Relaying Under Structured Interference

Kağan Bakanoğlu† Elza Erkip† Osvaldo Simeone‡ Shlomo Shamai (Shitz)⋄
†Dept. of ECE, Polytechnic Institute of NYU, kaganbakanoglu@gmail.com, elza@poly.edu
‡Center for Wireless Communications and Signal Processing Research, NJIT, New Jersey, osvaldo.simeone@njit.edu. Dept. of EE, Technion Institute of Technology, Haifa, Israel, email: sshlomo@ee.technion.ac.il.

***Abstract*—An orthogonal components relay channel subject to structured interference is studied. Modeling interference as a structured signal is accurate for scenarios in which the interferer is another source communicating with its own destination. It is assumed that interference is known non-causally at the source and the relay is used for both forwarding the source message and for cooperative interference mitigation. Several transmission strategies based on partial decode-and-forward relaying and leveraging the interference structure are proposed. Achievable rates are derived for discrete memoryless models and Gaussian channels. Finally, numerical results suggest several advantages of utilizing the interference structure at the source, relay or destination in the scenarios at hand.**

I. Introduction

In wireless networks, interference affects terminals participating in the same communication session in different ways. This gives rise to a number of key design challenges, which have been well studied studied in the context of medium access control protocols. An example is the so called exposed terminal problem, in which a node is incorrectly prevented from transmitting to its receiver when it overhears the transmission of another node that, however, does not affect the receiver.

A recent line of work has started to address the issue of locality of interference from an information-theoretic standpoint. Wireless nodes in the vicinity of the interferer can obtain *information about the interference* in a number of relevant scenarios. As an example, assume that the interferer employs retransmissions such as hybrid-ARQ on its link. A node in the vicinity may be able to decode a prior retransmission and use this information in order to facilitate interference mitigation. Another scenario where interference information is conventionally assumed is cognitive radio. A critical aspect of interference, from a physical layer standpoint, is that the interfering signal is not a purely random noisy waveform, but instead it has the *structure* provided by the specific codebook as it typically arises from the transmissions of other wireless users. These critical features of interfering signals can be leveraged to make the task of interference management more effective.

In information-theoretic terms, an i.i.d. interference can be modelled as the "state" of a channel. The capacity of a state-dependent memoryless channel, where the state sequence (i.e., the interference) is available non-causally at the transmitter, is established by Gel'fand Pinsker in [2]. Costa [3] applied Gel'fand and Pinsker's (GP) result to the Additive White Gaussian Noise (AWGN) model with additive Gaussian state, giving rise to the so called Dirty Paper Coding (DPC) technique. DPC achieves the state-free capacity even though the state is not known at the receiver. It was shown in [4], [5], that this principle continues to hold even if the state is not Gaussian.

Extensions to the multiuser case were performed by Gel'fand and Pinsker in [6] and by Kim *et al.* in [7][8]. In particular, in [7][8] it is proved that for multiple access channels (MACs) multi-user versions of GP and DPC, referred to as multi-user GP (MU-GP) and multi user DPC (MU-DPC) respectively, are necessary to achieve optimal performance. In [9], Somekh-Baruch *et al.* considered a memoryless two-user MAC, with the state available to one of the encoders but not to the other encoder or the receiver. The capacity region is characterized and shown to be obtained by schemes called generalized GP (GGP) and generalized DPC (GDPC), which perform both interference precoding and interference cancelation. The scenario studied in this paper, but with an i.i.d. state sequence is investigated in [1], [10] for a Discrete Memoryless (DM) and Gaussian relay channels with an in-band relay and [11] for a DM and Gaussian relay channel with an out-of-band relay where lower and upper bounds on the capacity are derived.

Utilizing the interference structure with a single dominating interferer is explored in [13] and a scenario in which a transmitter-receiver pair communicates in the presence of a single interferer is studied. It is shown therein that using GP coding, and hence treating the interference as if it were unstructured, it is generally suboptimal and *interference forwarding* with joint decoding at the destination can be beneficial [14]. This aspect is further studied in [15] for a MAC with structured interference available at one encoder, in [12] for a Gaussian relay channel with an out-of-band relay and in [16] for a cognitive Z-interference channel, where extensions of the techniques proposed in [13] are investigated.

In this paper, we investigate interference mitigation strategies for a cooperative communication scenario in which a source communicates via an out of band relay to a destination in the presence of an external interferer. The interferer is not willing, or not allowed, to change its transmission strategy to reduce interference on the destination. The source is able to obtain information about the interferer signal prior to transmission in the current block. We are interested in studying effective ways to use such interference information at the source. In particular, we explore the ones that leverage the structure of the interference.

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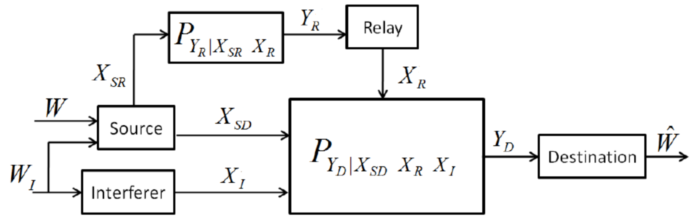

Fig. 1. Relay channel with orthogonal components under structured interference known at the source.

The interference structure can be exploited in a number of ways by the source. For instance, the structure of the interference signal potentially allows the source to reduce the amount of spectral resources necessary for communicating interference information to the relay. A second way to take advantage of the interference structure is for the source, possibly with the help of the relay, to help reception of the interfering signal at the destination so that the destination can decode and remove the interference. In this work, we will explore these possibilities and assess the advantages of strategies that exploit the interferer's structure with respect to the techniques studied in [1] that assume an unstructured interferer.

We adopt the relay channel with orthogonal components model [17] due to its ability to model half-duplex communications and availability of capacity achieving strategies [17]. We propose several techniques for discrete memoryless and AWGN channels that leverage interference structure to different degrees. Finally, numerical results bring insight into the advantages of interference mitigation techniques that exploit the interference structure.

The remainder of this paper is organized as follows. In Section II the system model is introduced. Achievable rates and proposed strategies are given in Section III. Numerical results and conclusions follow in Section IV and V.

II. System Model

We study a relay channel with an orthogonal source-to-relay link in the presence of an interferer. The source sends two different signals, one to the relay and one to the destination without mutual interference. The interference signal is available non-causally to the source as depicted in Fig. 1. We first consider a Discrete Memoryless Channel (DMC) version of the channel, which is described by the conditional probability mass functions (pmfs) $P_{Y_D|X_{SD}X_RX_I}$ and $P_{Y_R|X_{SR}X_R}$ where $Y_D \in \mathcal{Y}_D$, $Y_R \in \mathcal{Y}_R$, $(X_{SD}, X_{SR}) \in \mathcal{X}_{SD} \times \mathcal{X}_{SR}$, $X_R \in \mathcal{X}_R$ and $X_I \in \mathcal{X}_I$ are the destination (D) output, the relay (R) output, the source (S) input, the relay (R) input and the interference (I) signal, respectively. The source wishes to transmit a message W to the destination with the help of the relay in n channel uses. The message W is uniformly distributed over the set $\mathcal{W} = \{1, \ldots, 2^{nR}\}$, where R is the rate in bit/channel use. The interferer employs a fixed (and given) codebook that is not subject to design. In particular, the codebook of the interferer is assumed to be chosen by the interfering terminal independently to communicate with some other destination which is not modeled explicitly. The message W_I of the interferer is assumed to be uniformly distributed over the set $\mathcal{W}_{\mathcal{I}} = \{1, \cdots, 2^{nR_I}\}$, where R_I is the interferer's rate in bits/channel use. We assume that the interferer's codebook is generated according to a pmf P_{X_I}. The generated codebook of the interferer is known to all nodes. Furthermore, the interferer's message W_I is known to the source. In the paper we use the standard definitions of achievable rates and probability of error [18].

We also consider the AWGN scenario where the input and output relations at time instant i are given as

$$Y_{R,i} = h_{SR,i}X_{SR,i} + Z_{R,i} \tag{1a}$$
$$Y_{D,i} = h_{SD,i}X_{SD,i} + h_{RD,i}X_{R,i} + h_{I,i}X_{I,i} + Z_{D,i} \tag{1b}$$

where the noises $Z_{D,i}$ and $Z_{R,i}$ are independent zero mean complex Gaussian random variables with unit variance, and $h_{SR,i}$, $h_{SD,i}$, $h_{RD,i}$ and $h_{I,i}$ are the complex valued channel gains accounting for propagation from the source to the relay ($h_{SR,i}$), from the source to the destination ($h_{SD,i}$), from the relay to the destination ($h_{RD,i}$), and from the interferer to the destination ($h_{I,i}$), respectively. We assume that all channel gains remain constant over the entire coding block and channel gains are known to all nodes.

The codewords of the source X^n_{SR} and X^n_{SD} are subject to a total power constraint P_S and the codewords of the relay X^n_R is subject to power constraint P_R as

$$\frac{1}{n}\sum_{i=1}^{n}\left(\mathbb{E}\left[|X_{SR,i}|^2\right] + \mathbb{E}\left[|X_{SD,i}|^2\right]\right) \leq P_S \tag{2a}$$
$$\frac{1}{n}\sum_{i=1}^{n}\mathbb{E}\left[|X_{R,i}|^2\right] \leq P_R. \tag{2b}$$

We assume that the interferer codebook is generated i.i.d. with complex Gaussian distribution with zero mean and power P_I. We use the notation $\mathcal{C}(x) = \log_2(1+x)$.

III. Achievable Rates

El Gamal and Zaidi prove the optimality of partial decode-and-forward (PDF) for the relay channel with orthogonal components in Fig. 1 without interference in [17]. Motivated by this, we assume that the relaying strategy for the source message is based on PDF. Specifically, the source message W is split into two independent messages, $W = (W', W'')$, where W' is sent through the relay and W'' is sent directly to the destination. The messages W' and W'' are uniformly distributed over the set $\mathcal{W}' = \{1, \cdots, 2^{nR'}\}$ and $\mathcal{W}'' = \{1, \cdots, 2^{nR''}\}$, respectively, and the total rate is $R = R' + R''$.

We recall that the structure of the interference plays an important role in our problem. When the structure of the interference is ignored, it can be treated as i.i.d. state. For an i.i.d. interference signal X^n_I, the scenario at hand is studied in [1]. In this paper, we explore the techniques that exploit the interference structure, as modeled in the previous section. The advantages of leveraging interference structure will be discussed in Sec. IV via numerical results through comparison with the techniques proposed in [1].

Interference management is employed either by the source only or by both the source and the relay in a cooperative fashion. In order to perform cooperative interference mitigation, the source needs to share the interference information with the relay. The structure of the interference plays an important role for the two phases of informing the relay of the interference and of interference mitigation towards the destination. We categorize the possible strategies in both phases as follows.

Communication of interference to the relay: When the source chooses to inform the relay about the interfering signal, it has two options:

1) *Digital interference sharing*: The structure of the interference is exploited as follows. The source encodes the interference index W_I into a codebook (not necessarily the same as the interferer's codebook) and sends it to the relay through the orthogonal source-relay ($S-R$) channel. The relay then decodes the interference index W_I.
2) *Compressed interference sharing*: The structure of the interference is not used and the interference is treated as an i.i.d. sequence. Specifically, the source simply quantizes the interference sequence X_I^n and forwards the compressed description to the relay through the orthogonal source-relay channel. The relay hence recovers the interference sequence with some quantization distortion.

Interference mitigation at the destination: There are several interference mitigation scenarios applicable to our model depending on the availability of interference information at the relay. We mainly concentrate on two approaches:

1) *Structured approach*: The structure of the interference is exploited at the destination to decode and remove the interference signal. Decoding can be facilitated by having the source and/or the relay forward information about the interference to the destination. When the source does not inform the relay about the interfering signal, interference forwarding is performed by the source only. Otherwise, interference forwarding is done jointly by the source and the relay. In the AWGN channel, interference forwarding is performed by the source and/or relay by transmitting signals that are coherent with the interferer's signal, so that the correlation between transmitted signal and interference is positive;
2) *Unstructured approach*: The structure of the interference is ignored at the destination and the interference is treated as an i.i.d. state. Interference precoding via GP, MU-GP or GGP for the DMC model, and DPC, MU-DPC and GDPC for the AWGN model, are utilized by the source only or by the source and the relay jointly depending on the availability of interference information at the relay. This class of techniques was extensively explored in [1] and will be considered here only in combination with the digital approach mentioned above (not applicable in the unstructured model of [1]), and for reference.

A. No Relay

For comparison, here we provide an overview of the performance achievable when the relay is not present. The channel without the relay becomes a point to point communication channel with structured interference non-causally available at the source. An achievable rate for the DMC model is given by [13]

$$R_{NR} = \max\max\left\{\begin{array}{l} I(U;Y_D)-I(U;X_I), \\ \min\left\{\begin{array}{l} I(X_{SD};Y_D|X_I), \\ (I(X_{SD}X_I;Y_D)-R_I)^+ \end{array}\right. \end{array}\right. , \tag{3}$$

where the first maximum is over all joint input pmfs of the form $P_{UX_{SD}|X_I}$, where U is an auxiliary random variable. The achievability relies on GP coding (unstructured approach), which achieves the first term in the second max of (3), and interference forwarding (structured approach), which achieves the second term.

For the AWGN model, the unstructured approach consists of DPC and is known achieve the interference-free capacity [3]

$$R_{NR} = \mathcal{C}(|h_{SD}|^2 P_S). \tag{4}$$

Remark 3.1: For Gaussian channels, the source is able to remove the effect of interference completely by DPC [3] without the help of the relay. Thus, utilizing the relay for interference mitigation only is not beneficial and performance gains can only be attained if relay forwards information about the source message as well. Note that this conclusion does not hold for the general DMC model.

B. Cooperative Interference Mitigation

Below we consider scenarios in which the source sends the interference information and a part of the source message to the relay. This allows the source and the relay to manage interference jointly. We study two ways of communicating the interference information to the relay in conjunction with the interference mitigation approaches described above. Using the relay only for signal cooperation is a special case of the proposed schemes.

1) Scheme (D,U) (Digital interference sharing, Unstructured approach): In this scheme, the source sends the interference digitally to the relay, so that the relay is fully informed about the interference sequence. The source also forwards part of the source message to the relay according to PDF. Then, the source and the relay follow the unstructured approach by jointly employing MU-GP [8] to forward the source message.

Proposition 3.1: For Scheme (D,U), the following rate is achievable for the DM model:

$$R_{(D,U)} = \max\min\left\{\begin{array}{l} (I(X_{SR};Y_R|X_R)-R_I)^+ \\ +I(U_S;Y_D|U_R)-I(U_S;X_I|U_R), \\ I(U_SU_R;Y_D)-I(U_SU_R;X_I) \end{array}\right. \tag{5}$$

where the maximization is taken over the input pmfs $P_{U_SU_RX_RX_{SR}X_{SD}|X_I}$ of the form $P_{X_{SR}|X_RX_I}P_{U_SU_RX_RX_{SD}|X_I}$, where U_S, U_R are finite-alphabet auxiliary random variables.

Sketch of the proof: The message W is split into two messages W' and W''. The source conveys the message W' to the relay together with interference index W_I which leads to the constraint $R' \le I(X_{SR};Y_R|X_R)-R_I$. Since both the source and the relay have the interference knowledge, they are able to implement MU-GP [8] to send W' and W'' to the destination. Note that unlike [8], here the two encoders (source and relay) have the common message W', so that the channel from the source and the relay to the destination is equivalent to the state (interference) dependent MAC with common message and informed encoders. An achievable rate region can be derived by following similar steps in [8][9], obtaining the achievable rate region

$$R'' \leq I(U_S;Y_D|U_R) - I(U_S;X_I|U_R) \tag{6a}$$

$$R' + R'' \leq I(U_SU_R;Y_D) - I(U_SU_R;X_I) \tag{6b}$$

for some distribution $P_{U_SU_RX_RX_{SD}|X_I}$. Incorporating (6) with the constraint on R' above concludes the proof. □

Proposition 3.2: For Scheme (D,U), the following rate is achievable for the AWGN model (1):

$$R_{(D,U)} = \max_{\substack{\rho_{W'},\rho_{W''},\gamma: \\ |\rho_{W'}|,|\rho_{W''}|, \\ \gamma\in[0,1]}} \min \begin{cases} (\mathcal{C}\left(|h_{SR}|^2(1-\gamma)P_S\right) - R_I)^+ \\ +\mathcal{C}\left(P_{W''}\right), \\ \mathcal{C}\left(P_{W''}+P_{W'}\right) \end{cases}$$

$$\text{subject to } |\rho_{W'}|^2 + |\rho_{W''}|^2 \leq 1 \tag{7}$$

where $P_{W'} = (|h_{RD}|\sqrt{P_R} + |h_{SD}||\rho_{W'}|\sqrt{\gamma P_S})^2$ and $P_{W''} = |h_{SD}|^2|\rho_{W''}|^2\gamma P_S$.

Sketch of the proof: The result is obtained from (5), where all inputs are chosen according to Gaussian distribution. Specifically, the source input X_{SD} is allocated power γP_S, $0 \leq \gamma \leq 1$, and the remaining power $(1-\gamma)P_S$ is allocated to the source input X_{SR}. We set the inputs $X_{SD} = \rho_{W'}\sqrt{\gamma P_S}U_{W'} + \rho_{W''}\sqrt{\gamma P_S}U_{W''}$ and $X_R = \sqrt{P_R}U_{W'}$ where $U_{W'}$ and $U_{W''}$ are independent, zero mean, unit variance, complex Gaussian random variables and carrying the messages W' and W'', respectively. Furthermore, $U_{W'}$ and $U_{W''}$ are independent of X_I. The source conveys W' to the relay where $R' \leq (\mathcal{C}\left(|h_{SR}|^2(1-\gamma)P_S\right) - R_I)^+$. MU-DPC is used for transmission to the destination, where the auxiliary random variables U_S and U_R in (5) are chosen to be linear combinations of (X_{SD},X_I) and (X_R,X_I) as $U_S = X_{SD} + \alpha_S X_I$ and $U_R = X_R + \alpha_R X_I$ with inflation factors α_S and α_R and (X_{SD},X_R) jointly complex Gaussian and independent of X_I. When the inflation factors are optimized the effect of the interference is completely eliminated at the destination similar to [8], leading to (7). □

Remark 3.2: Once can also consider a scheme (D,S) in which the interference is digitally transmitted to the relay and the structured approach for decoding at the destination is used. Scheme (D,S) may lead to performance improvements over Scheme (D,U) for a DMC. However, for AWGN channels, Scheme (D,S) is inferior to Scheme (D,U). The reason is that in both Scheme (D,U) and Scheme (D,S) the relay obtains the interfering signal, but in the AWGN model, Scheme (D,U) is able to completely remove the effect of interference at the destination via MU-DPC.

2) Scheme (C,U) (Compressed interference sharing, Unstructured approach): With this scheme, studied in [1] [11] for the general relay channel and relay channel with orthogonal components respectively, the source sends the compressed interference signal and the part of the message to the relay and the unstructured approach is utilized for decoding at the destination. Achievable rate for Scheme (C,U) for our DM model can be obtained from [1, Corollary 1]. It can be extended to Gaussian case by using an approach similar to [1, Theorem 6] and taking the complex channel gains into account. The achievable rate for (C,U) for the AWGN model (1) can be written as

$$R_{(C,U)} = \max_{\substack{r_q,\rho_{W'},\rho_{W''}, \\ \rho_{W_I},\gamma:|\rho_{W'}|, \\ |\rho_{W''}|,|\rho_{W_I}|, \\ \gamma\in[0,1]}} \min \begin{cases} (\mathcal{C}\left(|h_{SR}|^2(1-\gamma)P_S\right) - r_q)^+ + \mathcal{C}\left(P_{W''}\right), \\ \mathcal{C}\left(P_{W'}\right) + \mathcal{C}\left(P_{W''}\right) \end{cases}$$

$$\text{subject to } 0 \leq r_q \leq \mathcal{C}\left(|h_{SR}|^2(1-\gamma)P_S\right)$$

$$|\rho_{W'}|^2 + |\rho_{W''}|^2 + |\rho_{W_I}|^2 \leq 1,$$

where $P_{W'} = (|h_{RD}|\sqrt{P_R} + |h_{SD}||\rho_{W'}|\sqrt{\gamma P_S})^2/(1+\xi^2 D + P_{W''})$, $P_{W''} = |h_{SD}|^2|\rho_{W''}|^2\gamma P_S$, $D = P_I 2^{-r_q}$ and $\xi = |h_I| - |h_{SD}||\rho_{W_I}|\sqrt{\gamma P_S/P_I}$.

3) Scheme (C,S) (Compressed interference sharing, Structured approach): In this scheme, the source informs the relay using compressed interference information, and the structured approach is used to mitigate interference at the destination. The compressed interference information is re-encoded by source and relay and decoded at the destination in a similar way as for standard compress-and-forward protocols for the relay channel (See, e.g., [19]).

Proposition 3.3: For Scheme (C,S), the following rate is achievable for the DM model:

$$R_{(C,S)} = \max\min \begin{cases} I(X_{SD};Y_D\hat{X}_I|X_RX_IU) \\ +(I(X_{SR};Y_R|X_R) - I(X_I;\hat{X}_I|UY_D))^+, \\ (I(X_{SD}X_I;Y_D\hat{X}_I|X_RU) - R_I)^+ \\ +(I(X_{SR};Y_R|X_R) - I(X_I;\hat{X}_I|UY_D))^+, \\ I(X_{SD}X_R;Y_D\hat{X}_I|X_IU), \\ (I(X_{SD}X_RX_I;Y_D\hat{X}_I|U) - R_I)^+ \end{cases}, \tag{8}$$

where the maximum is over all input pmfs $P_{U\hat{X}_IX_RX_{SR}X_{SD}|X_I}$ of the form $P_{\hat{X}_I|X_I}P_{X_{SR}|X_R\hat{X}_I}P_UP_{X_R|U}\ P_{X_{SD}|UX_RX_I}$ such that the inequality $I(U;Y_D) \geq I(X_I;\hat{X}_I|UY_D)$ holds.

Sketch of the proof: The source quantizes the interference signal X_I^n into a reconstruction sequence $\hat{X}_I^n$ by using a test channel $P_{\hat{X}_I|X_I}$. Moreover, random binning is performed according to the Wyner-Ziv strategy (See, e.g., [19]), reducing the rate of the compression codebook to $I(X_I;\hat{X}_I|UY_D)$. The source sends the index of the quantized interference and message W' to the relay. The relay recovers the compression index (but not $\hat{X}_I^n$) and W' successfully if the following constraint is satisfied

$$R' + I(X_I;\hat{X}_I|UY_D) \leq I(X_{SR};Y_R|X_R). \tag{9}$$

The relay then maps the index of the quantized interference received from the source into a codeword U^n from an independent codebook and forwards it along with the codeword that encodes message W' to the destination. The destination first decodes the codeword U^n, which is guaranteed if the following condition is satisfied

$$I(U;Y_D) \geq I(X_I;\hat{X}_I|UY_D). \tag{10}$$

From the compression index, the destination can now recover $\hat{X}_I^n$ via Wyner-Ziv decoding, since it has the side information Y_D^n and U^n. The decoded sequence $\hat{X}_I^n$ is then used to facilitate decoding at the destination. The resulting channel to the destination is thus a MAC with common messages in which the source and the relay have the message sets (W',W'',W_I)

and W', respectively. The destination has the knowledge of both $\hat{X}_I^n$ and U^n, which is used to jointly decode the messages set (W',W'',W_I). An achievable rate region is obtained as

$$R'' \leq I(X_{SD}; Y_D \hat{X}_I | X_R X_I U) \quad \text{(11a)}$$
$$R'' + R_I \leq I(X_{SD} X_I; Y_D \hat{X}_I | X_R U) \quad \text{(11b)}$$
$$R'' + R' \leq I(X_{SD} X_R; Y_D \hat{X}_I | X_I U) \quad \text{(11c)}$$
$$R'' + R' + R_I \leq I(X_{SD} X_R X_I; Y_D \hat{X}_I | U), \quad \text{(11d)}$$

for some input pmf $P_{U X_R X_{SD} | X_I} = P_U P_{X_R|U} P_{X_{SD}|U X_R X_I}$. Incorporating (9) and (10) into the obtained rate region gives us (8). □

Proposition 3.4: For Scheme (C,S), the following rate is achievable for the AWGN model (1):

$$R_{(C,S)} = \max_{\substack{r_q, \rho_{W'}, \rho_{W''}, \\ \rho_{W_I}, \rho_U, \bar{\rho}_{W'}, \\ \bar{\rho}_U, \gamma}} \min \begin{cases} \mathcal{C}\left(P_{W''}\right) + (\mathcal{C}\left(|h_{SR}|^2 (1-\gamma) P_S\right) - r_q)^+, \\ (\log_2\left((1+P_{W''})\frac{P_I}{D} + P_{W_I}\right) - R_I)^+ \\ +(\mathcal{C}\left(|h_{SR}|^2(1-\gamma)P_S\right) - r_q)^+, \\ \mathcal{C}\left(P_{W''} + P_{W'}\right), \\ (\log_2\left((1+P_{W''}+P_{W'})\frac{P_I}{D} + P_{W_I}\right) - R_I)^+ \end{cases}$$

$$\text{subject to } 0 \leq r_q \leq \min \begin{cases} \mathcal{C}\left(|h_{SR}|^2 (1-\gamma) P_S\right), \\ \mathcal{C}\left(\frac{P_U}{P_{W'}+P_{W''}+P_{W_I}+1}\right) \end{cases}$$

$$|\rho_{W'}|^2 + |\rho_{W''}|^2 + |\rho_{W_I}|^2 + |\rho_U|^2 \leq 1$$
$$|\bar{\rho}_{W'}|^2 + |\bar{\rho}_U|^2 \leq 1$$
$$|\rho_{W'}|, |\rho_{W''}|, |\rho_{W_I}|, |\rho_U|, |\bar{\rho}_{W'}|, |\bar{\rho}_U|, \gamma \in [0,1]$$

where $P_{W'} = (|h_{RD}||\bar{\rho}_{W'}|\sqrt{P_R} + |h_{SD}||\rho_{W'}|\sqrt{\gamma P_S})^2$, $P_{W''} = |h_{SD}|^2 |\rho_{W''}|^2 \gamma P_S$, $P_{W_I} = (|h_{SD}||\rho_{W_I}|\sqrt{\gamma P_S} + |h_I|\sqrt{P_I})^2$, $P_U = (|h_{SD}||\rho_U|\sqrt{\gamma P_S} + |h_{RD}||\bar{\rho}_U|\sqrt{P_R})^2$, $D = P_I 2^{-r_q} \frac{(1-x)}{1-x2^{-r_q}}$ and $x = P_{W_I}/(P_{W'} + P_{W''} + P_{W_I} + 1)$.

Sketch of the proof: The source quantizes the interference signal X_I with rate after binning, given by $r_q = I(X_I; \hat{X}_I | U Y_D)$. The quantization codebook is characterized by the equivalent test channel $\hat{X}_I = \rho X_I + Q'$, with $\rho = 1 - D/P_I$ and Q' being a complex Gaussian random variable with zero mean and variance $D(1 - D/P_I)$, independent of X_I. We obtain $D = P_I 2^{-r_q} \frac{(1-x)}{1-x2^{-r_q}}$ where x is defined above. The term $\frac{(1-x)}{1-x2^{-r_q}}$ represents the percentage of the decreased distortion due to side information about X_I at the destination. When $x = 0$, $D = P_I 2^{-r_q}$ which is the case where there is no side information about X_I at the destination. As $x \to 1$, $D \to 0$ for any nonzero r_q and the destination can completely recover X_I using the side information. The source inputs X_{SD} and X_{SR} are allocated power γP_S and $(1-\gamma)P_S$, respectively where $0 \leq \gamma \leq 1$. We set $X_{SD} = \rho_{W'}\sqrt{\gamma P_S} U_{W'} + \rho_{W''}\sqrt{\gamma P_S} U_{W''} + \rho_{W_I}\sqrt{\gamma P_S} U_{W_I} + \rho_U \sqrt{\gamma P_S} U$ and $X_R = \bar{\rho}_{W'}\sqrt{P_R} U_{W'} + \bar{\rho}_U \sqrt{P_R} U$ where $U_{W'}$, $U_{W''}$, U_{W_I} and U are independent, zero mean, unit variance, complex Gaussian random variables and carry the messages W', W'', W_I and the index of the compressed interference, respectively. Furthermore, $U_{W'}$, $U_{W''}$ and U are independent of X_I and $\hat{X}_I$ whereas $\mathbb{E}[U_{W_I} X_I] = \sqrt{P_I}$. The destination first decodes the codeword U and thus recovers $\hat{X}_I$, and then it decodes messages W', W'' and W_I jointly using the knowledge of U and $\hat{X}_I$. □

Remark 3.3: For comparison purposes, we also show the performance of the Scheme Analog Input Description, referred to as AID [1] [11]. In this scheme, the source generates the codeword to be transmitted by the relay as if the relay knew the interference and the message non-causally and they used DPC jointly. The source then quantizes this codeword and sends it to the relay through the source-relay link. The relay simply forwards a scaled version of the quantized signal received from the source. The achievable rate for DM and AWGN are given in [17, Theorem 2] and [17, Theorem 4], respectively. For the DMC model, [17, Theorem 2] can be easily modified by setting $V = X_{1R}$. For Gaussian case, we incorporate complex channel gains into [1, Theorem 4] and obtain

$$R_{AID} = \max_{\gamma: \gamma \in [0,1]} \mathcal{C}\left(\frac{(|h_{SD}|\sqrt{\gamma P_S} + |h_{RD}|\sqrt{P_R - D})^2}{1 + |h_{RD}|^2 D}\right) \quad (12)$$

$$\text{where } D = \frac{P_R}{|h_{SR}|^2 (1-\gamma) P_S + 1}.$$

IV. Numerical Results

In this section, we numerically evaluate the achievable rates for the AWGN models and compare them with two following simple schemes.

- *Scheme No Relay (NR)*: The achieved rate is given by (4) and denoted as R_{NR};
- *Scheme No Interference (NI)*: We set $P_I = 0$ and $R_I = 0$, so that the interference is not present. The capacity for this scenario, R_{NI}, is achieved by PDF [17] and is given by (7) with $R_I = 0$. Note that R_{NI} provides an upper bound to rates of the proposed achievable schemes.

The achievable rates as a function of the interference power P_I are illustrated in Fig. 2 for $P_S = P_R = 10dB$, $|h_{SD}| = |h_{RD}| = |h_I| = 1$, $|h_{SR}| = 2$ and $R_I = 1$ bits/channel use. Scheme (C,U) outperforms all others for low interference power, since in this case cooperative interference mitigation strategies are not worth the capacity needed on the source-relay link for digital interference informing. Moreover, leveraging the interference structure is not useful since interference decoding at the destination is hindered by the low interference power. For large P_I, Scheme (C,S) instead outperforms all others and eventually meets the upper bound R_{NI}. The larger P_I is, the less power the source and the relay need to make the interference decodable at the destination. In fact if P_I is sufficiently large, the destination is able to decode the interference without the help of the source or the relay leading to interference free rates. We also note that as the interference power increases, Scheme (C,S) has optimal $r_q = 0$ which means that the relay is utilized only for relaying the source message. Scheme (D,U) completely eliminates the interference by MU-DPC when R_I is greater than the capacity of the source-relay link, as is the case here, and hence, the performance of Scheme (D,U) is independent of the interference power. Even though Scheme (D,U) gives the highest rates for moderate P_I's, there is a gap between the performance of Scheme NI and Scheme (D,U) due to the source-relay capacity used for informing the relay about the interference. For similar reasons the performance of the scheme (AID) also does not depend on the interference power.

In Fig. 3, we increase the interference rate and set $R_I = 3$ bits/channel use by keeping the rest of the parameters the

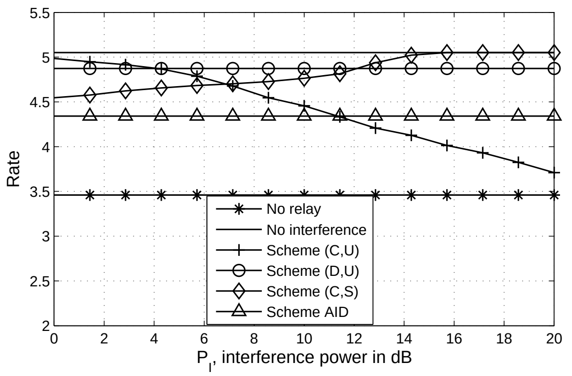

Fig. 2. Achievable rate as a function of P_I when $P_S = P_R = 10dB$, $|h_{SR}| = 2$, $|h_{SD}| = |h_{RD}| = |h_I| = 1$ and $R_I = 1$.

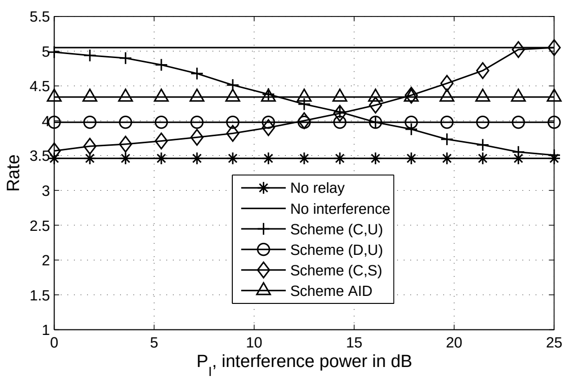

Fig. 3. Achievable rate as a function of P_I when $P_S = P_R = 10dB$, $|h_{SR}| = 2$, $|h_{SD}| = |h_{RD}| = |h_I| = 1$ and $R_I = 3$.

same as for Fig. 2. We observe that while the above arguments for schemes (C,U) and (C,S) continue to hold, Scheme (AID) outperforms Scheme (D,U) as well as all other schemes for moderate interference power. Since the interference rate is large compared to the source relay channel capacity, informing the relay about the interference in a digital fashion becomes too costly.

V. Conclusion

This paper studies a relay channel with orthogonal components that is corrupted by a single external interferer. The interference is non-causally available only at the source, but not at the relay or at the destination. The interference is assumed to be structured, since it corresponds to a codeword of the codebook of the interferer, whose transmission strategy is assumed to be fixed. Previous work that studied the model under the assumption of unstructured interference is complemented by establishing achievable schemes that leverage the interference structure. Effective interference management calls, on the one hand, for appropriate communication strategies towards the relay in order to enable cooperative interference management, and, on the other, for design of joint encoding/decoding strategies. The best available transmission strategies turn out to depend critically on the parameters of the interference signal and suggest that in AWGN channels exploiting the interference structure is beneficial when the interference power or the rate of the interferer is high enough. Further results on the relay channel subject to structured interference, including tight upper bounds and performance under fading can be found in [20].

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