A game-theoretic view on the interference channel with random access

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***Abstract*— As an important building block of cognitive radio networks, the interference channel with distributed and competing radio access is currently an active area of research. In this work, a basic two-by-two interference channel is studied by considering random packet arrivals and random access. In particular, each transmitter is assumed to select independently and concurrently a transmission probability based on the state of the system queues. Both the cases of perfect and partial information about the transmitters' backlogs are addressed.**

The system is analyzed using tools from game theory, and specifically from the theory of stochastic games. The main conclusion is that random packet arrival has a beneficial effect on the efficiency of decentralized random access. This result is achieved by comparing the efficiency of Nash equilibria for the case of backlogged users with the corresponding equilibria in presence of random packet arrivals via numerical simulations.

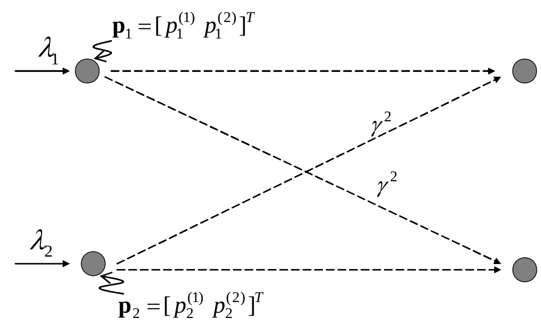

Fig. 1. Illustration of the interference channel with random access and random packet arrivals.

I. Introduction

Spurred by the interest of the Federal Communications Commission, unlicensed spectrum access (also referred to as cognitive radio) is envisaged to be a major component of next generation wireless networks [1]. A basic building block of cognitive wireless network is the interference channel [2] [3], that models the coexistence of different point-to-point links in the same bandwidth. Appropriate assumptions for a cognitive scenario include decentralization and competitiveness of the participating links, which have been considered in a number of recent papers [4] [5] [6], while focusing on power and rate allocation. A basic condition underlying the existing literature on the subject is that transmitters have an infinite backlog of data to transmit.

With secondary spectrum access, exploiting the dynamics of incoming traffic at the transmitters becomes of paramount importance in order to achieve satisfactory spectral efficiency. Towards the goal of modelling such a scenario, this paper assumes random packet arrivals at the transmitters. Furthermore, being interested in distributed schemes that do not require centralized control, we focus on random access. More specifically, it is assumed that each user selects independently and concurrently a transmission probability based on the state of the system with the aim of maximizing a properly defined utility function. Similar frameworks have been employed in order to study random access in the multi-access channel, see [8] [9] [10] [11] [12], and in the relay channel [13]. For simplicity, our work considers a simple interference channel with two competing links (see fig. 1).

The distributed system described above can be analyzed using tools from game theory [15] [16]. In particular, since we are interested in the dynamic behavior of the system, we focus on stochastic games [17] [18] [19]. This approach follows the recent works on multiaccess channel [11] and on relay channels [13]. In the framework of game theory, we are interested in evaluating the transmission policies of the two transmitters that are the most likely outcome of the decentralized optimization process. This amounts to evaluating the Nash equilibria (NEs) of the system.

In this work, we investigate the NEs of the interference channel with random packet arrival and random access in fig. 1. We consider two scenarios: 1) each transmitter has perfect information about the state of the queues on both links; 2) each transmitter is only aware of the state of its own queue. Based on the available information, the transmitters optimize their channel access probabilities toward the aim of maximizing their respective utility in a rational and selfish way. We are interested in assessing the efficiency of such a distributed scheme. In other words, we ask: what is the performance loss of concurrent optimization with respect to centralized (cooperative) solutions (see [5] for further discussion on efficiency)? By comparing NEs of a backlogged system with those of a system with random packet arrivals, we conclude that traffic dynamics has a beneficial effect on the efficiency of decentralized random transmission.

II. System model and problem formulation

We focus on an interference channel, consisting of two sources and two destinations, where the transmitters have no buffers (see fig. 1). The model follows the main assumptions made in [11] for the analysis of a multiaccess channel and [13] for a relay channel. Time is slotted and transmission of each packet takes one slot. The arrival processes of packets at the two sources are independent Bernoulli processes with parameters λ_1 for the first link and λ_2 for the second link ($0 \leq \lambda_1 \leq 1$, $0 \leq \lambda_2 \leq 1$, measured in packets per slot). As long as there is a packet at a source (i.e., as long as it has not been successfully transmitted), new packets arriving at the transmitter are discarded and lost[1]. Medium access control is operated in a decentralized fashion according to a random access protocol[2]. Each source with a backlogged packet transmits with a given probability, which is optimized by the transmitter. Since the system is assumed to lack a central controller, optimization has to be carried out concurrently at the two transmitters. In particular, we consider two scenarios:

1) *Perfect information*: at the beginning of each slot, say the tth, the ith transmitter ($i = 1, 2$) is aware of whether or not the other source has a packet to transmit. Accordingly, it transmits the backlogged packet with probability $p_i^{(1)}$ if the other transmitter has no packet in queue, and $p_i^{(2)}$ otherwise. It is apparent that, while in the former case there is no risk to incur in interference from a simultaneous transmission, in the latter case the opposite holds true. Therefore, in general each transmitter might want to select different values for $p_i^{(1)}$ and $p_i^{(2)}$. For notational convenience, we define the vectors $\mathbf{p}_i = [p_i^{(1)}\ p_i^{(2)}]^T, i = 1, 2$;
2) *Partial information*: at the beginning of each slot, say the tth, the ith transmitter ($i = 1, 2$) is aware only of the status of its own queue (i.e., of whether or not it has a packet to transmit). In this scenario, each source transmits with probability p_i if it has a backlogged packet (irrespective of the action of the second transmitter). Notice that the first setup boils down to the second if we constrain $p_i^{(1)} = p_i^{(2)} = p_i$ for $i = 1, 2$. Therefore, in the following, where not stated otherwise, we focus on the first scenario, with the understanding that imposing the above mentioned condition renders the presentation valid for the second scenario as well[3].

As discussed below, transmission of a given packet can incur in outage due to impairments on the fading channel. It is assumed that the transmitter is correctly informed of an erroneous reception (through transmission of a short Not-ACKnowledge message from the receiver), and, in such an event, considers the packet as not yet delivered and attempts retransmission in future slots using the same policy. A precise definition of the problem of decentralized optimization of the transmission probabilities discussed above is presented in Sec. II-B. Instrumental to the formulation of the problem is the brief discussion on the physical model of the interference channel in the next Section.

A. Notation, physical model and main assumptions

At each time slot t, the ith transmitter ($i = 1, 2$) decides its action, i.e., whether to transmit (T) or wait (W), according to the probabilities defined above. We denote as $\mathcal{A}_1 = \mathcal{A}_2 = \{\mathrm{T}, \mathrm{W}\}$ the set of actions available for the first and second transmitter, respectively, and the vector $\mathbf{a}(t)$ that contains the actions selected at the tth slot by the two transmitters, $\mathbf{a}(t) = [a_1(t)\ a_2(t)]^T \in \mathcal{A}$, where $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2$. Moreover, the state (backlog) of the system at the tth slot is described by a variable $S(t)$ that takes values in the set $\mathcal{S} = \{S_1, S_2, S_3, S_4\} = \{(0,0), (1,0), (0,1), (1,1)\}$, where the first entry m in each tuple (m, n) describes the backlog of the first transmitter ($m = 1, 0$ according to whether or not the first transmitter has a packet to transmit), and entry n has the same role for the second transmitter. Notice that not all the actions are available at all states, in particular, a user can transmit only if it has packets in queue.

Let the ith transmitter be transmitting at the tth slot, i.e., $a_i(t) =$T. Notice that throughout the paper we will denote a given link as i ($i = 1, 2$) and the other as j ($j = 1, 2$, $i \neq j$). The signal received by the destination of the ith link is

$$y_i(t) = h_{ii}(t)x_i(t) + f(a_j(t)) \cdot \gamma \cdot h_{ji}(t)x_j(t) + v_i(t), \quad (1)$$

where the fading channels $h_{ii}(t)$ and $h_{ij}(t)$ are independent complex circularly symmetric Gaussian variables, varying independently slot-by-slot (block-fading channel) with $E[|h_{ii}(t)|^2] = E[|h_{ji}(t)|^2] = 1$; the transmitted signals are assumed to have average power $E[|x_i(t)|^2] = E[|x_j(t)|^2] = P$; the power of the additive Gaussian noise $v_i(t)$ reads $E[|v_i(t)|^2] = N_0$; $\gamma \geq 0$ is a positive real number measuring the relative strength by which the interfering power is received (see fig. 1); function $f(\cdot)$ equals 1 or 0 according to whether its argument equals T or W (i.e., whether or not the other user is transmitting). From (1), defining the average signal-to-noise ratio as $SNR = P/N_0$, we can write the instantaneous signal-to-noise-plus-interference ratio (SINR) at receiver i as

$$SINR_i(a_j(t), t) = \frac{|h_{ii}(t)|^2}{\frac{1}{SNR} + f(a_j(t)) \cdot \gamma^2 |h_{ji}(t)|^2}. \quad (2)$$

Transmission of a given packet is assumed to be successful if the SINR is above a given threshold β. Therefore, the outage probability for the case where only the ith source is transmitting ($a_j(t) = \mathrm{W}$) is

$$P_{out}^{(1)} = P[SINR_i(\mathrm{W}, t) < \beta] = 1 - \exp\left(-\frac{\beta}{SNR}\right), \quad (3)$$

[1] Or, equivalently, no new packets are generated until the current packet is successfully transmitted.

[2] As discussed in [11], the considered model prescribing no buffers and random access is appropriate to study the sporadic transmission of signalling packets used for making reservation of a dedicated channel (e.g., according to the RTS/CTS access scheme implemented in the Decentalized Control Function mode of IEEE802.11 WLAN).

[3] A different model for the partial information case could be set up within the framework of partiall observable stochastic games [14]. Here we focus on the described framework where partial information is modelled as a limitation of the space of available strategies (see also [12]).

whereas if both sources are transmitting simultaneously ($a_j(t) = \text{T}$), we have (see [22], eq. (15))

$$P_{out}^{(2)} = P[SINR_i(\text{T}, t) < \beta] = 1 - \frac{\exp\left(-\frac{\beta}{SNR}\right)}{1+\beta} \geq P_{out}^{(1)}. \tag{4}$$

In order to obtain non-trivial solutions, we enforce a few intuitive conditions on the system parameters. First, we rule out the possibility that a user decides not to transmit: this requires the following conditions

$$p_i^{(1)} = 0 \text{ then } p_i^{(2)} > 0 \tag{5a}$$

$$p_i^{(2)} = 0 \text{ then } p_i^{(1)} > 0. \tag{5b}$$

Moreover, in order to prevent the system from getting stuck at the state where both transmitters have a packet to deliver (S_4) (i.e., to avoid absorbing states), we impose that probability $p_i^{(2)}$ is non-zero for at least one transmitter:

$$p_i^{(2)} = 0 \text{ then } p_j^{(2)} > 0. \tag{6}$$

Notice that in case of partial information, conditions (5) and (6) boil down to $p_i > 0$ for $i = 1, 2$. Finally, we denote the set of feasible transmission probabilities $\mathbf{p}_i$ for user i as the set

$$\mathcal{P}_i(\mathbf{p}_j) = \{\mathbf{p}_i = [p_i^{(1)}\ p_i^{(2)}]^T\text{: } 0 \leq p_i^{(1)}, p_i^{(2)} \leq 1 \text{ satisfying conditions (5) and (6)}\}. \tag{7}$$

Notice that the set of feasible transmission policies for a given transmitter depends on the strategy selected by the other through constraint (6). Finally, the set of all the feasible vectors $\mathbf{p}$ is defined as

$$\mathcal{P} = \{\mathbf{p} = [\mathbf{p}_1^T \mathbf{p}_2^T]^T\text{: } \mathbf{p}_1 \in \mathcal{P}_1(\mathbf{p}_2) \text{ and } \mathbf{p}_2 \in \mathcal{P}_2(\mathbf{p}_1)\}. \tag{8}$$

B. Optimizing the transmission probabilities

The goal of this work is to discuss decentralized optimization strategies for the transmission probabilities in the random access scheme illustrated in fig. 1. Following [9] [13] [11] [10], the optimization criterion used in this work assumes that the cost of each transmission for any source is given by a parameter $c \in (0, 1)$. This parameter is in general related to the energy spent at each transmission, which is in turn a function of the transmitted power P and may include (if relevant) the contribution of circuitry consumption. It represents essentially the cost of transmitting a packet relative to the value of a successful transmission, which is normalized to 1. In other words, at each slot t, the immediate payoff obtained by the ith source ($i = 1, 2$) reads $-c$ in case of unsuccessful transmission, $1 - c$ in case of successful transmission and 0 in case no transmission takes place.

From the discussion above, the immediate payoff $r_i(S(t), \mathbf{a}(t))$ experienced by the ith user in any tth slot is a function of the set of actions $\mathbf{a}(t) \in \mathcal{A}$ selected at time t and of the state $S(t) \in \mathcal{S}$. In particular, we consider as immediate payoff the average reward obtained with respect to fading, which reads

$$\rho^{(1)} = -c \cdot P_{out}^{(1)} + (1-c)(1 - P_{out}^{(1)}) = 1 - c - P_{out}^{(1)}, \tag{9}$$

when only the ith transmitter is using the channel (i.e., $r_1(S_k, [\text{T W}]) = \rho^{(1)}$ for $k = 2, 4$ and $r_2(S_k, [\text{W T}]) = \rho^{(1)}$ for $k = 3, 4$) and

$$\rho^{(2)} = -c \cdot P_{out}^{(2)} + (1-c)(1 - P_{out}^{(2)}) = 1 - c - P_{out}^{(2)}, \tag{10}$$

when the two users are transmitting simultaneously (i.e., $r_1(S_4, [\text{T T}]) = r_2(S_4, [\text{T T}]) = \rho^{(2)}$). For reference, the immediate payoffs $r_i(S_k, \mathbf{a})$ of the two transmitters for different states $S_k \in \mathcal{S}$ and actions $\mathbf{a} \in \mathcal{A}$ are shown in fig. 2 (crossed boxes correspond to non-feasible alternatives, i.e., to transmissions of packets from non-backlogged users). Notice that both expressions (9) and (10) easily follow from the law of total probability. Moreover, since $P_{out}^{(2)} \geq P_{out}^{(1)}$, we have

$$-c \leq \rho^{(2)} \leq \rho^{(1)} \leq 1 - c. \tag{11}$$

From (9)-(10), in order to ensure that transmission leads to positive payoff (so that waiting all the time is not the most advantageous strategy), we need to guarantee $\rho^{(1)} > 0$, which implies

$$c < 1 - P_{out}^{(1)}. \tag{12}$$

Each user is interested in choosing its transmission probabilities $\mathbf{p}_i$, $i = 1, 2$, in such a way to maximize its own payoff over time. Notice that the transmission probabilities determine both the sequence of actions $\mathbf{a}(t)$, $i = 1, 2$ and sequence of states $S(t)$. Actions and state sequence are then random process whose joint distribution depends on $\mathbf{p} = [\mathbf{p}_1^T\ \mathbf{p}_2^T]^T$. Two optimization criteria are usually considered. The first is the average discounted payoff [21] [18][4]

$$T_i(\mathbf{p}; S^0) = \lim_{T \to \infty} E_{\mathbf{p}} \left[\sum_{t=0}^{T} \delta^t \cdot r_i(S(t), \mathbf{a}(t)) \middle| S(0) = S^0 \right], \tag{13}$$

where $0 < \delta < 1$ is the discount factor; $E_{\mathbf{p}}[\cdot]$ denotes the ensemble expectation with respect to the distribution of the random process $S(t)$ and $\mathbf{a}(t)$; $S^0 \in \mathcal{S}$ is the initial state[5].

An alternative criterion is based on the observation that the state process $S(t)$ is a homogeneous finite Markov chain because of the Markovian transmission policy and the channel model employed. Accordingly, if the Markov chain $S(t)$ is irreducible for any choice of probabilities $\mathbf{p} \in \mathcal{P}$, it admits steady state probabilities $\pi_k(\mathbf{p}) = P[S(t) = S_k]$, $k = 1, 2, 3, 4$. We can then focus on the steady-state behavior, omit dependence on time t, and employ as performance criterion

[4]In [18] it is shown that, under appropriate conditions that are satisfied in our scenario, the solution with respect to the alternative criterion based on a time-average of the expected payoffs:

$$T_i(\mathbf{p}; S^0) = \lim_{T \to \infty} E_{\mathbf{p}} \left[\frac{1}{T} \sum_{t=0}^{T} r_i(S(t), \mathbf{a}(t)) \middle| S(t) = S^0 \right]$$

can be obtained as a limit case of the problem of maximizing (13) for δ that tends to 1.

[5]Notice that the limit in (13) is always finite since the absolute value of the immediate payoff satisfies $|r_i(S_k, \mathbf{a})| \leq 1$. In fact, this entails that (13) is a sum of terms that are bounded in absolute value by the decreasing geometric progression $\{\delta^t\}$.

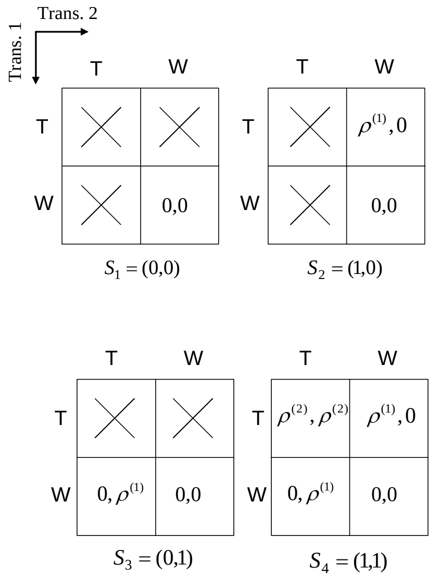

Fig. 2. Immediate payoffs $r_i(S_k, \mathbf{a})$ for the two transmitters (rows indicate the payoff of transmitter $i = 1$ and columns of transmitter $i = 2$). Definitions of $\rho^{(1)}$ and $\rho^{(2)}$ are given in (9)-(10).

the ensemble average payoff with respect to the steady state probabilities [11] [13]

$$R_i(\mathbf{p}) = E_{\mathbf{p}}[r_i(S, \mathbf{a})] = \sum_{k=1}^{4} \pi_k(\mathbf{p}) E_{\mathbf{p}}[r_i(S, \mathbf{a})|\, S = S_k]. \tag{14}$$

Differently from (13), criterion (14) does not depend on the initial state and applies only when the Markov chain $S(t)$ is ergodic (therefore, it admits steady state probabilities). As it will be shown in the following, this condition is satisfied under the assumptions (5) and (6).

The problem of maximizing payoffs (13) or (14) amounts to the optimal control of a Markov process, where the control policy (transmission probabilities) is independent of t (stationary policy) [20] [25]. However, in the considered scenario, optimal control has to be carried out in a decentralized fashion, whereby each transmitter of the interference channel strives to maximize its own utility (13) or (14). In this case, the system can be analyzed using tools from game theory [17] [18], as further discussed in the Sec. IV.

As discussed in [11] [13], criterion (14), because of its independence of the initial state, is a choice well suited for the case where there is imperfect knowledge of the state (see also footnote 3). Therefore, in order to allow comparison between the two scenarios of perfect and partial information, in the following we will focus on criterion (14). The problem of optimizing with respect to (13) is treated in Sec. V.

C. *Efficiency of decentralized optimization (Nash equilibrium and Pareto optimality)*

The scenario illustrated above prescribes a decentralized optimization problem, where each transmitter's goal is the optimization of its own utility. Towards this goal, each transmitters, say the ith, is assumed to decide its transmission policy $\mathbf{p}_i$ rationally and based on the knowledge of utilities and available strategies of the other transmitter. With such a model, an appropriate solution concept is that of Nash Equilibrium (NE) from game theory (see [16] for an introduction tailored to wireless engineers). A NE can be interpreted as the prediction of a likely outcome for a strategic scenario such as the one described above, characterized by selfishness and rationality of the participating agents (or players, in the game-theoretic language) [15]. More formally, a NE is given by a strategy vector $\mathbf{p}^* = [\mathbf{p}_1^{*T} \ \mathbf{p}_2^{*T}]^T$ (and corresponding payoffs $\{R_i(\mathbf{p}^*)\}_{i=1}^2$) such that no transmitter can profitably deviate from it through unilateral choice, i.e., such that there does not exist an alternative strategy $\mathbf{p}_1'$ such that $R_1([\mathbf{p}_1'^T \ \mathbf{p}_2^{*T}]^T) > R_1(\mathbf{p}^*)$, and similarly for the second player[6]. A strategic scenario (i.e., a non-cooperative game) can admit one, multiple or no NE. In cases where multiple NEs exist, it is of interest to select the *most reasonable* ones (according to some criterion), as predictions of the outcome of the game. This further specializations of NEs are referred to as *refinements*, and the Markov Perfect Equilibrium (MPE) discussed in Sec. V is an example.

Since NEs predict the outcome of a competitive (decentralized) optimization process, a natural question to ask is: how far is a NE from the best achievable performance through centralized optimization? In other terms, we are interested in quantifying the performance loss that arises because of the distributed (and selfish) nature of the considered transmission scheme. In a multiobjective optimization, as the one tackled here, there is generally no single optimal point $\mathbf{p}_{opt}$ that maximizes the utility for both agents simultaneously, i.e., for which $R_i(\mathbf{p}_{opt}) \geq R_i(\mathbf{p}')$ for every $\mathbf{p}' \in \mathcal{P}$ and $i = 1, 2$. Instead, optimality is attributed to a set of policies $\mathcal{P}_{opt}$ so that for each point $\mathbf{p}_{opt} \in \mathcal{P}_{opt}$, there is no other policy $\mathbf{p}' \in \mathcal{P}$ for which $R_i(\mathbf{p}') \geq R_i(\mathbf{p}_{opt})$, $i = 1, 2$, and at least one inequality is strict [24]. The set of points $\mathcal{P}_{opt}$ is referred to as *Pareto-efficient*. In words, a strategy $\mathbf{p}_{opt}$ is Pareto-efficient if, with respect to $R_i(\mathbf{p}_{opt})$, is not possible to improve the performance of a given link without degrading the performance of the other. Examples in the next section (see fig. 3) should clarify these concepts to a reader not familiar with them.

The question anticipated in the previous paragraph can now be rephrased as follows: how far is the performance of a NE from the Pareto-efficient strategies $\mathcal{P}_{opt}$? In short, and with the definitions above, the goal of the rest of the paper is to investigate the (Pareto) *efficiency* of decentralized random access policies through the analysis of the corresponding NEs. An interesting account on efficiency and fairness of distributed techniques in a related problem can be found in [5].

[6] For generalizations and details, the reader is referred to textbooks [15] [16].

III. The interference channel with backlogged users as a game

As a reference case, this section considers a simple scenario that deviates from the model in Sec. II in that users are assumed to be always backlogged. The motivation for this temporary detour from the general framework is that it will allows, by comparison, to assess the impact of traffic dynamic (random packet arrival) on the efficiency of decentralized random access.

When the users are backlogged, the system can be thought of being in state $S_4 = (1, 1)$ all the time. In this case, the problem can be formulated as a standard two-player non-cooperative strategic game $\langle \mathcal{N}, \mathcal{A}, \{r_i(S_4, \mathbf{a})\}_{i=1,2}\rangle$, where $\mathcal{N}$ ={1, 2} is the set of players (links) and the payoff $r_i(S_4, \mathbf{a})$ is shown in fig. 2. The transmission strategy employed corresponds to mixed strategies in the jargon of game theory, meaning that each user chooses a random action in $\mathcal{A}_i$ with given probabilities [15]. To simplify the notation in this section, we redefine the transmission probabilities of interest ($p_1^{(2)}$ and $p_2^{(2)}$) for the two transmitters as p_1 and p_2, and the transmission probability vector as $\mathbf{p} = [p_1\ p_2]^T$.

The average payoff (14) obtained by transmitter i when strategy $\mathbf{p} = [p_1\ p_2]^T$ is played can be written as

$$\begin{aligned} R_i(\mathbf{p}) &= E_{\mathbf{p}}[r_i(S_4, \mathbf{a})] = \\ &= p_i R_i(\mathrm{T}, p_j) + (1 - p_i) R_i(\mathrm{W}, p_j) = \\ &= p_i R_i(\mathrm{T}, p_j), \end{aligned} \tag{15}$$

where $R_i(\mathrm{T}, p_j) = E_{\mathbf{p}}[r_i(S_4, \mathbf{a})\,|a_i = \mathrm{T}]$ denotes, with a slight abuse of notation, the average payoff of transmitter i when transmitting:

$$R_i(\mathrm{T}, p_j) = p_j \rho^{(2)} + (1 - p_j)\rho^{(1)}, \tag{16}$$

and $R_i(\mathrm{W}, p_j) = E_{\mathbf{p}}[r_i(S_4, \mathbf{a})\,|a_i = \mathrm{W}] = 0$ is the average payoff corresponding to waiting. Notice that (16) follows from the observation that the payoff of user i is $\rho^{(1)}$ if it is the only one transmitting and $\rho^{(2)}$ if both transmitters are active.

If $\rho^{(2)} > 0$ (which implies $\rho^{(1)} > 0$), it is easy to see that each transmitter maximizes its own payoff (15) by selecting $p_i = 1$, irrespective of the choice of the other transmitter. Therefore, there exists a unique NE equilibrium corresponding to $p_1^* = p_2^* = 1$[7]. In other words, if the payoff is positive even in presence of collision, the two players decide to transmit in each slot. This is a very aggressive strategy, which is a result of the decentralized (competitive) mechanism employed for optimization. Such a solution is easily expected to be inefficient, as shown in the example in fig. 3. Here, we consider the following parameters: $SNR = 10dB$, $\beta = 5dB$, $\gamma^2 = 0dB$ and $c = 0.05$ that entail $\rho^{(2)} > 0$. Fig. 3 shows the region of achievable payoffs $\{R_i(\mathbf{p})\}_{i=1}^2$ with $\mathbf{p} \in \mathcal{P}$ and the NE (circle). The Pareto-efficient set $\mathcal{P}_{opt}$ is given by the boundary of the achievable region. As it can be seen, the NE is quite far from the optimal set.

[7] A degenerate mixed strategy like the one at hand is referred to as pure in game theory.

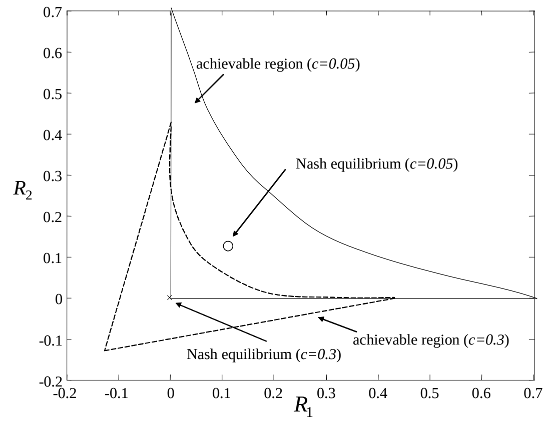

Fig. 3. Achievable utility region and NE for backlogged users and different transmission costs $c = 0.1$ and $c = 0.3$ ($SNR = 10dB$, $\beta = 5dB$, $\gamma^2 = 0dB$).

On the other hand, if $\rho^{(2)} < 0$, there exist a unique NE equilibrium that is obtained by imposing that the payoffs corresponding to the available actions (T and W) are the same [15]: $R_i(\mathrm{T}, p_j^*) = R_i(\mathrm{W}, p_j^*) = 0$. Therefore, the NE reads

$$p_i^* = p_j^* = \frac{\rho^{(1)}}{\rho^{(1)} - \rho^{(2)}}. \tag{17}$$

Following the example above, fig. 3 shows the achievable region and the corresponding NE for $c = 0.3$ ($\rho^{(2)} < 0$). Notice that in this case, utilities can be negative. Again, it can be seen that the decentralized solution (NE, cross) is quite inefficient.

IV. The interference channel with random access as a game

In this section, we consider the original problem discussed in Sec. II that includes traffic dynamics. In this case, the transmitters interact in a dynamic fashion adjusting their decision to their (either perfect or partial) knowledge of the current state of the system. Under the optimization criterion (13), the model lends itself to be cast into the framework of stochastic (or Markov) games [17] [18], as discussed in Sec. V. However, when considering the merit function (14), the problem can be equivalently seen as a non-cooperative strategic game, similar to the previous section. Notice that, as explained in Sec. II-B, in order to be able to apply criterion (13), we need to verify that the Markov chain $S(t)$ is ergodic for every strategy $\mathbf{p} \in \mathcal{P}$. This issue is dealt with in Appendix-A, where it is shown that, under assumptions (5)-(6), the state sequence $S(t)$ is indeed ergodic. Moreover, the steady state probabilities $\{\pi_k(\mathbf{p})\}_{k=1}^4$ are evaluated as a function of the probabilities $\mathbf{p}$ and the arrival rates $\{\lambda_i\}_{i=1}^2$ (see (27)).

The problem formulated in Sec. II-B with merit function (13) can be studied as a non-cooperative game $\langle \mathcal{N}, \mathcal{P}, \{R_i(\mathbf{p})\}_{i=1,2}\rangle$, with strategy space $\mathcal{P}$ in (8) and payoff function

$$R_i(\mathbf{p}) = \pi_{q(i)}(\mathbf{p}) \cdot p_i^{(1)} \rho^{(1)} + +\pi_4(\mathbf{p}) \cdot [p_i^{(2)}(1-p_j^{(2)})\rho^{(1)} + p_i^{(2)} p_j^{(2)} \rho^{(2)}], \quad (18)$$

where $q(1) = 2$ and $q(2) = 3$. Equation (18) is easily obtained from the law of total probabilities. In fact, the first term accounts for the state where only user i has a backlogged packet (the payoff is then $\rho^{(1)}$ if the user transmits), whereas the second corresponds to the state where both have a packet to transmit (the payoff is $\rho^{(1)}$ if only user i transmits, $\rho^{(1)}$ if both users transmit). Recalling (9) and (10), it is apparent that condition (12) is necessary in order to guarantee a positive utility $R_i(\mathbf{p}) \geq 0$ for non-null transmission probabilities.

In the following, we consider the case of perfect and partial information separately.

A. The case of perfect information

In the case of perfect information, each player maximizes its own utility (18) by always transmitting if it is the only one to have a packet in queue, i.e., by setting $p_1^{(1)} = p_2^{(1)} = 1$, irrespective of the strategy played by the other transmitter. Therefore, we are left with the problem of obtaining the transmission probabilities $p_1^{(2)}$ and $p_2^{(2)}$ that correspond to a NE equilibrium. This can be done numerically by evaluating the fixed point of the best responses of the two players [17] [18]. As an example, fig. 4 considers the same scenario as fig. 3 with cost $c = 0.3$ (only the positive part of the achievable region is shown). Moreover, the arrival rates are set as $\lambda_1 = \lambda_2 = 0.8$. The achievable region clearly shows that, with random access, obtaining utilities with large and similar values for both users is not feasible. Moreover, it should be noticed that there are two Pareto-efficient points corresponding to the corner points in the U-shaped part of the achievable region. The NE is found to correspond to $p_1^{(2)} = p_2^{(2)} = 0.6$, and provides utilities that are close to the boundary of the achievable region. Efficiency of the NE is expected to improve in an asymmetric case. This is confirmed by fig. 5, where the first transmitter has a smaller packet arrival rate $\lambda_1 = 0.2$ (the other parameters are selected as in fig. 4). In this scenario, the NE predicts transmission probabilities $p_1^{(2)} = 0.6$ and $p_2^{(2)} = 0.47$. The NE is qualitatively more efficient than in the backlogged case. As shown below, this effect is even more pronounced in the case of partial information.

B. The case of partial information

With partial information, only one transmission probability has to be selected per user and the problem is a special case of the game described above with $p_i^{(1)} = p_i^{(2)} = p_i$. NEs can be found numerically as the fixed point of the best responses of the two players [17] [18]. The effect of partial information is shown by comparison with the results discussed above for the case of perfect information in fig. 4 and 5. As it can be seen, partial information entails a smaller achievable region and a NE with smaller utilities. However, efficiency of the NE is enhanced, most notably in the asymmetric case. Notice that

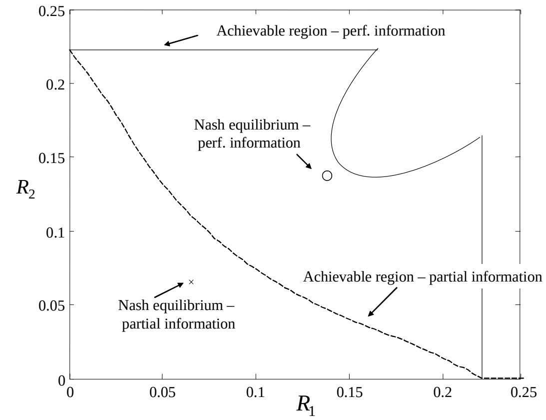

Fig. 4. Achievable utility region and NE with random packet arrivals and perfect/ partial information ($SNR = 10dB$, $\beta = 5dB$, $\gamma^2 = 0dB$, $\lambda_1 = \lambda_2 = 0.8$, $c = 0.3$).

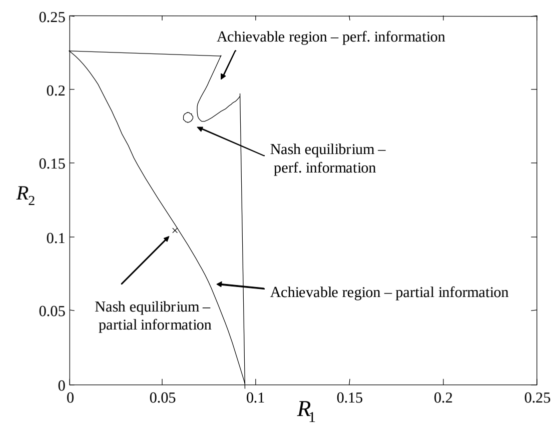

Fig. 5. Achievable utility region and NE with random packet arrivals and perfect/ partial information ($SNR = 10dB$, $\beta = 5dB$, $\gamma^2 = 0dB$, $\lambda_1 = 0.2, \lambda_2 = 0.8$, $c = 0.3$).

the NE in fig. 4 corresponds to $p_1^* = p_2^* = 0.88$ and the one in fig. 5 to $p_1^* = 1$, $p_2^* = 0.55$.

From the discussion above, random packet arrival has a beneficial effect on the efficiency of decentralized random access, especially in the case of partial information and in an asymmetric scenario. In a way, the selfishness of different transmitters is moderated by the fact that different links do not always have packets to transmit. Random packet arrivals then act as a sort of random scheduler that alleviate the inefficiency of purely competitive random access.

V. Optimizing the Average Discounted Payoff

In this section, we tackle the problem of decentralized optimization of the transmission probabilities for perfect information and optimization criterion (13). This scenario can be modelled as a stochastic game [19] $< \mathcal{S}, \mathcal{A}, G, \{r_i(S_k, \mathbf{a})\}_{i=1,2} >$, where we recall that $\mathcal{S}$ is the set of states, $\mathcal{A}$ the set of actions and $r_i(S_k, \mathbf{a})$ is the immediate payoff of user i in state $S_k \in \mathcal{S}$ for action $\mathbf{a} \in \mathcal{A}$ as defined in fig. 2. Moreover, $G(S_m|\mathbf{a}, S_n)$ is the transition probability of the Markov chain $S(t)$ from state S_n to S_m given that actions $\mathbf{a} \in \mathcal{A}$ are selected (see Appendix-A). Notice that at each state S_k, the actions $\mathbf{a}$ selected by the two transmitters determine: (*i*) the immediate payoff $r_i(S_k, \mathbf{a})$; (*ii*) the transition of the state from S_n to a state S_m with probability $G(S_m|\mathbf{a}, S_n)$.

In a general stochastic game, players can select a different strategy at each time according to the whole history of its own and other players' previous choices. Among all the NEs that can be established in this general framework, of particular interest is a refinement of the concept of NE referred to as perfect equilibrium. A strategy is a perfect equilibrium if it is a NE for every possible initial state S^0 in (13) [15][8]. The transmission policies discussed in Sec. II are behavior strategies in the jargon of stochastic game theory, since they randomize between possible actions (transmit and wait) at each time [15][9]. In particular, the transmission policy at hand is a Markov (or stationary) behavior strategy since it only depends on the past through the current state $S(t)$. For a stochastic game with a finite number of states and actions as the one of interest here, a Markov Perfect Equilibrium (MPE) is guaranteed to exist [18]. This section elaborates on the evaluation of the MPE.

To start with, we note again that each transmitter maximizes its own utility at each time by selecting $p_1^{(1)} = p_2^{(1)} = 1$, irrespective of the strategy played by the other transmitter. Therefore, the problem amounts to obtaining the transmission probabilities $p_1^{(2)}$ and $p_2^{(2)}$ that correspond to a MPE equilibrium. This can be done by imposing that the selected strategy is a NE for the game irrespective of the initial state S^0. In our case, this condition corresponds to ensuring that the payoff obtained when transmitting equals the payoff of waiting if starting in state S_4 (see Sec. III and [15] [16]):

$$T_i(\mathrm{T}, p_j^{(2)}; S_4) = T_i(\mathrm{W}, p_j^{(2)}; S_4), \tag{19}$$

where, with reference to equation (13),

$$T_i(\mathbf{p}; S_4) = p_i^{(2)} T_i(\mathrm{T}, p_j^{(2)}; S_4) + (1 - p_i^{(2)}) T_i(\mathrm{W}, p_j^{(2)}; S_4). \tag{20}$$

Notice that if the discount factor δ is zero, the problem boils down to the backlogged case treated in Sec. III (see also Appendix-B, equations (28) and (29)). In particular, the equilibrium probability $p_i^{(2)}$ equals (17). In this latter case, if the cost of transmission c is high (within the limits (12)) and/or the interference factor γ is sufficiently large (i.e., $\rho^{(1)} \simeq 0$ and/or $|\rho^{(2)}| \gg \rho^{(1)}$), the transmission probability $p_i^{(2)}$ (17) at the equilibrium becomes small. This is easily explained by noticing that, since $\delta = 0$, transmitters do not have the perspective of future payoffs to compensate for the present loss due to the large transmission cost and/or interference.

In the more general case with $\delta > 0$, payoffs $T_i(\mathrm{T}, p_j^{(2)}; S_4)$ and $T_i(\mathrm{W}, p_j^{(2)}; S_4)$ in (19) depend on the immediate payoffs $\rho^{(1)}$ and $\rho^{(2)}$ for $k = 1, 2, 3, 4$ through the discount factor δ (see Appendix-B, equations (28) and (29)). In fact, the payoff at each state depends also on the future payoffs that are conditioned on the current choice. As shown in Appendix-B, obtaining the probability $p_j^{(2)}$ that satisfies (19) then requires to solve a system of four non-linear equations

$$\mathbf{v}_i = \mathbf{f}_i(\mathbf{v}_i), \tag{21}$$

where $\mathbf{v}_i = [T_i(\mathbf{p}; S_1)\ T_i(\mathbf{p}; S_2)\ T_i(\mathbf{p}; S_3)\ T_i(\mathbf{p}; S_4)]^T$[10]. Existence of a solution (i.e., fixed point) for equation (21) is guaranteed by the existence of a Markov perfect equilibrium for the stochastic game at hand [18]. Iterative algorithms can be devised that aim at evaluating the fixed point in (21) as explained in [23]. Convergence of these algorithms in dependent on the choice of system parameters and not easily determined for our problem. In the next section, Jacobi iterations [23] are used to evaluate the solution to (21) showing fast convergence for the selected parameters.

A. Numerical results

Here we present some numerical results to corroborate the analysis above. Parameters are selected as in the examples in the previous section (symmetric case) with different costs c and discount factors δ, as detailed in the following. Fig. 6 and fig. 7 show the probability $p_i^{(2)}$ as it evolves during the Jacobi iterations[11] used to solve the non-linear system (21). Initialization is random. The two figures refer to scenarios where the discount factor δ has opposite effects of the probability at the MPE $p_i^{(2)}$. In particular, in the first, the cost of transmission is $c = 0.7$, which is large enough to entail $\rho^{(2)} < 0$ and a small (positive) $\rho^{(1)} (= 0.03)$. In this case, as discussed in Sec. IV-A, decreasing the memory δ renders the prospect of future revenues negligible and causes the transmitter to choose a decreasing $p_i^{(2)}$. On the other hand, in fig. 7 the dual case is depicted, which occurs when the cost is not large enough to "discourage" players to access the channel even in presence of interference ($c = 0.3$ in the example, which entails $\rho^{(1)} = 0.43$). In this latter scenario, decreasing δ increases $p_i^{(2)}$ at the MPE.

VI. Concluding remarks

In this work, a two-by-two interference channel with random packet arrival and random access has been studied under the assumption that transmission probabilities are concurrently and selfishly selected at the two transmitters. Using solution concepts from game theory (and in particular stochastic game

[8] In the case of centralized optimization, this condition corresponds to the principle of optimality in dynamic programming, that leads to the Bellman's equations [25].

[9] Behavior strategy are strictly related to mixed strategies in non-cooperative strategic games, see [15] for details.

[10] The system (21) has a correspondent in the Bellman's equations, that provide the solution to the centralized control problem [25].

[11] Denoting by k the iteration number and $\mathbf{v}_i(k)$ the corresponding solution to (21), the Jacobi iterations amount to $\mathbf{v}_i(k) = \mathbf{f}_i(\mathbf{v}_i(k-1))$, and the corresponding pobability $p_i^{(2)}$ is obtained from (30).

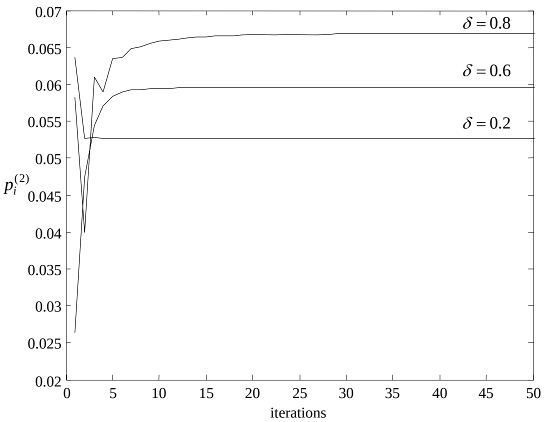

Fig. 6. Evolution of probability $p_i^{(2)}$ through the Jacobi iterations (21) for different discount factors δ ($SNR = 10dB$, $\beta = 5dB$, $\gamma^2 = 0dB$, $\lambda_1 = \lambda_2 = 0.8$, $c = 0.7$).

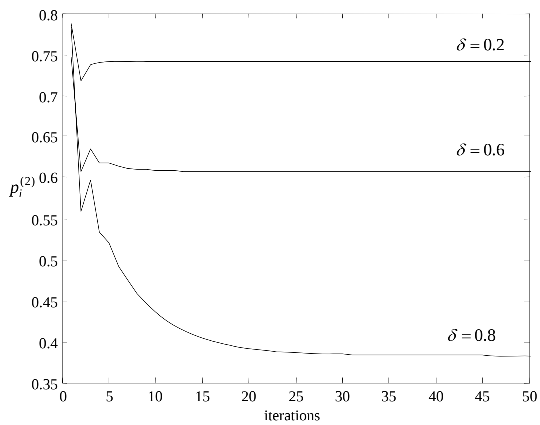

Fig. 7. Evolution of probability $p_i^{(2)}$ through the Jacobi iterations (21) for different discount factors δ ($SNR = 10dB$, $\beta = 5dB$, $\gamma^2 = 0dB$, $\lambda_1 = \lambda_2 = 0.8$, $c = 0.3$).

theory), efficiency of such a decentralized communication scenario has been investigated. The main conclusion is that random packet arrival alleviates the inherent drawbacks of purely competitive random access, especially in the case of partial information and in an asymmetric scenario. This result confirms traffic dynamics as an essential element to be accounted for in the analysis of secondary spectrum access solutions.

VII. Appendix

A. Analysis of the state sequence $S(t)$

Here we first show that for every strategy $\mathbf{p} \in \mathcal{P}$ the Markov chain $S(t)$ is ergodic and then evaluate the steady state probabilities $\boldsymbol{\pi}(\mathbf{p}) = [\pi_1(\mathbf{p})\ \pi_2(\mathbf{p})\ \pi_3(\mathbf{p})\ \pi_4(\mathbf{p})]^T$. Let us denote the transition probability function from state S_n to S_m given that the action $\mathbf{a} \in \mathcal{A}$ is played as $G(S_m|\mathbf{a}, S_n)$. Defining the 4×4 matrix $[\mathbf{G}(\mathbf{a})]_{mn} = G(S_m|\mathbf{a}, S_n)$, we easily get

$$
\begin{aligned}
\mathbf{G}([\mathrm{W\ W}]) &= \begin{bmatrix} (1-\lambda_1)(1-\lambda_2) & 0 & 0 & 0 \\ \lambda_1(1-\lambda_2) & 1-\lambda_2 & 0 & 0 \\ \lambda_2(1-\lambda_1) & 0 & 1-\lambda_1 & 0 \\ \lambda_1\lambda_2 & \lambda_2 & \lambda_1 & 1 \end{bmatrix} \\
\mathbf{G}([\mathrm{T\ W}]) &= \begin{bmatrix} \times & (1-P_{out}^{(1)})(1-\lambda_2) & \times & 0 \\ \times & P_{out}^{(1)}(1-\lambda_2) & \times & 0 \\ \times & (1-P_{out}^{(1)})\lambda_2 & \times & 1-P_{out}^{(1)} \\ \times & P_{out}^{(1)}\lambda_2 & \times & P_{out}^{(1)} \end{bmatrix} \\
\mathbf{G}([\mathrm{W\ T}]) &= \begin{bmatrix} \times & \times & (1-P_{out}^{(1)})(1-\lambda_1) & 0 \\ \times & \times & (1-P_{out}^{(1)})\lambda_1 & 1-P_{out}^{(2)} \\ \times & \times & P_{out}^{(1)}(1-\lambda_1) & 0 \\ \times & \times & P_{out}^{(1)}\lambda_1 & P_{out}^{(2)} \end{bmatrix} \\
\mathbf{G}([\mathrm{T\ T}]) &= \begin{bmatrix} \times & \times & \times & (1-P_{out}^{(2)})^2 \\ \times & \times & \times & P_{out}^{(2)}(1-P_{out}^{(2)}) \\ \times & \times & \times & P_{out}^{(2)}(1-P_{out}^{(2)}) \\ \times & \times & \times & (P_{out}^{(2)})^2 \end{bmatrix},
\end{aligned}
$$

where the symbol "$\times$" indicates that the given set of actions is not allowed in the state corresponding to the selected column[12].

Being a function of the action vector $\mathbf{a}$, the probability $G(S_m|\mathbf{a}, S_n)$ is a random variable under the randomized strategy prescribed by the transmission policy. The unconditional transition probability function $G(S_m|S_n)$ is then obtained by averaging with respect to the distribution of the actions $\mathbf{a}$, according to the law of total probability. The resulting transition probabilities $G(S_m|S_n)$ can be conveniently accommodated in a 4×4 transition matrix $\mathbf{G}(\mathbf{p})$ ($[\mathbf{G}(\mathbf{p})]_{mn} = G(S_m|S_n)$), where we have made explicit the dependence on the transmission probabilities $\mathbf{p}$ in (23). This can be easily calculated as

$$
\mathbf{G}(\mathbf{p}) = \begin{bmatrix} \bar{\lambda}_1\bar{\lambda}_2 & \Omega_1^{(1)}\bar{\lambda}_2 & \Omega_2^{(1)}\bar{\lambda}_1 & \Omega_1^{(2)}\Omega_2^{(2)} \\ \lambda_1\bar{\lambda}_2 & \chi_1\bar{\lambda}_2 & \Omega_2^{(1)}\lambda_1 & G_{24}(\mathbf{p}) \\ \lambda_2\bar{\lambda}_1 & \Omega_1^{(1)}\lambda_2 & \chi_2\bar{\lambda}_1 & G_{34}(\mathbf{p}) \\ \lambda_1\lambda_2 & \chi_1\lambda_2 & \chi_2\lambda_1 & G_{44}(\mathbf{p}) \end{bmatrix}, \quad (23)
$$

with $\bar{\lambda}_i = 1-\lambda_i$, $\chi_i = [p_i^{(1)}P_{out}^{(1)} + (1-p_i^{(1)})]$, $\Omega_j^{(i)} = p_j^{(i)}(1-P_{out}^{(i)})$ and

$$
\begin{aligned}
G_{24}(\mathbf{p}) &= (1-p_1^{(2)})p_2^{(2)}(1-P_{out}^{(1)}) + p_1^{(2)}\Omega_2^{(2)}P_{out}^{(2)} \\
G_{34}(\mathbf{p}) &= (1-p_2^{(2)})p_1^{(2)}(1-P_{out}^{(1)}) + p_1^{(2)}\Omega_2^{(2)}P_{out}^{(2)} \\
G_{44}(\mathbf{p}) &= (1-p_1^{(2)})(1-p_1^{(2)}) + (1-p_2^{(2)})p_1^{(2)}P_{out}^{(1)} \\
&\quad +(1-p_1^{(2)})p_2^{(2)}P_{out}^{(1)} + p_1^{(2)}p_2^{(2)}(P_{out}^{(2)})^2. \quad (24)
\end{aligned}
$$

Based on the expression of the transition matrix $\mathbf{G}(\mathbf{p})$, it is easy to show that $S(t)$ is guaranteed to be ergodic for each $\mathbf{p} \in \mathcal{P}$. In fact, by building the graph associated to the transition matrix $\mathbf{G}(\mathbf{p})$ in (23), it is clear that as long as (5) and (6) are satisfied, the chain is irreducible and aperiodic.

[12]Notice that in order for state S_4 to be non-absorbing, we need the condition $P_{out}^{(2)} < 1$, which is satisfied by (4).

The steady state probability $\boldsymbol{\pi}(\mathbf{p})$ of the Markov chain are defined by the fixed point of the transition matrix as:

$$\boldsymbol{\pi}(\mathbf{p}) = \mathbf{G}(\mathbf{p})\boldsymbol{\pi}(\mathbf{p}). \quad (25)$$

Recalling that $\boldsymbol{\pi}(\mathbf{p})$ has to lie in the probability simplex, we have $\mathbf{1}^T\boldsymbol{\pi}(\mathbf{p})$ =1, which added to (25) leads to

$$(\mathbf{I} - \mathbf{G}(\mathbf{p}))\boldsymbol{\pi}(\mathbf{p}) + \mathbf{1}\mathbf{1}^T\boldsymbol{\pi}(\mathbf{p}) = \mathbf{1}, \quad (26)$$

from which (27) we have:

$$\boldsymbol{\pi}(\mathbf{p}) = (\mathbf{I} - \mathbf{G}(\mathbf{p}) + \mathbf{1}\mathbf{1}^T)^{-1}\mathbf{1}. \quad (27)$$

B. Derivation of (21)

Let us consider the first user and drop the user index for simplicity of notation by defining the vector $\mathbf{v} = [T_1(\mathbf{p}; S_1)\ T_2(\mathbf{p}; S_2)\ T_3(\mathbf{p}; S_3)\ T_4(\mathbf{p}; S_4)]^T$. The second link can be treated in exactly the same way. As explained in Sec. V, in order to obtain the MPE for the stochastic game at hand, we need to impose condition (19). The discounted payoffs when transmitting and waiting can be written as

$$\begin{aligned} T_1(\mathrm{T}, p_j^{(2)}; S_4) &= p_2^{(2)}\rho^{(2)} + (1 - p_2^{(2)})\rho^{(1)} + \quad (28) \\ & +\delta \cdot p_2^{(2)} \sum_{k=1}^{4} G(S_k|[\mathrm{T\ T}], S_4)v_k \end{aligned}$$

and

$$\begin{aligned} T_1(\mathrm{W}, p_j^{(2)}; S_4) &= \delta p_2^{(2)} \sum_{k=1}^{4} G(S_k|[\mathrm{W\ T}], S_4)v_k + \quad (29) \\ & +\delta(1 - p_2^{(2)}) \sum_{k=1}^{K} G(S_k|[\mathrm{W\ W}], S_4)v_k, \end{aligned}$$

respectively. Probability $p_2^{(2)}$ can then obtained using (19) and recalling (22) as

$$p_2^{(2)} = \frac{-\delta(1 - P_{out}^{(1)})v_3 + \delta(1 - P_{out}^{(1)})v_4}{(\rho^{(2)} - \rho^{(1)}) + \sum_{k=1}^{4}\theta_k v_k}, \quad (30)$$

where $\theta_1 = \delta(1-P_{out}^{(2)})^2$, $\theta_2 = \delta[(1-P_{out}^{(2)})P_{out}^{(2)} - (1-P_{out}^{(1)})]$, $\theta_3 = \delta((1 - P_{out}^{(2)})P_{out}^{(2)} - (1 - P_{out}^{(1)}))$ and $\theta_4 = (P_{out}^{(2)})^2 - P_{out}^{(1)} - P_{out}^{(2)} + 1$. Since the probability $p_2^{(2)}$ in (30) depends on the unknown v_k $k = 1, 2, 3, 4$, in order to be able to evaluate it, we need to solve the following system of equalities. These conditions easily follow from the definition (13) by recalling that $p_1^{(1)} = p_2^{(1)} = 1$:

$$\begin{aligned} v_1 &= \delta \sum_{i=1}^{4} G(S_i|[\mathrm{W\ W}], S_1)v_i \\ v_2 &= \rho^{(1)} + \delta \sum_{i=1}^{4} G(S_i|[\mathrm{T\ W}], S_2)v_i \\ v_3 &= \delta \sum_{i=1}^{4} G(S_i|[\mathrm{W\ T}], S_3)v_i \\ v_4 &= \delta p_2^{(2)}(1 - P_{out}^{(1)})v_2 + \delta[p_2^{(2)}(P_{out}^{(2)} - 1) + 1]v_4, \end{aligned}$$

where the last equation is a consequence of (20) and (19). Substituting (30) in (31), we have defined the system of equality (21).

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