How Many Bits of Feedback is Multiuser Diversity Worth in MIMO Downlink?

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Abstract— **The impact of multiuser diversity on MIMO downlink is generally measured in terms of two asymptotic quantities derived from the sum-rate, namely the scaling law of the sum-rate versus the number of users n (for a fixed signal-to-noise ratio, SNR) and the multiplexing gain (i.e., asymptotic growth of the sum-rate versus SNR). Designing optimal strategies with respect to these two criteria requires the availability of Channel State Information (CSI) at the transmitter, which in turn demands feedback of information from receivers to the transmitter. An open question is: how many bits of feedback are really necessary to achieve optimality of these criteria, i.e., to fully exploit multiuser diversity?**

In this paper, the optimal scaling law of the sum-rate with respect to n, for fixed SNR, fixed number of transmit antennas M and any number of receiving antennas N (i.e., $M \log\log nN$), is proved to be achievable with a deterministic feedback of only one bit per user. The impact of adding feedback bits is also investigated. Furthermore, it is shown that the optimal multiplexing gain of M is guaranteed if the total feedback per cell is proportional to the SNR (in dB). The proofs build on opportunistic beamforming and binary quantization of the signal-to-noise-plus-interference ratio.

I. Introduction

The optimal transmission scheme for a multi-antenna (MIMO) downlink channel, that is able to achieve the corresponding capacity region [1], is known to be the Dirty Paper Coding (DPC) technique proposed in [2]. The method is based on interference pre-subtraction at the transmitter and requires full channel state information (CSI) at the base station (BS). Based on this side information, the BS can maximize the sum-rate by appropriately ordering the data streams to be transmitted before pre-subtraction and by power allocation. This strategy amounts to privileging users that experience the best fading channel at each time-instant, a property usually defined as multiuser diversity [3].

Multiuser diversity broadly refers to the benefits of exploiting the instantaneous CSI available at the BS in order to properly design the transmission strategy. From a mathematical standpoint, multiuser diversity is generally measured in terms of two asymptotic quantities derived from the sum-rate, namely the *scaling law* of the sum-rate versus the number of users n (for a fixed signal-to-noise ratio, SNR) and the *multiplexing gain* (i.e., growth of the sum-rate versus SNR).

As explained above, the optimal transmission scheme (DPC) requires full CSI, which in turns calls for a high rate feedback channel from the receivers (that measure their own CSI) to the BS. Therefore, the challenge for the designer of a MIMO downlink is to devise (suboptimal) schemes that reduce the amount of feedback with negligible loss in terms of multiuser diversity. Unlike single-user MIMO systems, where optimal singular-value decomposition and water-filling power allocation based on full CSI is known to provide marginal gains over CSI-unaware transmission schemes [4], multi-user MIMO broadcast channels are known to be very sensitive to the availability of CSI [5].

For fixed signal-to-noise ratio (SNR), fixed number of transmit antennas M and any number of receiving antennas (per user) N, the asymptotic measure of interest is the *scaling law* of the sum-rate. In [6] it was shown that zero-forcing beamforming coupled with an ad-hoc scheduling strategy is able to attain the optimal scaling law $M \log\log nN$ with less complexity than DPC, but with no reduction in terms of feedback load. Moreover, [7] proved that the asymptotic optimality of zero-forcing beamforming can be preserved even if the transmitter only knows the singular vectors corresponding to the largest singular value of the channel matrices for each user. The scaling law is also considered in [8], where an opportunistic scheme that prescribes transmission on a random set of M orthogonal beams to M users is investigated. Each user measures the signal-to-noise-plus-interference ratio (SINR) on the M beams and feeds back only the best SINR and the corresponding beam index (total feedback of N reals and N integer numbers). In spite of the significant reduction of feedback, the scheme in [8] is proved to achieve the optimal scaling law of $M \log\log nN$.

The *multiplexing gain* (i.e., growth of sum-rate with respect to the SNR, fixing the other parameters) of limited feedback techniques for the multi-antenna Gaussian broadcast channel has been addressed in [9]. Focusing the analysis on zero-forcing beamforming techniques, it was shown that the number of feedback bits per user has to grow linearly with the number of transmit antennas M and logarithmically with the SNR, in order to retain the optimal multiplexing gain of M. If the condition on the feedback bits is not satisfied, the system shows to be interference-limited and the multiplexing can be as low as 1, reducing the sum-capacity up to a factor of M. Here, it is shown that the same conclusion is valid in the framework of orthogonal random beamforming [8].

A. Main Contributions

In this paper, we first show that the optimal scaling law of the sum-rate of a MIMO Gaussian broadcast channel $M \log \log nN$, can be achieved with only one bit of feedback per user (Sec. III, Theorem 1).The transmission scheme used to prove this result is similar to [8] in that it applies the opportunistic beamforming principle [10] to a set of orthogonal beams. However, each user, instead of feeding back the SINR for the best beam for each receive antenna (N real plus N integer numbers), only transmits one bit to the BS, indicating whether or not the SINR on a pre-selected beam for any receive antenna is above a given threshold.

Finally, in Sec. V the multiplexing gain is considered. It is proved that in order for the one-bit feedback scheme to be non-interference limited (i.e., to guarantee the optimal scaling law of M), the average number of feedback bits has to scale logarithmically with the SNR. The latter result confirms the outcome of the analysis in [9] where zero-forcing beamforming was studied.

II. Multi-beam transmission with one-bit feedback per user

We consider a multi-antenna downlink system where the BS employs a set of M random orthonormal beams $\mathcal{U} = \{\mathbf{u}_1, \dots, \mathbf{u}_M\}$, generated from an isotropic distribution [11], and constructs the transmitted signal $\mathbf{x}$ as

$$\mathbf{x} = \sum_{m=1}^{M} \mathbf{u}_m s_m, \quad (1)$$

where $\mathbf{x} \in \mathcal{C}^{M\times 1}$ has a power constraint $\mathbb{E}[\|\mathbf{x}\|^2] = M$ and we assume that s_m's are letters from a Gaussian codebook. The signal received at the jth antenna of the ith user is given by

$$y_{i,j} = \sqrt{\rho}\mathbf{h}_{i,j}^T \mathbf{x} + w_{i,j}. \quad (2)$$

The AWGN $w_{i,j}$ has unit variance $\mathcal{CN}(0,1)$. The channel $\mathbf{h}_{i,j}$ is a $M \times 1$ dimensional vector of independent identically distributed zero mean circularly symmetric complex Gaussian random variables with unit variance $\mathcal{CN}(0,1)$, independent among different users and receive antennas. It is assumed that the channel is perfectly known at the receiver and that communication spans a large number of channel coherence periods (ergodic model).

Let us assume that the rth beam is intended for user i. The received signal in (2) may now be restated as

$$y_{i,j} = \sqrt{\rho}\mathbf{h}_{i,j}^T \mathbf{u}_r s_r + \sqrt{\rho}\sum_{m\neq r}^{M} \mathbf{h}_{i,j}^T \mathbf{u}_m s_m + w_{i,j}. \quad (3)$$

Treating each antenna independently, the SINR from beam r at the jth antenna of user i is given by,

$$\mathcal{S}_{i,r,j} = \frac{|\mathbf{h}_{i,j}^T \mathbf{u}_r|^2}{1/\rho + \sum_{\substack{m=1\\ m\neq r}}^{M} |\mathbf{h}_{i,j}^T \mathbf{u}_m|^2}. \quad (4)$$

A. Scheduling

The scheduling of transmission onto the M beams is carried out at the BS aided by one bit of feedback per user. Each user, say the ith, measures the SINR only on *one* beam, say the rth, previously assigned to it by the BS in a pseudo-random fashion. The same beam is measured for all the receive antennas of a given user.

The maximum SINR among the receive antennas, i.e $\tilde{\mathcal{S}}_{i,r} = \max_j \mathcal{S}_{i,r,j}$, is compared to a given threshold α, which is a network parameter known by the BS and all users. One bit of feedback from user i informs the BS on whether or not $\tilde{\mathcal{S}}_{i,r}$ is above the threshold ($\tilde{\mathcal{S}}_{i,r} > \alpha$). After receiving feedback from all users, the BS schedules for each beam one user picked randomly among those who have signaled the SINR on the corresponding beam to be above the threshold. The scheme is similar to the one proposed in [12], but the latter only employs one beam at each time, proving to be unable to attain the optimal scaling law.

Notice that there is a small probability that a certain beam is not *requested* by any user, i.e., that no user measures a strong enough SINR on the beam. In this event, we assume that the BS communicates through the *unrequested* beam to a user picked randomly from the entire set of users. This assumption ensures that equation (4) holds at all times, which is mathematically convenient for our analysis.

B. Sum-rate

Let P_m be the probability that beam m is requested, i.e., the probability that at least one user measures a SINR above the threshold on that beam. Because of the symmetry of the setup P_m is equal for all beams and henceforth the index is dropped. The sum-rate of the multi-beam scheme with one bit feedback described above satisfies the following.

Lemma 1: Let P be the probability of any beam to be requested (by at least one user) and α be the pre-determined threshold. Then, the achievable sum-rate of the multi-beam transmission scheme with one-bit feedback is lower bounded by

$$R_1 \geq MP \cdot \mathbb{E}[\log(1+\tilde{\mathcal{S}})|\tilde{\mathcal{S}} > \alpha], \quad (5)$$

where

$$P = \left(1 - (F(\alpha))^{\frac{nN}{M}}\right), \quad (6)$$

and $F(x)$ is the cumulative distribution function of the SINR (4) [8]

$$F(x) = 1 - \frac{e^{-x/\rho}}{(1+x)^{M-1}}. \quad (7)$$

Proof of Lemma 1 is provided in Appendix-A. For the case of $M = 1$ and $N = 1$, (5) reduces to the expression obtained in [13]. As elaborated in the following sections, the key parameter in (5) is the threshold α: fixing the other parameters, increasing the threshold is expected to decrease the probability P of the beam being requested (that represents the pre-log term in (5)), and, on the other hand, increase the rate $\mathbb{E}[\log(1+\tilde{\mathcal{S}})|\tilde{\mathcal{S}} > \alpha]$. Therefore, an appropriate selection of the threshold α is of crucial importance in order to obtain the optimum performance.

III. Scaling law of the sum-rate with one-bit feedback and fixed M

In this section we show that the optimal scaling law of the sum-rate achieved by DPC with full CSI is also attainable by the multi-beam transmission scheme with only one bit of feedback per user described in section II.

Theorem 1: Let M and ρ be fixed. The sum-rate R_1 of the multi-beam transmission scheme with one bit feedback per user, satisfies for any N

$$\lim_{n\to\infty} \frac{R_1}{M \log\log nN} = 1. \quad (8)$$

Proof: According to Lemma 1, the sum-rate is bounded by (5), which can be further lower bounded by substituting $\tilde{\mathcal{S}}$ by its minimum value α, i.e., exploiting the inequality $\mathbb{E}\left[\log\left(1+\tilde{\mathcal{S}}\right)|\tilde{\mathcal{S}}>\alpha\right] \geq \log(1+\alpha)$:

$$R_1 \geq M\left(1-(F(\alpha))^{\frac{nN}{M}}\right)\log(1+\alpha) = R_{lb}. \quad (9)$$

If we choose the threshold α as follows

$$\alpha = \rho\log nN - \rho M \log\log nN, \quad (10)$$

then it is easy to verify that

$$\lim_{n\to\infty}\left(1-\frac{e^{-\alpha/\rho}}{(1+\alpha)^{M-1}}\right)^{\frac{nN}{M}} = \lim_{n\to\infty}\left(1-\frac{\log n}{n\rho^{(M-1)}}\right)^{\frac{nN}{M}} = 0. \quad (11)$$

As a result, the pre-log term of R_{lb} tends to M when n goes to infinity. Therefore, the asymptotic behavior (with respect to n) of the lower bound in (9) coincides with the upper bound set by the performance of DPC with full CSI:

$$\lim_{n\to\infty}\frac{R_{lb}}{M\log\log nN} = \lim_{n\to\infty}\frac{R_1}{M\log\log nN} = 1, \quad (12)$$

thus concluding the proof. ∎

The amount of feedback that guarantees the optimal scaling law of the sum-rate can be reduced even further applying the selective feedback principle [14] [15]. Selective feedback prescribes that each user feeds back information to the BS only when the measured SINR is above the threshold. The resulting average feedback rate for this opportunistic scheme, derived in [16], is later used for the multiplexing gain analysis in section V.

Next section shows that, even though the first bit of feedback captures most of the multiuser diversity, additional bits of feedback are very valuable because they yield significant gains in sum-rate for the regime of "not so large" number of users. In the following sections, we focus on the case $N = 1$ with the understanding that extension to the MIMO scenario follows the guidelines explained in this section. Hence, the SINR of the scheduled user $\tilde{\mathcal{S}}_{i,r} = \max_j \mathcal{S}_{i,r,j}$, is simply referred as $\mathcal{S}$.

IV. Impact of increasing feedback bits on sum-rate

As discussed in the previous sections, one bit of feedback is enough to guarantee the optimal scaling law of the sum-rate for large n. Here we want to quantify the impact of increasing the feedback to $b > 1$ bits. A way to increase the number of bits is to modify the transmission scheme described in Sec. II-A by allowing each user to measure the SINR on more than one pre-determined beam. In particular, if $b = \log_2(K+1)$ bits are granted for any user in the uplink channel, the SINRs on K beams can be measured by each user. Then, the index of the beam with the maximum SINR, provided that it crosses the threshold, is fed back to the BS (the all-zero string is sent if no beam crosses the threshold). Scheduling on each beam is carried out at the BS by selecting randomly a user among the ones that signaled the corresponding index.

As shown in Appendix-C, if $\alpha > 1$, the result in Lemma 1 can be generalized to this scenario, leading to the following lower bound on the rate of the proposed scheme with b bits of feedback ($b = \log_2(K+1)$):

$$R_K \geq M\left(1-(F(\alpha))^{\frac{nK}{M}}\right)\mathbb{E}[\log(1+\mathcal{S})|\mathcal{S}>\alpha] \quad \alpha > 1. \quad (13)$$

Since R_K with $K > 1$ is lower bounded by the sum-rate with one-bit feedback R_1 (5), the optimal scaling law $M\log\log n$ clearly holds also for R_K. In [16], this result was proved for the case of $K = M$, showing that the constraint $\alpha > 1$ in the sum rate expression (13) does not constitute an impediment for the proof.

The impact of increasing number of feedback bits is depicted in fig. 1, where the lower bound on the sum-rate (13) is evaluated versus the number of users n for a threshold α obtained through numerical maximization of (13). Moreover, as a performance reference, the sum-rate of the scheme proposed in [8] is plotted as well. It is seen that the first bit of feedback captures most of the multiuser diversity because the growth of the sum-rate with one-bit feedback resembles that of the scheme proposed in [8]. However, a significant gap in sum-rate exists between the two schemes. This gap is partially bridged by the second and the third bit of feedback, with decreasing gain for each extra bit.

V. Multiplexing Gain

Subject of the previous sections has been the asymptotic behavior of the sum-rate of limited feedback transmission schemes for increasing number of users n and fixed SNR. Here, we focus on the asymptotic performance of such schemes with respect to the SNR (i.e., on the multiplexing gain). It is known that, for fixed M and n, opportunistic transmission techniques are interference-limited, that is the rate R satisfies $R/\log_2(\rho) \to 0$ for $\rho \to \infty$ [5]. Notice that, fixing M and n, the amount of feedback per cell for the opportunistic schemes proposed in both [8] and in this paper is independent of the SNR.

It has been recently shown in [9], that in order to avoid interference-limited behavior of a MIMO broadcast channel with limited feedback, the feedback load must approximately scale linearly with M and the SNR (in dB). Therein, the argument is based on the performance of a zero-forcing beamforming precoder. Each user feeds back the index of the best channel quantization point, selected from a set of 2^b vectors of a random vector quantizer, where b is the number of bits of feedback per user. Clearly, increasing the feedback load improves the quality of the channel estimate available at the transmitter and the ability of zero-forcing beamforming to invert the channel.

In this section we show that a similar feedback rate can prevent interference-limited behavior of the considered opportunistic scheme with binary feedback as well. However, since in our scheme the feedback load per user is fixed, here the feedback rate is increased by increasing the number of users n with the SNR. In the following theorem we find a sufficient condition for the growth rate of n with SNR that allows to guarantee the optimal multiplexing gain.

Theorem 2: Consider the multi-beam transmission scheme described in Sec. IV, where M is fixed and $K = M$. If the number of users is $n = \rho^M$, then

$$\lim_{\rho\to\infty} \frac{R_M}{\log(\rho)} = M. \tag{14}$$

This implies that the optimal multiplexing gain of M is guaranteed and that the system is not interference limited.

Proof: The sum-rate is lower bounded for $\alpha > 1$ by (13), which is further lower bounded substituting $\mathcal{S}$ by its lowest possible value α . Now, choosing $\alpha = \rho\log\rho - M\rho\log\log\rho$, which implies $\alpha > 1$ for ρ large enough, we have

$$\lim_{\rho\to\infty} \frac{\log(1+\alpha)}{\log(\rho)} = \lim_{\rho\to\infty} 1 + \frac{\log\log\left(\frac{\rho}{(\log\rho)^M}\right)}{\log\rho} = 1 \tag{15}$$

and for the pre-log term of (13)

$$\begin{aligned} \lim_{\rho\to\infty}\left(1-\frac{e^{-\alpha/\rho}}{(1+\alpha)^{M-1}}\right)^{\frac{nK}{M}} &= \\ \lim_{\rho\to\infty}\left(1-\frac{(\log\rho)^M}{\rho}\frac{1}{\rho^{M-1}\left(\log\left(\frac{\rho}{(\log\rho)^M}\right)\right)^{M-1}}\right)^{\frac{K\rho^M}{M}} &= \\ \lim_{\rho\to\infty}\left(1-\frac{\log\rho}{\rho^M}\right)^{\frac{K\rho^M}{M}} &= 0, \end{aligned} \tag{16}$$

thus concluding the proof. ■

Corollary 1: The multi-beam scheme with selective feedback [16] preserves the optimal multiplexing gain (14) with an average number of feedback bits per cell that satisfies

$$\lim_{\rho\to\infty} \frac{\bar{b}_n}{\log(\rho)} = M\log_2(M). \tag{17}$$

Proof: According to [16], the average number of feedback bits per cell of the selective feedback scheme reads

$$\bar{b}_n = nM(1-F(\alpha))\log_2 M \tag{18}$$

From Theorem 2, a number of users $n = \rho^M$ guarantees the optimal scaling law. Moreover, from equation (16) we have that $\lim_{\rho\to\infty}(1 - F(\alpha)) = (\log\rho)/\rho^M$. Substituting these results into (18) concludes the proof. ■

The total feedback required in (17) is proportional to $M\log_2 M$ and the logarithm of the SNR. This result is similar to the one proved in [9] in the context of zero-forcing beamforming. In fig. 2, the sum-rate of the selective feedback scheme with $K = M$ is plotted versus the SNR for number of antennas $M = \{2, 3\}$ and users $n = \{100, 1000\}$. It can be seen that for the same number of users, deploying more transmit antennas is advantageous at low SNR but causes a more pronounced performance floor due to interference at high SNR.

VI. Conclusion

In this paper, we tackled the problem of quantifying the amount of feedback that guarantees the fulfillment of the asymptotic criteria usually associated with the concept of multiuser diversity, namely scaling law of the sum-rate with respect to the number of users and multiplexing gain. It has been proved that a feedback as small as one bit per user is enough to obtain the optimal scaling law. Moreover, in accordance with what has been recently reported in the context of zero-forcing beamforming [9], the optimal multiplexing gain of orthogonal opportunistic beamforming requires a number of feedback bits that scales with the SNR (in dB).

VII. Appendix

A. Proof of Lemma 1

The achievable sum-rate of the multi-beam scheme with one bit feedback described in Sec. II is

$$R_1 \geq \sum_{m=1}^{M}\sum_{i=1}^{n} R_{i,m} \tag{19}$$

where $R_{i,m}$ is the rate to user i through beam m that is achievable by coding only during the scheduling intervals where the ith user has actually requested the beam (i.e., when $\tilde{\mathcal{S}}_{i.m} > \alpha$). Expression (19) is only a lower bound on the actual achievable rate since it neglects the time instants where user i is scheduled on a beam that no user has requested (recall the description of the scheduling process in Sec. II-A). Notice that the probability that no user measures a strong enough SINR on a given beam (for an appropriately selected threshold α) is expected to be small for a large number of users n.

The rate $R_{i,m}$ reads

$$R_{i,m} = \Phi_{m,i}\mathbb{E}[\log(1+\tilde{\mathcal{S}}_{i,m})|\tilde{\mathcal{S}}_{i,m} > \alpha], \tag{20}$$

where $\Phi_{m,i}$ is the percentage of time in which the BS schedules user i to beam m (provided that the user has requested the beam). The SINR $\mathcal{S}_{i,m}$ is identically distributed for all users and beams, henceforth the indexes are dropped. Then, (19) becomes

$$R_1 \geq \mathbb{E}[\log(1+\tilde{\mathcal{S}})|\tilde{\mathcal{S}} > \alpha]\sum_{m=1}^{M}\sum_{i=1}^{n}\Phi_{m,i}, \tag{21}$$

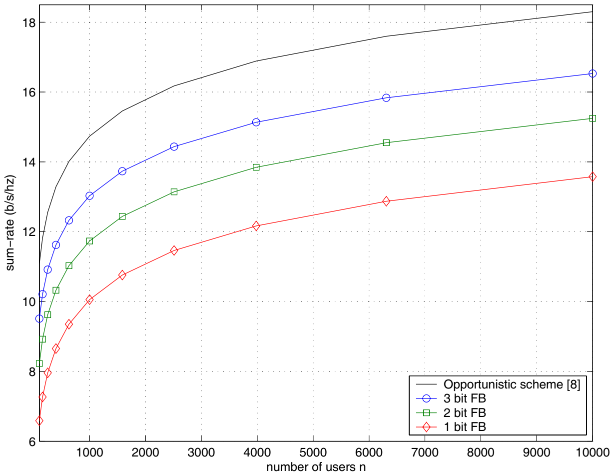

Fig. 1. Sum-rate with $b = 1, 2, 3$ bits of feedback per user and sum-rate of the opportunistic scheme in [8] ($M = 7$, $N = 5$, $\rho = 10dB$).

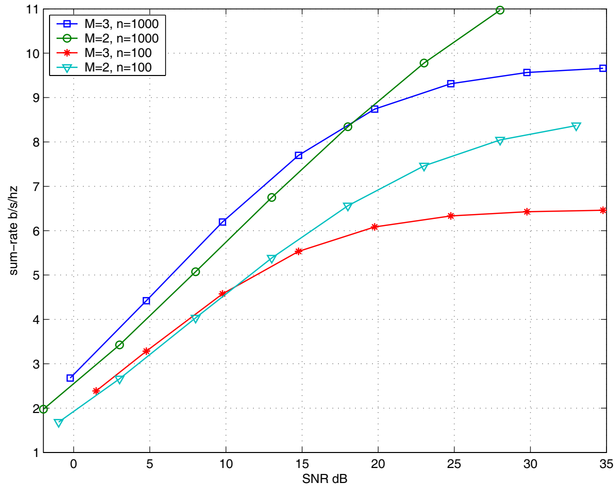

Fig. 2. Sum-rate of the selective scheme with $K = M$ versus SNR for $M = 2, 3$ and $n = 100, 1000$.

where the inner sum term is clearly the probability that a given beam m is requested (by at least one user):

$$\sum_{i=1}^{n} \Phi_{m,i} = P_m. \tag{22}$$

By symmetry $P_m = P$, thus substituting (22) in (21) leads to the sum-rate expression in (5). Moreover, the probability that any given beam is requested reads

$$P = \left(1 - (P_1)^L\right), \tag{23}$$

where L is the number of users measuring the SINR on the beam, and P_1 is the probability that a user does not request transmission after measuring the beam (i.e., the probability of the event $\tilde{\mathcal{S}} < \alpha$): $P_1 = F(\alpha)^N$. Assuming that the number of users measuring each beam is the same, we have $L = n/M$ and (6), thus completing the proof.

B. Proof of (13)

According to the scheme described in Sec. IV, the number of users measuring a certain beam is nK/M. Therefore, the probability P of a beam being used follows (23), whereas P_1 is given by

$$P_1 = \mathbb{P}\{\mathcal{S} < \alpha\} + \mathbb{P}\{(\mathcal{S} > \alpha) \cap (\mathcal{S} \text{ is not the maximum})\}. \tag{24}$$

For $\alpha > 1$, $\mathbb{P}\{(\mathcal{S} > \alpha) \cap (\mathcal{S} \text{ is not the maximum})\} = 0$, because the SINR can not be greater than one in more than one beam [8]. Therefore, $P_1 = F(\alpha)$ and

$$P = 1 - (F(\alpha))^{\frac{nK}{M}} \qquad \alpha > 1. \tag{25}$$

Therefore, following the same reasoning as in the previous sections, we finally get (13).

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