Robust Communication Against Femtocell Access Failures

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***Abstract*—[1]A single macrocell serving a number of outdoor users, overlaid with a femtocell which includes several home users, is studied. The home users in the femtocell are served by a home base station (HBS) that is connected to the macrocell base station (BS) via an unreliable connection (e.g., DSL). The unreliable link may take several possible capacity values. Robust communications strategies for the home users are investigated accounting for the facts that: (*a*) the home users (served by the HBS) may not be aware of the current state of the HBS-BS link; and that (*b*) the performance of the outdoor users (served directly by the BS) should not be disrupted by possible outages on the HBS-BS link. The problem is formulated in information-theoretic terms and inner and outer bounds are given to achievable sum-rates for outdoor and home users. Expected sum-rates with respect to the distribution over the HBS-BS link states are studied as well, and conditions are identified under which the proposed schemes are optimal.**

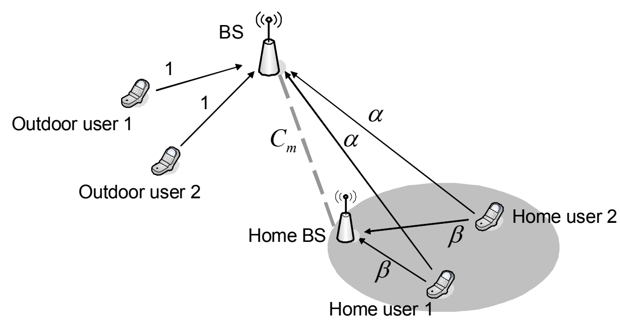

Fig. 1. A macrocell BS serving K_O outdoor users overlaid with a femtocell consisting of a home BS (HBS) and K_H home users (in this figure, $K_O = K_H = 2$). The HBS is connected to the macrocell BS via an unreliable link with variable capacity with M possible values $C_1, ..., C_M$ (e.g., a DSL connection).

I. Introduction

The increase in the throughput of cellular systems has been so far mostly driven by the reduction of the cell size. This has allowed transmission with smaller powers and the possibility to reuse the spectrum more aggressively, thus boosting the system capacity. The latest development in this evolutionary process is the idea of femtocells [1] [2]. A femtocell consists of a *home base station (HBS)* serving only the customer's premises. The short-range low-cost HBS is installed by the customer and is connected to the provider via a separate connection, such as DSL, cable model or an orthogonal RF channel. A typical network configuration consists of macrocells with embedded femtocell "hot-spots", which can be seen as a two-tier network structure [2] [3]. Femtocells are particularly attractive if one considers that 50 percent of all voice and 70 per cent of data originate indoors, and that, via a femtocell, the user would experience seamless connectivity employing the same (e.g., 3G) handset both indoors and outdoors. Femtocells have already been deployed by some providers in the US [2].

Among the major challenges for a successful deployment of femtocells, it is worthwhile to mention: (*i*) *Inter-tier interference:* Due to the aggressive frequency reuse, the system throughput in the presence of femtocells is ultimately limited by the inter-tier interference between femto and macrocells. This calls for effective interference management strategies, such as distributed power allocation or interference avoidance techniques (see [2][4] and references therein)*; (ii) Reliability of the connection between HBS and provider*: Being installed by the user in the customer's premises, HBSs do not enjoy the same reliability guarantees of other multi-tier network structure such as microcells [3]. In particular, the access connection between HBS and provider, e.g., a DSL link, is typically unreliable. For instance, recent trials have shown that, on a DSL link shared with Wi-Fi, femtocell connectivity was severely degraded even for low-bandwidth services [2].

In this work, we study the two issues mentioned above by focusing on a basic system with one macrocell, served by a single-antenna base station (BS), overlaid with one femtocell, served by a single-antenna HBS and characterized by an unreliable connection to the provider (see Fig. 1). We cast the problem in information-theoretic terms by accounting for the facts that: (*a*) the home users (served by the HBS) may not be aware of the current state of the HBS-BS link; and that (*b*) the performance of the outdoor users (served directly by the BS) should not be disrupted by possible outages on the HBS-BS link. The proposed transmission strategies for the home users are *robust* to the unknown HBS-BS link state and are proved to be optimal in some scenarios of interest.

Beside the connection to the literature on femtocells mentioned above, the considered information-theoretic framework ties in with a number of works related with robust communications and generalized definitions of capacity. Specifically,

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the proposed transmission scheme is related to the broadcast coding approach proposed in [5] for use over quasi-static fading channels. Moreover, the notion of *average capacity* used here, also studied in [5], has been the subject of [6] (for non-ergodic "composite" channels) and has been more recently formalized and generalized in [7].

Notation: $\mathcal{C}(x) \triangleq 1/2\log_2(1+x)$; $[1, N]$ represents the interval $(1, ..., N)$ and $a_{[1,N]}$ the vector $(a_1, ..., a_N)$.

II. System Model

We focus on the uplink channel sketched in Fig. 1, which consists of a macrocell with K_O outdoor ("O") users, each with power constraint P'_O, and K_H home (femtocell, "H") users, each with power constraints P'_H. Notice that considering different power constraints for the users could be easily accommodated in our framework. Both outdoor and home users are active on the same bandwidth. The signals transmitted by the home users $X_{H,k,i}$, $k \in [1, K_H]$ and by the outdoor users $X_{O,k,i}$, $k \in [1, K_O]$, at time instant i are received by the BS and the HBS as, respectively,

$$Y_i = \sqrt{\alpha}\sum_{k=1}^{K_H} X_{H,k,i} + \sum_{k=1}^{K_O} X_{O,k,i} + N_{B,i} \quad \text{(1a)}$$

$$\text{and } Z_i = \sqrt{\beta}\sum_{k=1}^{K_H} X_{H,k,i} + N_{H,i}, \quad \text{(1b)}$$

with unit-power white Gaussian noises $N_{B,i}$ and $N_{H,i}$. We assume that the channel power gain β between home users and HBS satisfies $\beta \geq 1$, while the gain α between home users and BS is such that $\alpha \leq 1$, where normalization is with respect to the channel power gain of the outdoor users. We do not account explicitly for interference from other cells that is not already modelled in the Gaussian noises and for interference from outdoor users to the home BS (see [5] for extensions).

The HBS is connected to the BS via an unreliable finite-capacity link (e.g., DSL) with variable capacity. The current link capacity C_m [bits/ channel use] can be measured at the two link ends, HBS and BS, but is assumed to be unknown to all other nodes. This is due, e.g., to generally unpredictable DSL channel conditions and absence of a feedback channel from HBS or BS to the users. Moreover, the current link state is considered to remain constant for the entire duration of the current codeword (non-ergodic link state). The number of possible states (link capacities) is M and we order them as $C_m > C_{m-1}$. We assume that home users are informed about the *possibility* of different HBS-BS connectivity conditions and about the corresponding available link states $(C_1, ..., C_M)$. They may therefore design their communications strategy so as to be *robust* with respect to the different realizations of the link state. In particular, as discussed below, indoor users accept *variable-rate data delivery* that is dependent on the link state. In contrast, the outdoor users expect *fixed-rate data delivery* irrespective of the current link condition within the femtocell. This constraint is related to the typical assumption of absence of coordination between the tiers (here macro and femtocell) of a multi-tier network [3].

The above is formalized as follows. Each kth outdoor user has a message $W_{O,k} \in [1, 2^{nR_{O,k}}]$, $k \in [1, K_O]$, and each kth home user has M messages (or "layers" of information) $W_{H,m,k} \in [1, 2^{nR_{H,k,m}}]$, $k \in [1, K_H]$ and $m \in [1, M]$, for the BS. The message layers of the home users are to be decoded at the BS according to the current link capacity C_m following a degraded message structure: In state m (i.e., link capacity C_m is realized), the BS decodes messages $\mathcal{W}_{H,m} = (W_{H,m,1}, ..., W_{H,m,K_H})$ corresponding to the mth layer of all home users and all the "lower" layers $\mathcal{W}_{H,1}, ...,$ $\mathcal{W}_{H,m-1}$. For instance, the mth layer may be refinement information for the previous layers $1, ..., m-1$ in a multimedia transmission. Given the above, while any kth outdoor user operates at a fixed rate $R_{O,k}$, any kth home user, aware of the unreliability of the connection between the femtocell BS and the macrocell BS, operates at a variable rate, delivering rate $\sum_{i=1}^{m} R_{H,k,i}$ *when the HBS-BS link is in state* m.

Encoding for the kth outdoor user takes place as $X^n_{O,k} = f_{O,k}(W_{O,k})$ and for the kth home user as $X^n_{H,k} = f_{H,k}(W_{H,k,1}, ..., W_{H,k,M})$ for given encoding functions $f_{O,k}(\cdot)$ and $f_{H,k}(\cdot)$. Notice that the encoding functions at the users are not dependent on the current link state m, since this information is not available to them. Encoding at the HBS is instead dependent on the state m, since the HBS is aware of the state of the connection to the BS and we have $V_m = f_{HBS,m}(Z^n)$ with $V_m \in [1, 2^{nC_m}]$. Decoding at the BS is also dependent on m and defined as $(\widehat{\mathcal{W}}_{H,1}, .., \widehat{\mathcal{W}}_{H,m}, \widehat{\mathcal{W}}_O) = g_m(Y^n, V_m)$, with $\mathcal{W}_O = (W_{O,1}, ..., W_{O,K_O})$. The probability of error is defined as $P^n_e = \max_{m\in[1,M]} \Pr[g_m(Y^n, V_m) \neq (\mathcal{W}_{H,1}, .., \mathcal{W}_{H,m}, \mathcal{W}_O)]$, where messages are assumed to be uniformly distributed in their respective sets. Rates $(\{R_{H,k,m}\}_{k\in[1,K_H],\, m\in[1,M]}, \{R_{O,k}\}_{k\in[1,K_O]})$ are said to be achievable if there exists a sequence of encoders and decoders such that $P^n_e \to 0$ for $n \to \infty$.

A. Sum-Rates and Remarks

We are interested in the *sum-rate* that home users can deliver successfully to the BS at each layer m, which is defined as

$$R_{H,m} = \sum_{k=1}^{K_H} R_{H,k,m}, \; m \in [1, M], \quad (2)$$

and in the sum-rate of the outdoor users

$$R_O = \sum_{k=1}^{K_H} R_{O,k}. \quad (3)$$

The sum-rate tuple $(R_{H,1}, ..., R_{H,M}, R_O)$ is said to be achievable if there exists a tuple of achievable component rates $(\{R_{H,k,m}\}_{k\in[1,K_H],\, m\in[1,M]}, \{R_{O,k}\}_{k\in[1,K_O]})$ satisfying (2)-(3). Notice that, by the symmetry of the model, an achievable sum-rate tuple implies the achievability of the corresponding equal rate point, i.e., $R_{H,k,m} = R_{H,m}/K_H$ for all $k \in [1, K_H]$, $m \in [1, M]$ and $R_{O,k} = R_O/K_O$.

Remark 1: It is sometimes appropriate to assume a probability distribution over the M possible link states. Where this is of interest, we will denote the probability of having a link of rate C_m by p_m for $m \in [1, M]$, with $\sum_{m=1}^{M} p_m = 1$. In this case, an interesting figure of merit is the *average* sum-rate

$$\bar{R}_H = \sum_{m=1}^{M} p_m \sum_{i=1}^{m} R_{H,i} = \sum_{m=1}^{M} R_{H,m} \sum_{i=m}^{M} p_i \qquad (4)$$

that the home users can achieve for a given sum-rate of the outdoor users R_O (3). We will say that a pair of sum-rates $(\bar{R}_H, R_O)$ is achievable if there exists a tuple of sum-rates $(R_{H,1}, ..., R_{H,M}, R_O)$ that is achievable (according to the definition given above) and such that (4) is satisfied. It is emphasized again that in the considered network, the home users achieve variable-rate delivery, whose individual sum-rates are given for different layers by (2) and the corresponding average sum-rate by (4), whereas the outdoor users target fixed-rate transmission, with sum-rate (3) (to be always guaranteed irrespective of the realized m).

Remark 2: An important special case of the model presented above is the *link failure scenario,* where the connection to the base station may either be active with some capacity C [bits/channel use], or be in outage with some probability P_{fail}. This corresponds to setting $M = 2$, $C_1 = 0$, $C_2 = C > 0$, $p_1 = P_{fail}$.

III. Fully Reliable HBS-BS Connection ($M = 1$)

In this section, we consider the case where the link from HBS to BS is fully reliable, i.e., $M = 1$. As discussed below, even for this simple special case of our model, capacity results are available only for some scenarios. In fact, if only one home user is present and no outdoors users are active (i.e., $K_H = 1$ and $K_O = 0$), the model reduces to the so called *primitive relay channel* [8], where the home user is the source, the HBS plays the role of the relay and the BS of the final destination. Different achievable strategies have been proposed for this channel including Decode-and-Forward (DF), Compress-and-Forward and "Extended-Hash-and-Forward" [8]. None of these strategies is strictly better than the others and the capacity is known only for special cases. Throughout this work, we focus on DF techniques at the HBS given the favorable channel from the home users to the HBS. The following proposition extends the results available in this regard for the primitive relay channel [8] to the case $K_H \geq 0$ and $K_O \geq 0$.

Proposition 1: For the case of fully reliable HBS-BS link ($M = 1$), the sum-rate region (R_H, R_O) characterized by the conditions

$$\begin{aligned} 0 \leq R_{H,1} < R_{H,1}^{DF} &\triangleq \min\{\mathcal{C}(\beta P_H),\ \mathcal{C}(\alpha P_H) + C_1\} \\ 0 \leq R_O &< \mathcal{C}(P_O) \\ R_{H,1} + R_O &< \mathcal{C}(P_O + \alpha P_H) + C_1 \end{aligned}$$

with sum-powers $P_H = K_H P'_H$ and $P_O = K_O P'_O$ is achievable via DF at the HBS. Moreover, an outer bound to the achievable sum-rate region is given by the inequalities

$$\begin{aligned} R_{H,1} \leq R_{H,1}^{UB} &\triangleq \min\{\mathcal{C}((\alpha+\beta) P_H),\ \mathcal{C}(\alpha P_H) + C_1\} \\ R_O &\leq \mathcal{C}(P_O) \\ R_{H,1} + R_O &\leq \mathcal{C}(P_O + \alpha P_H) + C_1. \end{aligned}$$

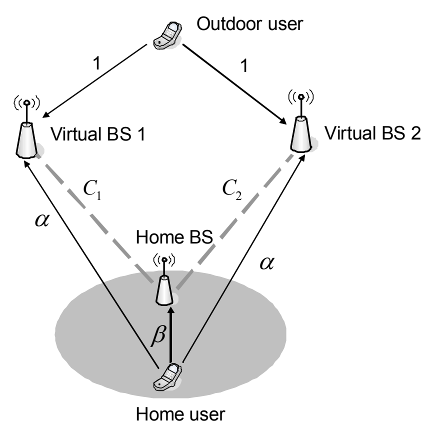

Fig. 2. Equivalent channel seen by a given home user for $K_O = 1$, $K_H = 1$ and $M = 2$.

The considered DF scheme is thus optimal if the condition

$$\mathcal{C}(\beta P_H) \geq \mathcal{C}(\alpha P_H) + C_1 \qquad (5)$$

holds.

Proof: The upper bound follows from standard cut-set arguments, accounting for the fact that the inputs of different users are constrained to be independent. Here we provide a brief sketch of the proof of achievability (more details can be found in [9]). Each kth home user generates the coding function $f_{H,k}(W_{H,k,1})$ by constructing an i.i.d. Gaussian codebook of average power P'_H and rate $R_{H,k,1}$ (i.e., number of codewords $2^{nR_{H,k,1}}$). Similarly, each kth outdoor user employs a randomly generated Gaussian codebook $f_{O,k}(W_{O,k})$ with rate $R_{O,k}$ and power P'_O. The HBS and BS agree on a partition of the whole space of the home users' messages $[1, 2^{nR_{H,1,1}}] \times ... \times [1, 2^{nR_{H,K_H,1}}]$ into 2^{nC_1} bins (selected randomly). The HBS decodes all the K_H messages of the home users and sends to the BS the index V_1 corresponding to the bin the decoded messages have been found to lie in. The BS decodes all messages of outdoor and indoor users jointly, based on the received signal Y^n, by looking only among the codewords of the home users within the bin indexed by the HBS. ■

IV. Unreliable HBS-BS Connection ($M \geq 1$)

We now turn to the general scenario where the HBS-BS link is unreliable ($M \geq 1$).

A. Achievable Rate Region

In this section, we study an achievable strategy based on *superposition (broadcast) coding* at the home users, inspired by the technique proposed in [5], and DF at the HBS. The basic idea is to study an equivalent system where each home BS sees the possible link states (i.e., link capacities C_m) as corresponding to different "virtual users", here "virtual BSs", of a broadcast channel. The equivalent channel seen by a single home user is shown in Fig. 2 for the case with one outdoor user. Notice that we have M virtual BSs, where the mth is connected to the HBS via a link of capacity C_m. We refer to the proposed scheme based on Broadcast coding and DF as BDF.

Proposition 2: The sum-rate region $(R_{H,1}, ..., R_{H,M}, R_O)$ obtained as the convex hull of the union of all rates satisfying the conditions

$$0 \le R_{Hm} < R_{H,m}^{BDF}(\underline{\gamma}, \underline{a}) \triangleq \min\left\{ \mathcal{C}\left(\frac{\beta\gamma_m P_H}{1+\beta P_H \sum_{i=m+1}^{M}\gamma_i}\right), \mathcal{C}\left(\frac{\alpha\gamma_m P_H}{1+\alpha P_H \sum_{i=m+1}^{M}\gamma_i}\right) + a_m \right\}, \tag{6}$$

for $m = 1, 2, ..., M$, and

$$0 \le R_O < R_O^{BDF}(\underline{\gamma}) \triangleq \mathcal{C}\left(\frac{P_O}{1+\alpha P_H(1-\gamma_1)}\right) \tag{7a}$$

$$R_O + R_{H1} < R_{OH}^{BDF}(\underline{\gamma}, \underline{a}) \triangleq \mathcal{C}\left(\frac{\alpha\gamma_1 P_H + P_O}{1+\alpha P_H(1-\gamma_1)}\right) + a_1 \tag{7b}$$

for some set of parameters $a_i, \gamma_i \ge 0$ with

$$\sum_{i=1}^{M}\gamma_i = 1 \text{ and } \sum_{i=1}^{m} a_i \le C_m, \;\; m \in [1, M] \tag{8}$$

is achievable via BDF.

Proof: We provide here a sketch of the proof by describing the encoding/ decoding operations to be carried out at the different nodes. (*i*) *Home users*: The broadcast coding approach used at the *home users* consists in transmitting a superposition of M Gaussian codewords, one destined to each "virtual BS" m, with power allocation dictated by the vector $\underline{\gamma}= [\gamma_1, ..., \gamma_M]$ (the same for all home users). The mth layer of the kth home user encodes message $W_{H,k,m}$ and needs to be decoded at the BS in all link states with $m' \ge m$, i.e., by all m'th virtual BSs with $m' \ge m$; (*ii*) *HBS*: The HBS decodes all the layers of all home users starting with the first $m' = 1$ (corresponding to messages $\mathcal{W}_{H,1}$) and ending with the layer corresponding to the current channel state $m' = m$ (messages $\mathcal{W}_{H,m}$) by *successive interference cancellation* (higher layers are treated as Gaussian noise). Communications between HBS and BS consists, as described in the previous section, in the transmission of the bin index of the tuple of all decoded messages, according to a predefined binning function shared by HBS and BS. Specifically, the HBS selects a priori a vector $\underline{a}= [a_1, ..., a_M]$ with $a_i \ge 0$ and $\sum_{i=1}^{m} a_i \le C_m$. Suppose now that the current link state is m. Having decoded the set of messages $\mathcal{W}_{H,1}, ..., \mathcal{W}_{H,m}$, the HBS first bins messages $\mathcal{W}_{H,1}$ with rate a_1, producing the bin index $V_1' \in [1, 2^{na_1}]$, then bins the messages $\mathcal{W}_{H,2}$ with rate a_2, producing the bin index $V_2' \in [1, 2^{na_2}]$ and so on, up to $\mathcal{W}_{H,m}$, producing the bin index $V_m' \in [1, 2^{na_m}]$. The signal V_m sent to the BS is then $V_m = [V_1' \, V_2' \cdots V_m']$. It is noted that the rate allocation a used to bin the different message layers is selected independently of m, which is not required, since the HBS knows m. (*iv*) *BS*: When in state m, the BS, or in other words the mth virtual BS, decodes successively in m stages as follows: In the first stage, messages $\mathcal{W}_{H,1}$ of the home users (first layer) and all messages $\mathcal{W}_O$ of the outdoor users are decoded jointly. In doing so, search is restricted to the messages $\mathcal{W}_{H,1}$ in the bin indexed by V_1'. Notice that the messages $\mathcal{W}_O$ are decoded at this first stage so that they are recovered for any link state, as required by the problem formulation. Decoded messages are stripped off from the received signal and the procedure repeats for the following layers. ■

B. Outer Bound

An outer bound can be found, as proved in [9], as follows.

Proposition 3: Any achievable tuple of sum-rates $(R_{H,1}, ..., R_{H,M}, R_O)$ must satisfy the conditions

$$R_{Hm} \le \mathcal{C}\left(\frac{\alpha\gamma_m P_H}{1+\alpha P_H \sum_{i=m+1}^{M}\gamma_i}\right) + a_m, \tag{9}$$

for $m = 1, 2, ..., M$, and

$$R_O \le \mathcal{C}(P_O) \tag{10a}$$

$$R_O + R_{H1} \le \mathcal{C}\left(\frac{\alpha\gamma_1 P_H + P_O}{1+\alpha P_H(1-\gamma_1)}\right) + C_1 \tag{10b}$$

$$\sum_{m=1}^{M} R_{Hm} \le \mathcal{C}((\alpha+\beta)P_H), \tag{10c}$$

for some choice of parameters $a_i, \gamma_i \ge 0$ verifying (8).

Remark 3: While the achievable sum-rate region of Proposition 2 and the outer bound of Proposition 3 do not coincide in general, we will see below that the given outer bound is useful to attain conclusions about the optimality of the considered scheme in important special cases.

C. Expected Sum-Rates in the Link Failure Scenario

To provide more insight into the system performance, we now turn to the analysis of the achievable sum-rate pairs $(\bar{R}_H, R_O)$ with $\bar{R}_H$ is the average sum-rate (4) of the home users with respect to a probability distribution p_m over the link states. For lack of space, we focus here only on the link failure scenario (recall Remark 2) and we concentrate on finding the maximum *equal sum-rate* that can be achieved by both home and outdoor users. An achievable rate and a corresponding upper bound can be found from Proposition 2 and 3, respectively, and are given as follows.

Proposition 4: Any equal sum-rate $R_{sym} < R_{sym}^{BDF}$ is achievable by the proposed BDF scheme with

$$R_{sym}^{BDF} \triangleq \max_{0\le\gamma\le1} \min \left\{ \begin{array}{l} \mathcal{C}\left(\frac{\alpha\gamma P_H}{1+\alpha P_H(1-\gamma)}\right) + (1-P_{fail})\mathcal{K} \\ \mathcal{C}\left(\frac{P_O}{1+\alpha P_H(1-\gamma)}\right), \\ \frac{1}{2}\left(\mathcal{C}\left(\frac{\alpha\gamma P_H + P_O}{1+\alpha P_H(1-\gamma)}\right) + (1-P_{fail})\mathcal{K}\right) \end{array} \right. \tag{11}$$

and definition $\mathcal{K} = \min(\mathcal{C}\left(\beta(1-\gamma)P_H\right), \mathcal{C}\left(\alpha(1-\gamma)P_H\right) + C)$. Moreover, the following is an upper bound on the achievable equal rate R_{sym}

$$R_{sym} \leq \max_{0\leq\gamma\leq 1} \min \left\{ \begin{array}{l} \mathcal{C}\left(\frac{\alpha\gamma P_H}{1+\alpha P_H(1-\gamma)}\right) + (1-P_{fail})\mathcal{K}') \\ \mathcal{C}\left(P_O\right), \\ \frac{1}{2}\left(\mathcal{C}\left(\frac{\alpha\gamma P_H+P_O}{1+\alpha P_H(1-\gamma)}\right) + (1-P_{fail})\mathcal{K}'\right) \end{array} \right. \quad (12)$$

with $\mathcal{K}'\!=\!\min(\mathcal{C}\left((\alpha+\beta)P_H\right), \mathcal{C}\left(\alpha(1-\gamma)P_H\right)+C)$.

Remark 4: Achievability and upper bound coincide in the important special case in which

$$\mathcal{C}\left(\beta(1-\gamma^*)P_H\right) \geq \mathcal{C}\left(\alpha(1-\gamma^*)P_H\right)+C \quad (13a)$$

$$\text{and } \mathcal{C}\left(\frac{P_O}{1+\alpha P_H(1-\gamma^*)}\right) > R_{sym}^{BDF}, \quad (13b)$$

where γ^* is the parameter that maximizes (11). Notice that this condition extends (5), which is valid for the case $M=1$.

V. Numerical Results

In this section, we provide some numerical results related to the link failure scenario studied above. The channel gain β is typically $30-80dB$ larger than α, depending on the propagation environment [2]. Here we consider the conservative $\beta=1000\alpha$ (i.e., a $30dB$ gain) and set $\alpha=1/d^4$, thus assuming a path loss exponent of four, where d represents the normalized distance between home users and BS (normalization is with respect to the distance between outdoor users and the BS). We also set $P_O=2$, $P_H=2$, $C=\mathcal{C}(P_O)$ and $d=1.5$. Beside the achievable equal sum-rate (11) and the upper bound (12), we consider for reference the equal sum-rate achievable: (*i*) When the BS treats the signals received over the wireless channel from the home users as noise, thus decoding only based on the message received from the HBS over the link: $R_{sym}^{INT}=\min\{(1-P_{fail})\min(\mathcal{C}\left(\beta P_H\right), C),$ $\mathcal{C}\left(\frac{P_O}{1+\alpha P_H}\right)\}$; (*ii*) With "best-case" design ((11) with $\gamma=0$), which corresponds to allocating all the power to the second layer to be decoded only in the best case where the HBS-BS link is active; (*iii*) With "worst-case" design ((11) with $\gamma=1$), whereby all power is devoted to the first layer, thus not exploiting at all the presence of the HBS-BS link. Fig. 3 shows the mentioned rates versus the probability of link failure P_{fail}. In this example, conditions (4) are satisfied, so that the considered BDF scheme is optimal. Moreover, the advantages of a *robust design that uses both layers* are clear from the given performance. In fact, it is noted that in this example, best-case design is not optimal for any value of P_{fail}, while worst-case design is optimal only for $P_{fail}=1$. In particular, the optimal γ^* (not shown) for (11) turns out to be close to 1 (in the range $0.93-1$) for all values of P_{fail}. That, even for small $P_{fail}\rightarrow 0$, the best-case design is not optimal follows from the fact that, when $M=2$ states are possible, there is always a, possibly vanishing, chance of failure (that is, of $m=1$). In this state, choosing $\gamma=0$ would cause (inter-tier) interference to the detection of the outdoor users (The case $P_{fail}=0$ is better handled by setting $M=1$ and $C_1=C$, in which case, clearly the best-case design would be optimal).

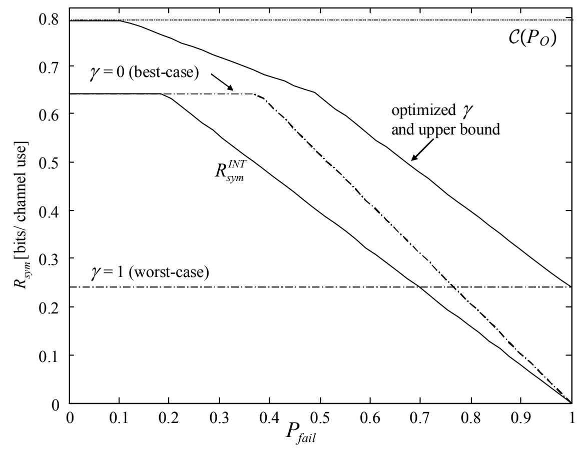

Fig. 3. Achievable equal sum-rates along with the upper bound of Proposition 4 versus the probability of link failure P_{fail} ($\beta=1000\alpha$, $\alpha=1/d^4$, $P_O=2$, $P_H=2$, $C=\mathcal{C}(P_O)$, $d=1.5$).

VI. Concluding Remarks

A number of issues still have to be resolved for an effective deployment of femtocells, including the interference created to regular macrocell users and the unreliability of the access connection between the home BS and the provider (e.g., DSL). We have proposed a first information-theoretic look at the problem, by accounting for the need of robust communications strategies at the home users that are resilient to the variable HBS-provider link capacity. The proposed strategy, based on superposition (broadcast) coding and DF, has been proved to be sum-rate optimal in a number of scenarios of interest.

References

[1] H. Claussen, L. T. W. Ho, L. G. Samuel: "An overview of the femtocell concept," *Bell Labs Technical Journal*, vol. 13, no. 1, pp. 221-245, 2008.

[2] V. Chandrasekhar, J. G. Andrews and A. Gatherer, "Femtocell networks: a survey," *IEEE Comm. Magazine*, vol. 46, no. 9, pp. 59-67, Sept. 2008.

[3] A. Ganz, C.M. Krishna, D. Tang, Z.J. Haas, "On optimal design of multitier wireless cellular systems," *IEEE Commun. Magazine*, vol. 35, no. 2, pp. 88-93, Feb 1997.

[4] V. Chandrasekhar, M. Kountouris and J. G. Andrews, "Coverage in multi-antenna two-tier networks," submitted [arXiv:0902.3210].

[5] S. Shamai and A. Steiner, "A broadcast approach for a single-user slowly fading MIMO channel," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2617-2635, Oct. 2003.

[6] M. Effros, A. Goldsmith and Y. Liang, "Capacity definitions for general channels with receiver side information," submitted [arXiv:0804.4239].

[7] S. Verdú and S. Shamai (Shitz), "Variable-rate channel capacity," presented at *IEEE ITA Workshop*, La Jolla, CA, Feb. 2009.

[8] Y.-H. Kim, "Coding techniques for primitive relay channels," in *Proc. Forty-Fifth Annual Allerton Conf. Commun., Contr. Comput.*, Monticello, IL, Sept. 2007.

[9] O. Simeone, E. Erkip and S. Shamai (Shitz), "Robust coding for femtocells with unreliable network access," in preparation.