On the Energy Efficiency of Hybrid-ARQ Protocols in Fading Channels

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***Abstract*— As the distance between terminals in modern wireless networks tends to decrease, the energy consumption issue, conventionally assumed to be exclusively dominated by the transmission power, needs to be revaluated. In particular, retransmission (ARQ) protocols that typically reduce the transmission energy required to obtain a given error probability on the channel (at the expense of a larger delay), also increase the energy consumed by the circuitry other than the power amplifier.**

In this paper, the energy efficiency of Hybrid-ARQ Type I, Chase Combining and Incremental Redundancy protocols in Rayleigh fading channels, is analyzed by accounting for the energy consumed by the transmitting and receiving electronic circuitry. It is shown that the advantages of Hybrid-ARQ protocols in terms of energy consumption strictly depend on the transmission range.

I. Introduction

In wireless ad hoc and sensor networks, inter-terminal distances are typically constrained to be small enough to allow low-power communication and, as a result, increase the network lifetime. In such a scenario, the energy consumed by power amplifiers becomes comparable to the energy spent by other electronic components, such as analog/digital converters, frequency synthesizers, mixers and so on [1]-[3]. The contribution of circuits other than the power amplifier to the total energy expenditure depends in general on the amount of time the terminal remains active (transmitting or receiving). Therefore, it is particularly relevant for transmission schemes employing low-rate modulation ([1]-[5]) or multiple retransmissions.

Retransmission protocols, such as Automatic-Repeat-ReQuest (ARQ) and Hybrid-ARQ (HARQ) are known to provide energy-efficient transmission for delay insensitive applications, due to their inherent ability to exploit time-diversity [6]-[8]. This conclusion, however, is drawn by neglecting the energy consumed by the terminals' circuitry, and thus it does not necessarily hold in low-power networks.

In this work, we account for the energy spent by both power amplifier and electronic circuitry and analyze the energy-efficiency of three classes of Hybrid-ARQ (HARQ) protocols, namely: (i) HARQ Type I (HARQ-TI), that prescribes retransmissions of copies of the same packet and memoryless detection (i.e., erroneous packets are discarded at the destination); (ii) Chase Combining (HARQ-CC), whereby erroneous packets at the destination are preserved for soft combining [9]; and (iii) Incremental-Redundancy (HARQ-IR) protocol, where retransmissions consist of extra parity bits that are appropriately combined at the destination.

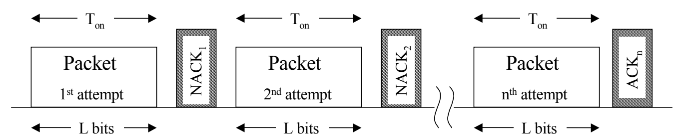

Fig. 1. Illustration of the transmission protocol.

Similarly to [1]-[2], the presented analysis considers the packet transmission time as a degree of freedom that can be optimized in order to maximize energy-efficiency. In particular, it is shown that the advantages of Hybrid-ARQ protocols in terms of energy consumption strictly depend on the transmission range.

The paper is organized as follows. The system model is provided in Section II. The performance analysis of HARQ protocols is presented in Section III. Numerical results in Section IV corroborate the analysis and provide insight into the problem at hand. Finally, section V concludes the paper.

II. System Model

We consider single-link communication over a block-fading Rayleigh channel. In such an environment, the fading channel is constant during each transmission slot and changes independently with each retransmission. All signalling messages, such as Acknowledge (ACK) and Not-Acknowledge (NACK), are assumed to be significantly shorter than the user data packets, and therefore transmitted with the negligible overall energy consumption. Following [1]-[2], we assume that each packet carries L bits and that transmission of each packet lasts T_{on} seconds (with a maximum value T, $T_{on} \leq T$, fixed by design constraints), where T_{on} is a design parameter of the system (see fig. 1).

The quality-of-service requirement to be met by the communication link is defined in terms of the maximum tolerated probability of outage p_{out} (i.e., probability of incorrect detection of a packet). Then, by letting n be the number of transmissions of a packet, including the original, this condition can be expressed as:

$$P_{out}(n) \leq p_{out}, \quad (1)$$

where the probability of outage $P_{out}(n)$ (after n transmissions) will be derived in Section III for different HARQ schemes. The problem amounts to the following: given the number of transmissions n, find the optimal packet duration T_{on} that minimizes the total energy consumption per bit $E_{bt}(n)$ under condition (1). The total energy expenditure per bit $E_{bt}(n)$ (measured in Joules) reads:

$$E_{bt}(n) = n \times \left(K_t E_b + P_c \frac{T_{on}}{L} \right), \quad (2)$$

where:

- the first term in parentheses accounts for power consumed by the *power amplifier* at each transmission of a packet: E_b is the energy per bit to be received at each retransmission so as to guarantee the quality-of-service requirement (1), K_t is a constant that depends on physical characteristics of the link and power amplifier [1]:

$$K_t = (1+\alpha) \frac{(4\pi)^2 d^\gamma}{G_t G_r \lambda^2} M_l N_f. \quad (3)$$

  In (3) we have defined: $\alpha = \xi/\eta - 1$, with η being the drain efficiency of the RF power amplifier and ξ the peak-to-average power ratio (PAPR) of the transmitted signal; d in (in meters) is the transmission distance; γ is the propagation path loss exponent; G_t and G_r are the transmitter and the receiver antenna gains, respectively; λ (in meters) is the carrier wavelength; M_l is the link margin compensating the hardware process variations and other additive background noise or interference; and N_f is the receiver noise figure defined as $N_f = N_r/N_o$ with N_0 being the single-sided thermal noise power spectral density (PSD) at room temperature and N_r is the PSD of the total effective noise at the receiver input;
- the second term $P_c = P_{ct} + P_{cr}$ measures the power (in Watts) consumed by the *circuitry at the transmit (P_{ct}) and receive (P_{cr}) side* (see [1]-[3] for details).

As a final remark, notice that the block fading assumption considered in this paper constitutes a major difference with respect to [1]-[3], where the average probability of error is used as performance metric, thus implying an ergodic fading scenario.

III. Performance Analysis

We first address the reference case where no retransmissions occur ($n = 1$) in Sec. III-A, and then extend the analysis to HARQ protocols.

A. The reference case: one transmission ($n = 1$)

The maximum rate (in bit/s) achievable in one transmission reads

$$C(1) = B \times \log_2 \left(1 + |h_{SD}^{(1)}|^2 \frac{P_r}{N_0 B} \right), \quad (4)$$

where P_r is the received power, B (in Hertz) is the available bandwidth and $h_{SD}^{(n)}$ denotes the (zero-mean unit-power complex Gaussian) fading channel between the source and the destination at the nth retransmission. Notice that (4) assumes optimal Gaussian coding; however, more practical transmission schemes could be easily accommodated in the proposed framework by using the approach in [10]. Writing the received power P_r in terms of the received energy per bit E_b, that is $P_r = LE_b/T_{on}$, the probability of outage, defined as the event when the achievable maximum rate is smaller than the transmission rate, becomes:

$$\begin{aligned} P_{out}(1) &= P\left\{C(1) < L/T_{on}\right\} \\ &= P\left\{ |h_{SD}^{(1)}|^2 < \left(2^{\frac{L}{BT_{on}}} - 1\right) \frac{N_0 B T_{on}}{E_b L} \right\} \\ &= F_{\chi^2}\left[\mu_1/E_b, 2\right], \end{aligned} \quad (5)$$

where $F_{\chi^2}[x,\nu]$, $\nu = 1, 2, ...$ is the cumulative distribution function of a chi-square variable with ν degrees of freedom, taken at value x, while the coefficient $\mu_n = \left(2^{\frac{L}{nBT_{on}}} - 1\right) N_0 \frac{BT_{on}}{(L/n)}$ is introduced for convenience of notation. Imposing the equality in condition (1), we can easily get the minimum received energy per bit as a function of the packet transmission time T_{on}:

$$E_b = \frac{\mu_1}{F_{\chi^2}^{-1}[p_{out}, 2]}, \quad (6)$$

with $F_{\chi^2}^{-1}[y,\nu]$ denoting the inverse of $F_{\chi^2}[x,\nu]$ taken at value y. The total energy per bit then reads from (2):

$$E_{bt}(1) = K_t \frac{\mu_1}{F_{\chi^2}^{-1}[p_{out}, 2]} + P_c \frac{T_{on}}{L}. \quad (7)$$

The energy consumption in (7) can be minimized by appropriately selecting T_{on} (see Sec. IV for further discussion). Notice that, strictly speaking, the minimization of (7) should also take into account the dependence of K_t on T_{on} through the PAPR ξ of the transmitted signal (recall (3)). In fact, reducing the transmission time T_{on} requires the increase of the data rate, which may imply a larger constellation size, and thus a larger PAPR (this is the case, for instance, if M-QAM modulation is used [1]). However, for the sake of simplicity, here we set PAPR to $\xi = 1$ as in M-PSK or FSK modulation, thus making K_t constant (not dependent on T_{on}). Considering the dependence of PAPR on T_{on} yields slightly different results but does not modify the main conclusions discussed in Sec. IV.

B. HARQ-TI

With HARQ-TI, packets received in previous retransmissions are discarded, so that the probability of outage after the nth retransmission reads:

$$\begin{aligned} P_{out}(n) &= P\left\{C(1) < L/T_{on}, ..., C(n) < L/T_{on}\right\} \quad (8a) \\ &= P\left\{C(i) < L/T_{on}\right\}^n, \quad (8b) \end{aligned}$$

where $C(i)$ is the maximum rate achievable in the ith transmission ($i = 1, .., n$):

$$C(i) = B \times \log_2 \left(1 + |h_{SD}^{(i)}|^2 \frac{P_r}{N_0 B} \right). \quad (9)$$

Following the same reasoning as in the previous Section, we obtain the minimum received energy per bit as a function of the packet transmission time T_{on}

$$E_b = \frac{\mu_1}{F_{\chi^2}^{-1}\left[p_{out}^{1/n}, 2\right]}. \quad (10)$$

The total energy per bit then reads from (2):

$$E_{bt}(n) = n \times K_t \frac{\mu_1}{F_{\chi^2}^{-1}\left[p_{out}^{1/n}, 2\right]} + n \times P_c \frac{T_{on}}{L}. \quad (11)$$

As for (7), the energy per bit (11) can be minimized with respect to T_{on} (see Sec. IV).

C. HARQ-CC

With HARQ-CC, previously received copies of the same packets are soft combined (Maximum Ratio Combining) at the receiver, so that the maximum achievable rate at the nth transmission reads

$$C(n) = B \times \log_2 \left(1 + \sum_{i=1}^{n} |h_{SD}^{(i)}|^2 \frac{P_r}{N_0 B} \right). \quad (12)$$

Taking into account the fact that conditions $C(1) < L/T_{on}, ..., C(n) < L/T_{on}$ are implied by $C(n) < L/T_{on}$ (since $C(n) \geq C(n-1), .., C(n) \geq C(1)$), the outage probability, defined in (8a), becomes:

$$P_{out}(n) = P\left\{C(n) < L/T_{on}\right\}. \quad (13)$$

Furthermore, the overall gain $\sum_{i=1}^{n} |h_{SD}|^2$ in (12) is a chi-square distributed variable with $2n$ degrees of freedom and, imposing equality in (1), we can write

$$P_{out}(n) = F_{\chi^2}\left[\mu_1/E_b, 2n\right] = p_{out}. \quad (14)$$

The required received energy is now

$$E_b = \frac{\mu_1}{F_{\chi^2}^{-1}[p_{out}, 2n]}, \quad (15)$$

and the total consumed energy per bit finally becomes

$$E_{bt}(n) = n \times K_t \frac{\mu_1}{F_{\chi^2}^{-1}[p_{out}, 2n]} + n \times P_c \frac{T_{on}}{L}. \quad (16)$$

D. HARQ-IR

In [11] a tight upper bound on the rate achievable by the HARQ-IR protocols after n transmissions was derived as:

$$C(n) \leq B \times n \log_2 \left(1 + \frac{1}{n} \sum_{i=1}^{n} |h_{SD}^{(i)}|^2 \frac{P_r}{N_0 B} \right). \quad (17)$$

Using the maximum value in (17), the outage probability of HARQ-IR can be written, similarly to the previous Section, as

$$\begin{aligned} P_{out}(n) &= P\left\{C(n) < L/T_{on}\right\} \\ &= F_{\chi^2}\left[\mu_n/E_b, 2n\right]. \end{aligned} \quad (18)$$

Notice that the only difference in the analytical expression for the outage probability of HARQ-CC and HARQ-IR protocols, given by (14) and (18) respectively, is in the coefficient μ_n that changes with the number of transmission attempts n for HARQ-IR protocol. The required received energy per bit per transmission becomes

$$E_b = \frac{\mu_n}{F_{\chi^2}^{-1}[p_{out}, 2n]}, \quad (19)$$

while the the total consumed energy per bit reads

$$E_{bt}(n) = n \times K_t \frac{\mu_n}{F_{\chi^2}^{-1}[p_{out}, 2n]} + n \times P_c \frac{T_{on}}{L}. \quad (20)$$

We remark that being based on the bound (17), the results of this section have to be interpreted as upper bounds on the performance of HARQ-IR. However, the validity of the analysis is attested by the tightness of the bound (17), as discussed in [11].

IV. Numerical Results

Following the energy consumption model described in [1]-[2], we use the following setting: $P_c = 210.8\ mW$, divided as $P_{ct} = 98\ mW$ (transmit side) and $P_{cr} = 112.4\ mW$ (receive side), $\gamma = 2$, $G_t G_r = 5\ dBi$, $\lambda = 12\ cm$, $M_l = 40\ dB$, $N_0 = -171 dBm/Hz$ and $N_f = 10\ dB$. The outage probability requirement is set to $p_{out} = 10^{-6}$, while the bandwidth is normalized to $B = 1\ Hz$, without loss of generality. We remark that we assume the same value of the circuitry power expenditure P_c (P_{ct} and P_{cr}) for all HARQ protocols. This is arguably unfair towards simpler protocols that require less complexity, but, for lack of a better model, and in order to be consistent with the existing literature on the subject, we will enforce this condition.

Fig. 2 shows the total energy consumption per bit E_{bt} versus transmission time per bit T_{on}/L for HARQ-TI, HARQ-CC and HARQ-IR (using the upper bound (17)) protocols with $n = 2$ and $n = 3$ transmissions and $d = 1m$. Also shown is the reference case of $n = 1$. As fig. 2 shows, increasing the number of transmissions n reduces the optimal transmission time T_{on}, but not necessarily the minimum energy consumption E_{bt}. For HARQ-TI and HARQ-CC protocols, the required energy actually increases, whereas HARQ-IR improves the energy efficiency with more transmissions n.

Fig. 3 and fig. 4 investigate the impact of the transmission range d on the energy efficiency of HARQ protocols. The optimal bit transmission time T_{on}/L versus d is shown in fig. 3 for HARQ-CC and HARQ-IR protocols with $n = 1, 2, 3$ transmissions, while fig. 4[1] illustrates the corresponding

[1]The performance of HARQ-TI is not shown in fig. 4 for the sake of clarity, but, according to the previous discussion, its performance is similar and slightly worse than that of HARQ-CC.

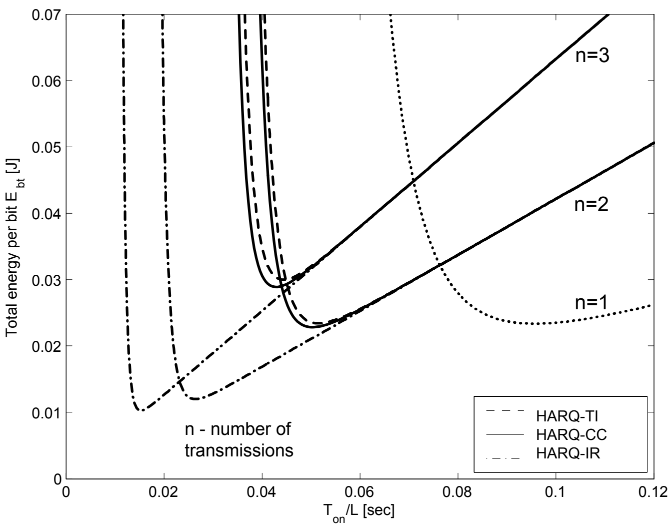

Fig. 2. Total energy per bit E_{bt} versus the transmission time T_{on}/L per bit ($d = 1m$).

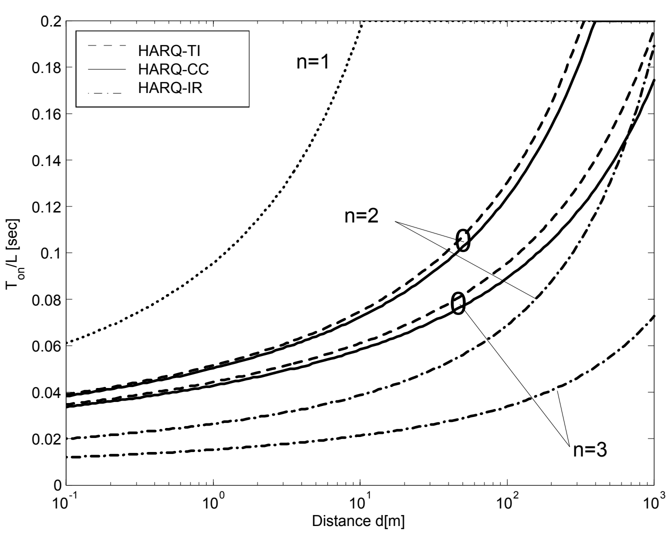

Fig. 3. Optimal transmission time per bit T_{on}/L versus the transmission range d.

minimum total energy per bit E_{bt}. In the example of fig. 2 (for $d = 1m$), only HARQ-IR was proved to be able to decrease the energy per bit E_{bt} with increased number of retransmissions n, whereas HARQ-TI and HARQ-CC were shown to be disadvantageous from an energy consumption standpoint. From fig. 4 we can conclude that, while the statement on HARQ-IR remain valid for any distance d, increasing the number of retransmissions n through HARQ-CC (and HARQ-TI, not shown) improves the energy efficiency for d sufficiently large ($d > 1m$ for $n = 2$ and $d > 4m$ for $n = 3$). This can be explained by noticing that, from (2), increasing d beyond a given value renders the energy expenditure due to the power amplifiers more relevant than the contribution of the transmit and receive circuitry. In this regime, HARQ protocols are known to be able to improve the energy efficiency of the system.

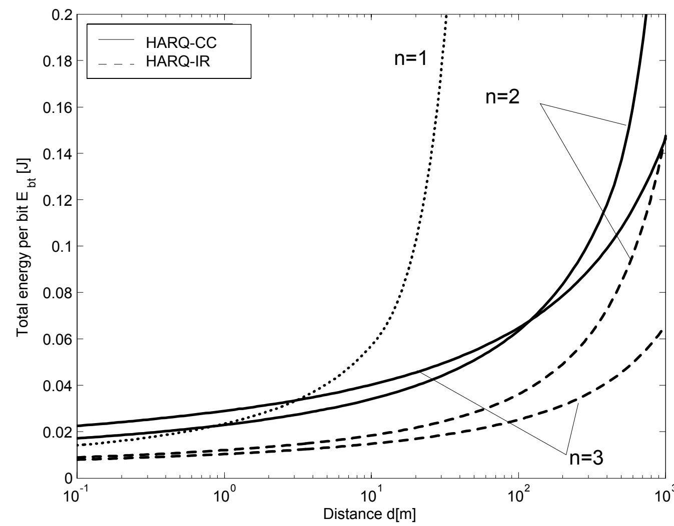

Fig. 4. Minimum total energy per bit E_{bt} versus distance d.

V. CONCLUDING REMARKS

In this work, the advantages of HARQ protocols have been reconsidered from the standpoint of energy efficiency, accounting for scenarios where the energy consumption due to the electronic circuitry is comparable to the transmission power. It was shown that the energy consumption can be reduced by optimizing the packet transmission time (i.e., in practice the transmission rate), and that for transmission ranges large enough (of the order of a few meters) HARQ protocols are generally advantageous in terms of energy efficiency.

VI. ACKNOWLEDGMENT

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