," *IEEE Trans. Comm.*, vol.37, no.2, pp.125-137, Feb. 1989.

[6] B. Nilsson, L. Bengtsson, B. Svensson, "An application dependent medium access protocol for active RFID using dynamic tuning of the back-off algorithm," in Proc. RFID Conf. 2009, Orlando, FL, USA, Apr. 2009.

[7] W. Chonggang, M. Daneshmand, K. Sohraby; L. Bo, "Performance analysis of RFID Generation-2 protocol," *IEEE. Trans. Wireless Comm.*, vol.8, no.5, pp.2592-2601, May 2009.

[8] M. Kodialam and T. Nandagopal, "Fast and reliable estimation schemes in RFID systems," in Proc. MOBICOM 2006, Los Angeles, CA, USA, Sep. 2006.

[9] R. Gallager, *Discrete stochastic processes*. Kluwer Academic Publisher, 1995.

[10] B. Knerr, M. Holzer, C. Angerer, M. Rupp, "Slot-by-slot maximum likelihood estimation of tag populations in framed slotted aloha protocols," in Proc. SPECTS 2008, Edinburgh, UK, Jun. 2008.

[11] C. Floerkemeier, "Bayesian Transmission Strategy for Framed ALOHA Based RFID Protocols," in Proc. RFID Conf. 2007, Grapevine, TX, USA, Mar. 2007.