Throughput and Energy Efficiency of Opportunistic Routing with type-I HARQ in Linear Multihop Networks

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***Abstract*—Opportunistic routing is a well-known technique that exploits the broadcast nature of wireless transmissions and path diversity to form the route in an adaptive manner based on current channel conditions. This paper studies the throughput advantages of opportunistic routing over conventional multihop routing for linear multihop wireless networks with type-I Hybrid Automatic Repeat reQuest (HARQ) and quasi-static Rayleigh fading channels. The end-to-end throughput of opportunistic routing is derived using Markov chain tools and accounting for fading statistics. Both fixed-rate and optimal-rate transmissions are considered. Moreover, an investigation of the throughput using standard information-theoretic performance metrics for asymptotic signal-to-noise ratio regimes is provided. Specifically, the multiplexing gain and energy efficiency (i.e., minimum energy per bit) of both opportunistic and multihop routing are analyzed. Numerical results are given to corroborate the analysis.**

***Index Terms*—Opportunistic routing, energy efficiency, end-to-end throughput, spectral efficiency**

I. INTRODUCTION

Multihop routing is a conventional strategy used to forward a packet from source to destination through a number of hops in wireless ad hoc networks. Analysis of this class of protocols from a communication and information-theoretic standpoint has been pioneered by [1] under the assumption of links affected by additive white Gaussian noise only (i.e., no fading). This work shows that multihop transmission, with or without spatial reuse, performs very well in the *power-limited regime* (i.e., for *low signal-to-noise ratios*, SNRs), but becomes inefficient in the *bandwidth-limited regime* (i.e., for *high* SNRs). Analysis in the power-limited regime is performed using the standard measure of minimum energy per bit (over noise spectral density) required for a reliable transmission, $E_b/N_0|_{\min}$. Reference [2] extends these results to non-ergodic fading channels by studying the end-to-end outage probability, while ergodic fading is considered in [3]. Finally, [4] studies the end-to-end throughput for the same class of networks of [2] by assuming HARQ protocols to combat channel outages. However, no asymptotic SNR analysis is provided in [4].

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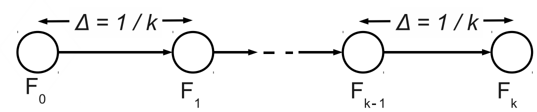

Fig. 1. A linear multihop network with k hops over quasi-static fading channels.

In a parallel line of investigation, a new routing paradigm has been introduced that potentially improves on standard multihop routing by exploiting the availability of multiple possible next hops in an adaptive manner: The next relay is selected based on the current channel conditions (and thus reception outcomes), as well as the distance to the destination [5]–[9]. These works focus on proposing different protocols to select the next hop based on alternative metrics.

In this work, we consider a linear multihop network over quasi-static fading channels as in [2], [4]. Our contributions are as follows: (*i*) We derive the end-to-end throughput of opportunistic routing with type-I HARQ[1]; (*ii*) We address the asymptotic regimes of high SNR (i.e., bandwidth-limited) and low SNR (i.e., power-limited) for both multihop and opportunistic routing, by studying the multiplexing gain and minimum energy per bit $E_b/N_0|_{\min}$ of the two schemes [10]. Throughout, we consider both the cases where the transmission rate is fixed and where the transmission rate can be optimized based on channel statistics. It is noted that concurrent work [11] also reports the average number of transmissions necessary to deliver a packet correctly (i.e., (*i*)), but without providing the asymptotic analysis of the end-to-end throughput (*ii*).

II. SYSTEM MODEL

We consider a linear multihop network, where the source F_0 wants to communicate with the destination F_k at a normalized distance of one, possibly taking advantage of a set of $k-1$ relays, $F_1, ..., F_{k-1}$, equally spaced with inter-node distance

[1]In Type-I HARQ error correction coding is used, but previous undecodable transmissions are discarded and detection is done only based on the current transmission [4].

$\Delta = 1/k$, as depicted in Fig. 1. All nodes work in half-duplex mode (i.e., they cannot receive and transmit at the same time). Transmission is organized in blocks of n (complex) channel uses each. Recall that, from standard theory, n (complex) channel uses over a bandwidth of $W\ Hz$ amount to n/W seconds in the absence of bandwidth expansion (see, e.g., [12]).

In each block, only one node (source or relay) is active, i.e., no spatial reuse is allowed. In the first block, the source F_0 transmits a packet of nR bits, where R is the rate of the first transmission in $bits/c.u.$ (channel use), or equivalently (in the absence of bandwidth expansion) in $bits/s/Hz$[2]. In the following blocks, the source may retransmit the packet or else the relays, upon decoding previous transmissions, may forward or retransmit the packet, until the final destination F_k correctly receives it. We will discuss different transmission policies in the next sections. In general, it is assumed that transmission of each block is followed by some signaling, such as ACKnowledgement/ Not ACKnowledgement messages. When the current packet is successfully received, a new packet is transmitted by the source F_0, and the procedure repeats. Notice that this amounts to assuming the source is always backlogged.

Let $y_j(b,t)$ be the discrete-time (complex) baseband sample received by node j, $j \in \{F_1, \ldots, F_k\}$ during the b-th block, at channel use t, $t = 1, \ldots, n$:

$$y_j(b,t) = \left(\frac{k}{|j-i|}\right)^{\eta/2} h_{ij}(b) x_i(b,t) + z_j(b,t), \quad (1)$$

where $z_j(b,t)$ is the complex white Gaussian noise term with zero mean and power $\mathbb{E}[|z_j(b,t)|^2] = N_0$ and $x_i(b,t)$ is the symbol transmitted by the currently active node i, $i \in \{F_0, \ldots, F_{k-1}\}$. We enforce the per-block power constraint

$$\frac{1}{n}\sum_{t=1}^{n} \mathbb{E}\left[|x_i(b,t)|^2\right] \leq P. \quad (2)$$

The channel coefficient between the i-th transmitter and the j-th receiver in (1), $h_{ij}(b)$, models quasi-static Rayleigh fading, i.e., it is a complex Gaussian random variable with zero mean and unit power, which is assumed to be constant within each block. Moreover, it is assumed to vary independently from block to block. Channels are known to the receivers but not to the transmitters. Finally, the term $(k/|j-i|)^{\eta/2}$ in (1) represents the path loss over the distance $d = |j-i|/k$ (i.e., $|j-i|$ hops) between transmitter i and receiver j with path-loss exponent η.

From (1), we define the signal-to-noise ratio γ as the ratio between the maximum average power received by F_k directly from source F_0 and the noise power N_0, $\gamma = P/N_0$. With this definition, we have that γ/d^η is the SNR for a transmission that covers distance $d \leq 1$.

Let $P_{out}(d)$ denote the probability that a certain packet transmitted by node i is not decoded correctly by node j with $d = |j-i|/k$. It is well known that this probability is given by the outage probability (see, e.g., [12]):

$$\begin{aligned} P_{out}(d) &= \Pr\left\{\log_2\left(1 + |h_{ij}|^2 \gamma d^{-\eta}\right) \leq R\right\} \\ &= 1 - \exp\left(-\frac{2^R - 1}{\gamma d^{-\eta}}\right), \end{aligned} \quad (3)$$

where we have used the fact that fading is Rayleigh.

We are interested in comparing the throughput, measured in $bits/s/Hz$, of multihop and opportunistic routing, both coupled with type-I HARQ. The next section defines the throughput and evaluates it for these two strategies.

III. Throughput Analysis

The goal of this section is to determine the end-to-end throughput for both multihop and opportunistic routing. We define the throughput $T(k,R)$ as the average number of *successfully* delivered bits per second per Hz, given the total number of hops k and the transmission rate R. Using renewal theory, it is possible to show that (see, e.g., [13]):

$$T(k,R) = \frac{nR}{n\,\mathbb{E}[N]} = \frac{R}{\mathbb{E}[N]} \quad [bits/s/Hz], \quad (4)$$

where N is the number of transmission blocks necessary to transmit a given packet correctly, starting from the original transmission by the source F_0 until correct decoding at the destination F_k.

While definition (4) applies to the case where the transmission rate R is selected by the application and fixed, in many scenarios devices can tune their transmission rate. Therefore, we also consider an alternative definition of throughput $T^*(k)$, in which the transmission rate R is optimized:

$$T^*(k) = \sup_{R \geq 0} T(k,R). \quad (5)$$

Notice that there is a clear and well-known trade-off in the optimization of R: Increasing R allows to send more bits to the destination (increasing the numerator in (4)), but also leads to an increased error probability and therefore to more transmissions (thus increasing the denominator in (4)). We also emphasize that optimization of rate R in (5) only requires knowledge of the channel statistics (and not of the instantaneous values) at the source.

The rest of the section derives the end-to-end throughput (4) and (5) for both multihop and opportunistic routing with type-I HARQ.

A. Multihop Routing

With multihop routing, each packet goes through all the k hops: The source F_0 retransmits the packet until relay F_1 decodes it successfully; Then, the link between F_1 and F_2 is operated in the same way, and so on, until the destination F_k decodes correctly. Assuming type-I HARQ on each hop, previous retransmissions are discarded at each receiver and the probability of outage for any transmission is given by (3) with

[2]Throughout the paper, we will use $bits/c.u.$ or $bits/s/Hz$ interchangeably.

$d = \Delta$. Thus, we easily evaluate the throughput (4) with fixed rate R as[3]:

$$T_{mh}(k, R) = \frac{R}{k}\left(1 - P_{out}(\Delta)\right) = \frac{R}{k}\exp\left(-\frac{2^R - 1}{\gamma k^\eta}\right). \quad (6)$$

Note that the result in Eq. (6) can also be interpreted by observing that each slot contains a successful transmission with probability $1 - P_{out}(\Delta)$, and since all transmitters use the same fraction of slots on average, the probability that a successful slot corresponds to a packet reaching the final destination is $1/k$.

For the throughput with optimized rate, we easily extend the result in [14] for a single-hop network ($k = 1$) to obtain the following Lemma.

Lemma 1. *The end-to-end throughput for a multihop scheme with k hops and optimized rate is given by*

$$T^*_{mh}(k) = \frac{R^*_{mh}}{k}\exp\left(-\frac{2^{R^*_{mh}} - 1}{\gamma k^\eta}\right), \quad (7)$$

$$\text{with} \quad R^*_{mh} = \frac{\mathcal{W}_0\left(k^\eta \gamma\right)}{\log_\mathrm{e} 2}, \quad (8)$$

where $\mathcal{W}_0(z)$, known as the Lambert $\mathcal{W}$ function, is the unique solution of the equation $\mathcal{W}(z)\,\mathrm{e}^{\mathcal{W}(z)} = z$, for $z > 0$.

B. Opportunistic Routing

With opportunistic routing, after each (re)transmission of the current packet, all decoding nodes issue an ACK message. If the destination is among the decoding nodes, transmission of the current packet is terminated and the next packet is transmitted by the source F_0. Otherwise, the transmitter for the next hop is selected opportunistically as the decoding relay that is the closest to the destination. The exact mechanism as to where and how the decision is made is not of concern here, and has been studied in [6]–[9]. With opportunistic routing, the average number $\mathbb{E}[N]$ of hops per packet can potentially be greatly reduced with respect to standard multihop routing, thus boosting the throughput (4) and (5).

To derive the throughput of opportunistic routing, we use the theory of Markov chains. Specifically, there are $k+1$ states in the chain, one for each node in the linear network, with state S_0 referring to scenarios where the current packet is at the source F_0, states S_i, $i = 1, \ldots, k-1$, similarly defined, and S_k representing the state where the destination has successfully decoded. Recalling that we assume type-I HARQ, the current transmitter retransmits the packet until at least one of the downstream nodes has successfully decoded. Based on this, the transition matrix can be found as

$$\mathbf{P} = \begin{bmatrix} P_{S,S}(0,0) & \ldots & P_{S,S}(0,k) \\ 0 & \ddots & \vdots \\ 0 & 0 & P_{S,S}(k,k) \end{bmatrix}, \quad (9)$$

where $P_{S,S}(i,j)$ is the probability that, given that the current state is i (i.e., the transmitter is node $i = 0, ..., k-1$), the next state is j (i.e., the next relay is j for $j = i, i+1, ..., k-1$ or the destination has decoded for $j = k$). The first k states are transient and the last state, corresponding to the packet having been received at the destination, is absorbing. The transition probabilities are given by:

$$P_{S,S}(k,k) = 1;$$
$$P_{S,S}(i,j) = \left(1 - P_{out}((j-i)\Delta)\right)\prod_{\ell=j+1}^{k} P_{out}((\ell - i)\Delta),$$
$$i = 0, \ldots, k-1, \quad j = i, \ldots, k;$$

$P_{S,S}(i,j) = 0$ otherwise.

Proposition 1 (End-to-end Throughput of Opportunistic Routing)**.** *The throughput (4) for fixed transmission rate R of opportunistic routing is given by*

$$T_{opp}(k, R) = \frac{R}{v_0}, \quad (10)$$

where v_0 is the first entry of vector $\mathbf{v} = [v_0, \ldots, v_{k-1}]$, which is evaluated as

$$\mathbf{v} = \left(\mathbf{I} - \mathbf{Q}\right)^{-1}\mathbf{1}, \quad (11)$$

where $\mathbf{1}$ is a $k \times 1$ vector with all entries equal to 1, matrix $\mathbf{Q}$ is obtained from $\mathbf{P}$ by removing the last row and the last column and $\mathbf{I}$ is the $k \times k$ identity matrix.

Proof: The proposition follows from the theory of absorbing Markov chains (see, e.g., [15, Sec. 4.5]). Let v_i be the expected number of steps before the chain is absorbed given that the chain starts in state S_i, $i = 0, \ldots, k-1$. Then, from standard first-step analysis, we have the set of equations (which is recursive, due to the triangular form of matrix (9)):

$$v_i = 1 + \sum_{j \neq k} P_{S,S}(i,j)\, v_j \quad \text{with} \quad i \neq k, \quad (12)$$

which is the same result of [11, Eqs. (11-12)]. Equation (11) is readily obtained from the matrix formulation for such set of equations (see, e.g., [15, Sec. 4.5]). ■

Remark 1. When k is large, closed-form (i.e., non-recursive) expressions for (11) are very involved. Here, we report the throughput for $k = 2$:

$$T_{opp}(2,R) = \frac{R\left[1 - \left(1 - \mathrm{e}^{-\frac{2^R-1}{2^\eta \gamma}}\right)\left(1 - \mathrm{e}^{-\frac{2^R-1}{\gamma}}\right)\right]}{2 - \mathrm{e}^{-\frac{2^R-1}{\gamma}}}. \quad (13)$$

Unfortunately, a closed-form expression for $T^*_{opp}(k)$ appears to be hard to find. In the next section, we shed some light on the performance analysis by focusing on asymptotic SNR regimes for both multihop and opportunistic routing.

IV. Asymptotic Analysis

In this section, we focus on the asymptotic regimes of high and low SNR.

[3]The number of retransmissions N is a geometric random variable with success probability given by $(1 - P_{out}(\Delta))$.

A. *High* SNR *(Bandwidth-Limited Regime)*

Consider first the case where the SNR is large (i.e., the bandwidth-limited regime). For a fixed transmission rate R, it is meaningful to consider the value of the throughput (4) as $\gamma \to \infty$, since the throughput remains necessarily finite, being bounded by R. However, when optimizing the transmission rate R, the throughput (5) scales with SNR, so that it is more meaningful to study the multiplexing gain, defined as $\lim_{\gamma\to\infty} T^*(k)/\log_2\gamma$ (see, e.g., [12]).

Proposition 2 (High-SNR Characterizations)**.** *The high-*SNR *throughput (4) with fixed transmission rate R is given by:*

$$\lim_{\gamma\to\infty} T_{mh}(k,R) = \frac{R}{k}, \tag{14}$$

for multihop routing, wheras for opportunistic routing we have

$$\lim_{\gamma\to\infty} T_{opp}(k,R) = R. \tag{15}$$

When rate R is optimized, the multiplexing gain of throughput (5) is

$$\lim_{\gamma\to\infty} \frac{T^*_{mh}(k)}{\log_2\gamma} = \frac{1}{k}, \tag{16}$$

for multihop routing, whereas for opportunistic routing we have the bounds

$$\lim_{\gamma\to\infty} \frac{T^*_{opp}(k)}{\log_2\gamma} = 1. \tag{17}$$

Proof: The results (14) and (15) follow easily from the fact that $P_{out}(d) \to 0$ for a fixed rate R and any d. Specifically, for opportunistic routing, this implies that direct transmission from source to destination is successful with high probability. The multiplexing gain in (16) is derived from eq. (9) in [14], conveniently adapted. Finally, to show (17), we prove that:

$$1-\epsilon \le \lim_{\gamma\to\infty} \frac{T^*_{opp}(k)}{\log_2\gamma} \le 1, \tag{18}$$

where $\epsilon > 0$ is arbitrarily small. The upper bound follows from cut-set arguments: The throughput of opportunistic routing cannot be larger than the throughput of a system where relays and destination fully cooperate for decoding. Moreover, the throughput of the latter system is upper bounded by its ergodic capacity, i.e., by $\mathbb{E}[\log_2(1+\sum_{j=1}^{k}|h_{0j}|^2\gamma(k/j)^\eta)]$ [4], whose multiplexing gain is one [12]. To obtain the lower bound in (18) it is enough to consider the following suboptimal transmission scheme: Set the rate at $R(\gamma) = \log_2\gamma^{(1-\epsilon)}$ and consider only the link between source and destination. This scheme clearly sets a lower bound on the achievable throughput, namely $T^*_{opp}(k) \ge R(\gamma)\exp\left(-\frac{2^{R(\gamma)}-1}{\gamma}\right)$. Letting ϵ be arbitrarily small in (18) we conclude the proof. ■

Discussion of the results of Proposition 2 is postponed to Section V.

[4]This follows immediately from the definition of throughput, noting that the latter can be written as $\mathbb{E}[R\cdot 1(\log_2(1+\sum_{j=1}^{k}|h_{0j}|^2\gamma(k/j)^\eta)) < R)] \le \mathbb{E}[\log_2(1+\sum_{j=1}^{k}|h_{0j}|^2\gamma(k/j)^\eta))]$, where $1(\cdot)$ is the indicator function.

B. *Low* SNR *(Power-Limited Regime)*

We now focus on the energy efficiency of multihop and opportunistic routing. Specifically, we evaluate the minimum energy per bit required for reliable transmission, which is defined as [10]

$$\left.\frac{E_b(k,R)}{N_0}\right|_{\min} = \inf_\gamma \frac{\gamma}{T(k,R)} \tag{19}$$

for transmission with fixed rate R and

$$\left.\frac{E_b(k)^*}{N_0}\right|_{\min} = \inf_\gamma \frac{\gamma}{T^*(k)} \tag{20}$$

for transmission with optimized rate. As already explained in Section I, this is a standard measure on the performance of transmission schemes [10] and has been considered in related routing scenarios in [1], [2].

Proposition 3 (Energy Efficiency of Multihop Routing)**.** *The minimim energy per bit for multihop routing with fixed transmission rate is given by:*

$$\left.\frac{E_b(k,R)}{N_0}\right|_{\min,mh} = \inf_\gamma \frac{\gamma}{T_{mh}(k,R)} = \mathrm{e}\,k^{1-\eta}\frac{2^R-1}{R}, \tag{21}$$

whereas for optimized rate, we have:

$$\left.\frac{E_b(k)^*}{N_0}\right|_{\min,mh} = \inf_\gamma \frac{\gamma}{T^*_{mh}(k)} = \mathrm{e}\,k^{1-\eta}\log_e 2. \tag{22}$$

Proof: The first equality, (21), is obtained by noticing that the convexity of the exponential function implies that $\gamma/T_{mh}(k,R)$ is also convex, and therefore the minimum is found where its derivative is zero. The optimal value of the SNR, which maximizes energy efficiency, is

$$\gamma = (2^R-1)k^{-\eta}. \tag{23}$$

For (22) we first note that the quantity

$$\frac{\gamma}{T^*_{mh}(k)} = \frac{k\gamma\log_\mathrm{e}2}{\mathcal{W}_0(k^\eta\gamma)}\exp\left(\frac{1}{\mathcal{W}_0(k^\eta\gamma)} - \frac{k^{-\eta}}{\gamma}\right), \tag{24}$$

where we have exploited the equation $\exp(\mathcal{W}(z)) = z/\mathcal{W}(z)$, is an increasing function of the SNR and therefore:

$$\inf_\gamma \frac{\gamma}{T^*_{mh}(k)} = \lim_{\gamma\to0}\frac{\gamma}{T^*_{mh}(k)}. \tag{25}$$

We then expand (24) using the first two terms of the Taylor expansion of $\mathcal{W}_0(z) = \sum_{n=1}^{\infty} z^n(-n)^{n-1}/n!$ (see, e.g. [14]) to obtain the approximation for small SNR

$$\frac{\gamma}{T^*_{mh}(k)} \cong \frac{k\log_\mathrm{e}2}{k^\eta - k^{2\eta}\gamma}\exp\left(\frac{1}{1-k^\eta\gamma}\right).$$

The proof is concluded by evaluating the limit (25). ■

Remark 2. We emphasize that, from the proof given above, the value of SNR that maximizes the energy efficiency (19) for multihop routing with fixed transmission rate is $\gamma = (2^R-1)k^{-\eta}$, whereas if one allows optimal rate selection (20) the optimal $\gamma \to 0$. This is due to the well-known fact that energy efficiency is maximized at vanishing spectral efficiencies, that

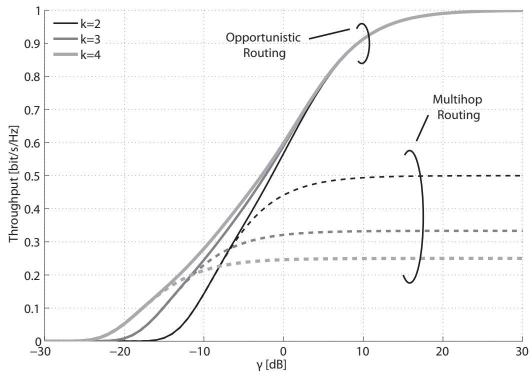

Fig. 2. End-to-end throughput of opportunistic routing, $T_{opp}(k,R)$ (solid lines), and multihop routing, $T_{mh}(k,R)$ (dashed lines), versus SNR, γ, for transmission rate $R=1$ $[bits/s/Hz]$ and $\eta=3$.

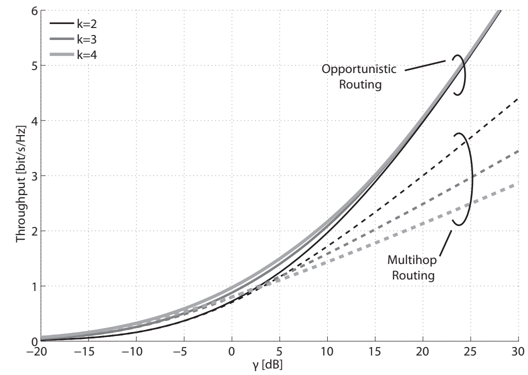

Fig. 3. End-to-end throughput of opportunistic routing, $T^*_{opp}(k,R)$ (solid lines), and multihop routing, $T^*_{mh}(k,R)$ (dashed lines), versus SNR, γ, for optimized rate and $\eta=3$.

is $R \to 0$ and $\gamma \to 0$, if one can optimize the transmission rate [10].

Remark 3 (Wideband slope for multihop routing). Beside the minimum energy per bit $E_b/N_0|_{\min}$, reference [10] defines also the slope $\mathcal{S}_0$ of the spectral efficiency at $E_b/N_0|_{\min}$ in order to provide a more complete description of the rate behavior in the power-limited regime. This can be easily calculated for multihop routing with optimized rate and is given by [10]:

$$\begin{aligned}\mathcal{S}_{0,mh} &= -\frac{2\left[\dot{T}^*_{mh}(k)\Big|_{\gamma=0}\right]^2}{\ddot{T}^*_{mh}(k)\Big|_{\gamma=0}} \\ &= \frac{2}{\mathrm{e}\,k} \quad [bits/s/Hz/(3dB)], \end{aligned} \tag{26}$$

where $\dot{T}^*_{mh}(k)|_{\gamma=0}$ and $\ddot{T}^*_{mh}(k)|_{\gamma=0}$ denote the first and second derivative of the end-to-end throughput curve evaluated in $nats/s/Hz$ with $\gamma=0$. Note that the slope of the throughput of the multihop (and opportunistic) routing for fixed rate turns out not to be well-defined due to the fact that (see Remark 2) the energy $E_b/N_0|_{\min}$ is not attained for vanishing throughput (see also discussion in [10]).

General expressions for energy efficiency in the case of opportunistic routing are difficult to obtain. In the rest of section, we consider some approximation for $k=2$.

1) Energy-Efficiency Approximations for Opportunistic routing: Here we consider analytical approximations for the minimum energy per bit with fixed rate and $k=2$, $E_b(2,R)/N_0|_{\min,opp}$, and optimized rate, $E_b(2)^*/N_0|_{\min,opp}$ (recall (19) and (20), respectively). The approximations are based on suboptimal choices for the optimal SNR in (23) and optimal rate in (8), that are expected to be close-to-optimal. The key observation is that, for low SNR, opportunistic routing will be often forced to use all the hops like multihop routing. Therefore, setting the optimal SNR and rate to the corresponding optimal SNR and rate for multihop routing (namely $\gamma=(2^R-1)k^{-\eta}$ and $R^*_{mh}=\mathcal{W}_0\left(k^\eta\gamma\right)/\log_{\mathrm{e}}2$) leads to potentially good approximations. Using these choices, we obtain

$$\frac{E_b(2,R)}{N_0}\bigg|_{\min,opp} = \inf_\gamma \frac{\gamma}{T_{opp}(2,R)} \simeq \frac{\mathrm{e}\left(2\,\mathrm{e}^{2^\eta}-1\right)\left(2^R-1\right)}{2^\eta\left(\mathrm{e}+\mathrm{e}^{2^\eta}-1\right)R} \tag{27}$$

for fixed rate, and

$$\frac{E_b(2)^*}{N_0}\bigg|_{\min,opp} = \inf_\gamma \frac{\gamma}{T^*_{opp}(2)} \simeq \frac{\mathrm{e}\left(2\,\mathrm{e}^{2^\eta}-1\right)\log_{\mathrm{e}}2}{2^\eta\left(\mathrm{e}+\mathrm{e}^{2^\eta}-1\right)}, \tag{28}$$

for optimized rate. Our numerical results show that indeed these are good approximations.

V. Numerical Results

Here we provide some numerical results to corroborate the analysis above. We first study the end-to-end throughput versus SNR for both fixed rate (Fig. 2) and optimized rate (Fig. 3). Starting with fixed rate, Fig. 2 shows the end-to-end throughput versus SNR for different numbers of hops $k=\{2,3,4\}$, path loss $\eta=3$ and transmission rate $R=1$ $[bits/s/Hz]$. It is seen that for sufficiently large SNR (i.e., $\gamma \gtrsim 10$ dB), the asymptotic results of Proposition 2 apply. In particular, as per (14) and (15), while opportunistic routing is able to attain the maximum throughput of $T_{opp}(k,1)=1$ $[bits/s/Hz]$ for every k, multihop routing shows the well-known performance degradation in the bandwidth-limited regime (recall Section I), which amounts here to a factor of k, i.e., $T_{mh}(k,1)=1/k$ $[bits/s/Hz]$. Turning to the performance with optimized rates, Fig. 3 confirms the advantages of opportunistic routing and validates the results in Proposition 2: When the SNR is large enough (here, $\gamma \gtrsim 20$ dB), the throughput of opportunistic routing increases with a slope independent of k and larger by a factor of k with respect to multihop routing.

We now explicitly consider the throughput ratio $\rho = T^*_{mh}(k)/T^*_{opp}(k)$ between multihop and opportunistic routing in Fig. 4 for different path loss exponents $\eta=\{2.5,3,4\}$ and $k=\{2,4\}$ versus SNR for optimized rate. The figure points out that opportunistic routing has better gains for smaller path loss exponents ($\eta=2.5$), due to the larger number of relays

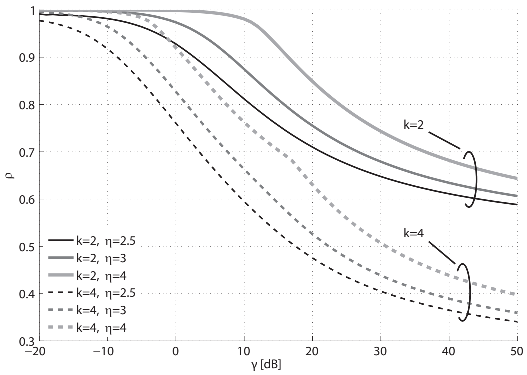

Fig. 4. Ratio $\rho = T^*_{mh}(k)/T^*_{opp}(k)$ between the throughput of multihop and opportunistic routing versus SNR, γ, for $k = \{2, 4\}$ and $\eta = \{2.5, 3, 4\}$ and optimized rate.

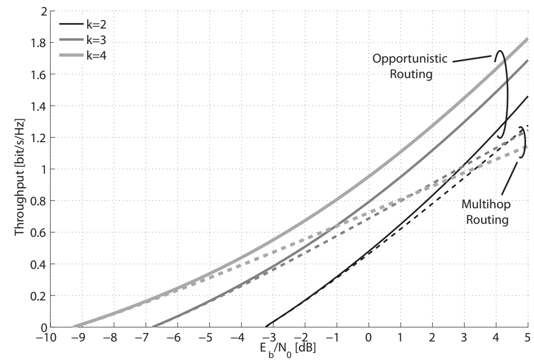

Fig. 5. End-to-end throughput of opportunistic routing, $T^*_{opp}(k)$ (solid lines), and multihop routing, $T^*_{mh}(k)$ (dashed lines), versus E_b/N_0 for $\eta = 3$ and optimized rate.

that are potentially reachable at each transmission and may thus serve as next hop.

Finally, Fig. 5 focuses on the energy efficiency of the considered schemes by showing the throughput versus the energy per bit over noise power spectral density, i.e., $E_b/N_0 = \gamma/T^*(k)$ (recall (20)), for $k = \{2, 3, 4\}$, path loss $\eta = 3$ and optimized rate. First of all, we note that the simulation results confirm the values analytically derived in (22) for the minimum energy per bit $(E_b(k)^*/N_0|_{\min,mh} = \{-3.27, -6.79, -9.29\}\ dB$ for $k = \{2, 3, 4\})$ and in (26) for the slope $\mathcal{S}_{0,mh}$ of the multihop case (see Fig. 5). We also note that the approximation given by (28) is close to the value found in the simulation, which uses a brute-force approach to find the optimum rate R^*_{opp} $(E_b(2)^*/N_0|_{\min,opp} \simeq -3.27\ dB)$[5]. It is also concluded that opportunistic routing fails to outperform multihop routing in the power-limited regime, being unable to exploit the path diversity, unlike in the bandwidth-limited regime.

VI. Concluding Remarks

This paper studied the end-to-end throughput of opportunistic routing with type-I HARQ over a linear multihop wireless network. Special emphasis was given on the impact of the number of hops and the path loss on the performance advantages with respect to multihop routing, by studying the asymptotic performance in the bandwidth-limited and power-limited regimes. Our study quantifies the gains achievable in the high-SNR (bandwidth-limited) regime for both fixed and optimized transmission rates, while showing negligible advantages in the low SNR (power-limited regime). Given the effectiveness of multihop routing in the power-limited regime, these results make opportunistic routing an excellent candidate to solve the performance limitations identified in [1], [2] for multihop routing in the bandwidth-limited regime.

We finally remark that this work relies on several simplifying assumptions, and does not account for spatial reuse, and thus interference, [1], [3]. Moreover, it should be noted that opportunistic routing, when considering other criteria such as delay and congestion, may not always be desirable (see, e.g. [16]). These aspects will be considered in future work.

[5]Also for the fixed rate case the simulation results are consistent with (27).

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