On Exploiting the Interference Structure for Reliable Communications

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***Abstract*—[1]Consider an additive Gaussian noise channel affected by an additive interference sequence, taken from a given codebook, which is known non-causally at the transmitter (e.g., via prior decoding). It is known that in this case optimal performance is attained by Dirty Paper Coding, which treats the interference signal as unstructured. In other words, for this example, the knowledge of the specific interferer's codebook at the decoder is not useful in terms of capacity. In this paper, two variations of this basic scenario are presented in which treating interference as unstructured is instead generally suboptimal. In the first case, a second encoder of the source message is present in the system that is not aware of the interferer's sequence, and source and interference messages are uncorrelated; In the second case, the sources encoded by the informed transmitter and interferer are correlated (and an uninformed encoder may or may not be present). Results are given in terms of conditions for achievability for both discrete and Gaussian models of the scenarios discussed above, and corroborated by numerical results. Optimal strategies are also identified in special cases. The conclusions herein point to the importance of exploiting the interfererence structure in multiterminal and source-channel coding scenarios.**

I. Introduction

Interference is among the main limiting performance factors in many popular communication scenarios, such as wireless cellular or ad hoc networks, thus making interference management a critical task. In some cases, interference management is facilitated by the presence of certain transmitters that, prior to encoding the current information message, have learned the interference signals that will impair reception of the intended destination. This is true, for instance, if the interferer employs a retransmission strategy (ARQ), and the given transmitter was able to decode a prior transmission of the current interfering packet. Another example is given by transmitters broadcasting different messages to different destinations: In this case, interference (from the standpoint of a given destination) is generated by the transmitter itself. A third example is related to the abstraction of cognitive radio system often made in information-theoretic analyses [1].

The problem of designing optimal transmission strategies in the presence of an interference non-causally known at the transmitter, as in the examples listed above, is in general open, and was recently proposed and studied in [2][3] (see Fig. 1-(a)). The key difficulty of the problem at hand lies in the choice of how to use the knowledge of the interferer's codebook at the receiver. To see this, consider the two diametrically opposite approaches of *interference precoding* and *interference relaying*. With precoding, interference is treated as unstructured and transmission takes place following the standard coset-coding strategy by Gelfand and Pinsker [4]: In this case, the receiver does not attempt decoding of the interference. On the contrary, with interference relaying, the transmitter attempts to boost the interference signal, along with the useful signal, in order to help decoding of both at the receiver, exploiting the interference codebook structure.

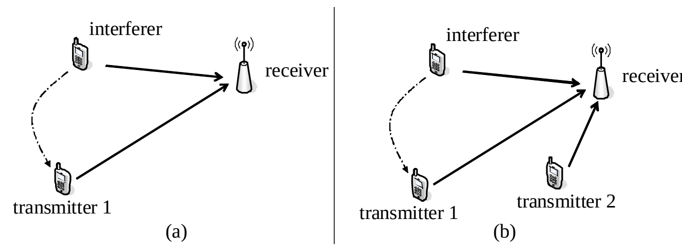

Fig. 1. (a) Communication against an interferer whose codeword is known at the transmitter; (b) Communication against an interferer whose codeword is known only at one transmitter.

In general, neither of the two approaches discussed above dominates the other [2]. Interference precoding is optimal if the interference is independent identically distributed (i.i.d.), as shown by Gelfand and Pinsker [4]. However, the codewords of a capacity-achieving code are not i.i.d., unless communication takes place over a noiseless channel [5] (see also [3]). Interference relaying is instead clearly optimal if the rate of the interference is so small that it can be decoded at the destination while treating the useful signal as noise – In this case, the interference has no impact on the achievable rate of the signal [2].

While the above is true for a general channel, in the special case of additive Gaussian noise channels, interference precoding is known to be optimal, and thus there is no need to exploit the codebook structure. In fact, a special version

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of Gelfand-Pinsker precoding, known as Dirty Paper Coding (DPC), is able to achieve the interference-free capacity [6][7][2], which is clearly an upper bound on the performance of the interference-impaired channel. The major contribution of this paper is to present two variations of the basic scenario in which a destination is affected by an additive interferer, in which interference precoding is instead generally suboptimal. In the first, a second transmitter is present in the system that is not informed about the interferer's sequence (see Fig. 1-(b)), while in the second the sources encoded by the informed and, if present, uniformed transmitters and interferer are correlated.

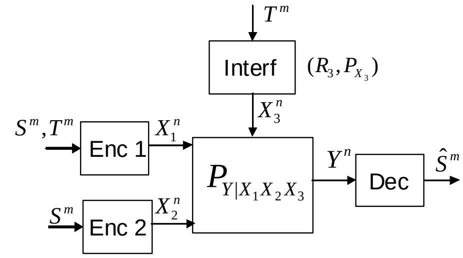

Fig. 2. System model corresponding to Fig. 1-(b).

II. System Model

The general scenario under study, sketched in Fig. 1-(b) and Fig. 2, consists of a single destination, to which two transmitters (terminals 1 and 2) wish to communicate an i.i.d. finite and discrete-alphabet source[3] S^m, of m samples, over n uses of a memoryless channel. Communication takes place in the presence of an interferer (terminal 3) which employs a *fixed (and given) codebook* $X_3^n(T^m)$ to transmit a second i.i.d. source T^m, which is correlated with S^m according to a joint probability mass function (pmf) P_{ST}. Source S^m is available at both transmitter 1 and 2. Moreover, transmitter 1 is also informed about the sequence T^m encoded by the interferer (or equivalently of codeword $X_3^n(T^m)$), so that encoding takes place at encoder 1 as $X_1^n(S^m, T^m)$ and encoder 2 as $X_2^n(S^m)$. We define the *bandwidth* ratio (between the channel and source bandwidths) as $b = n/m$.

The scenario at hand models a situation in which terminal 1 has been able to acquire both the signal source sequence S^m, intended for the destination, and the interfering source sequence T^m, prior to the current transmission block. This may take place as explained in the previous section. We aim at proving insight into the main issue as to whether one should design the system by exploiting the structure of the interference signal X_3^n, via interference relaying, or instead treat the interference as unstructured via interference precoding.

Similar to [2], the codebook $X_3^n(T^m)$ sent by the interferer is not subject to design and is assumed to be chosen by the interfering terminal autonomously to communicate with some other destination, not modelled explicitly. We will assume that such codebook is randomly generated according to a distribution[4] P_{X_3} known to all nodes. Specifically, the interferer uses separate source-channel coding and generates a codebook of 2^{nR_3} codewords $X_3^n(w)$, $w \in [1, 2^{nR_3}]$, where each codeword is generated i.i.d. according to P_{X_3}. The index $w(T^m) \in [1, 2^{nR_3}]$ associated to a source sequence T^m (and thus the mapping $X_3^n(w(T^m))$, or $X_3^n(T^m)$ for short) is selected randomly and uniformly in $[1, 2^{nR_3}]$ [5]. We remark that the operation at the interferer can be equivalently interpreted as Slepian-Wolf random binning (i.e., $w(T^m)$) followed by channel coding of the bin index (i.e., $X_3^n(w(T^m))$). Overall, an interference signal is described by (R_3, P_{X_3}) [6].

Communication takes place over a memoryless channel. We will study both a discrete memoryless version of the channel in Fig. 2, which is described by the tuple $(\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, P_{Y|X_1X_2X_3}, \mathcal{Y})$, according to standard notation, and the corresponding Gaussian model, which is of main interest, and is characterized by the received signal

$$Y = X_1 + X_2 + X_3 + Z, \tag{1}$$

with $Z \sim \mathcal{N}(0, 1)$ (alphabets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, \mathcal{Y} = \mathbb{R}$). In the Gaussian case we will assume that the interferer's codebook is generated according to a P_{X_3} given by $\mathcal{N}(0, P_3)$ (i.e., a "Gaussian codebook"). Standard block power constraints are P_1 and P_2 for the two transmitters.

Given the sources pmf P_{ST}, the channel $(\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, P_{Y|X_1X_2X_3}, \mathcal{Y})$ and the interferer (R_3, P_{X_3}), we are interested in identifying conditions on the bandwidth ratio b so that the destination can recover the signal S^m losslessly for sufficiently large n. Lossless reconstruction is defined with respect to the probability of error $\Pr[\hat{S}^m \neq S^m]$, calculated *on average over the distribution of source, channel and codebook of the interferer*[7]. Notice that the destination is not interested in decoding the interferer source T^m.

As a final note, we remark that the considered model with correlated sources fits a scenario in which sensors (the transmitters) measure correlated phenomena but report at different readers or fusion centers. In such networks, the bandwidth ratio b is the same for both useful signal and interferer, but no joint decoding of transmitter and interferer may be allowed.

III. Independent Sources

We start by considering the case where the sources (S^m, T^m) are independent, i.e., $P_{ST} = P_S P_T$. In this case, we will concentrate on the scenario where S^m is first converted

[2]Reference [7] extends the results of [6] to arbitrary state (interference) sequences, not necessarily i.i.d. Gaussian.

[3]X^n represents the sequence $(X_1, X_2, ..., X_n)$.

[4]We will denote with the same symbol both pmf and probability density functions (pdf) with an abuse of notation.

[5]We do not consider other forms of encoding such as superposition [2].

[6]One can assume $R_3 \leq H(T)/b$, since otherwise, for large n (or equivalently m), given the asymptotic equipartition property, some codewords in the codebook $X_3^n(w)$ would be used with vanishingly small probability and could thus be eliminated from the codebook.

[7]This is different from [2] in which stronger conditions are imposed for achievability.

into a message W_1, uniformly distributed in the set $[1, 2^{nR_1}]$, via source coding[8], and then channel encoded, while the interferer chooses the codeword $X_3^n(w)$ from the codebook uniformly in $w \in [1, 2^{nR_3}]$. We can then focus on the achievability (in the usual sense) of rate R_1, measured in bits/ channel use, given the interferer rate R_3.

Under these assumptions, if transmitter 2 is absent, the model in Fig. 2 reduces to the problem of encoding against an interference codebook, for which achievable rates were recently put forth by [2]. The scheme proposed therein switches between the two extreme solutions of interference precoding and interference relaying: Specifically, transmitter 1 either encodes "over" the interference using [4][6] *or* boosts the interference to enable decoding at the destination (interference relaying)[9]. In the first case, the signal is treated as an unstructured i.i.d. sequence, while in the second interference relaying is deployed to help the receiver leverages the interference structure via joint decoding.

Assume now that the transmitter 2 is also present. It is noted that this model is related to the scenario studied in [8], that differs from the current one in that the interference sequence X_3^n there is unstructured. This prevents interference decoding at the destination and thus rules out interference relaying. Reference [8] finds the capacity region with unstructured interference to be achieved with a scheme at transmitter 1 that combines superposition (for signal relaying) and a generalized interference precoding scheme that allows for interference cancellation.

Here, we study the performance of an achievable scheme, inspired by [2], that combines at transmitter 1 the two extremes of interference precoding/ cancellation as in [8] and interference relaying. We have the following result.

Proposition 1: The following rate is achievable for the discrete memoryless model of Fig. 2 for an interferer rate R_3:

$$R_1 = \max \min \left\{ \begin{array}{l} I(X_1X_2;Y|X_3), \\ \max \left\{ \begin{array}{l} I(UX_2;Y) - I(UX_2;X_3), \\ I(X_1X_2X_3;Y) - R_3 \end{array} \right\} \end{array} \right\}, \tag{2}$$

where maximization is taken over distribution $P_{X_2}P_{UX_1|X_2X_3}$, mutual informations are calculated with respect to $P_{X_3}P_{X_2}P_{UX_1|X_2X_3}P_{Y|X_1X_2X_3}$ and $|\mathcal{U}| \leq |\mathcal{X}_1||\mathcal{X}_2||\mathcal{X}_3|$.

Remark 1: Rate (2) can also be written as

$$R_1 = \max_{P_{X_2}P_{UX_1|X_2X_3}} \{R_u, R_s\}, \tag{3}$$

where:

$$R_u = I(UX_2;Y) - I(UX_2;X_3), \tag{4}$$

when maximized over $P_{X_2}P_{UX_1|X_2X_3}$, is the capacity of the channel at hand if the interference is unstructured, i.e., if X_3^n is an i.i.d. sequence [8], and is thus achieved via interference precoding/ cancellation; while rate

$$R_s = \min \left\{ \begin{array}{l} I(X_1X_2;Y|X_3), \\ I(X_1X_2X_3;Y) - R_3 \end{array} \right\} \tag{5}$$

is obtained via interference forwarding and joint decoding, using capacity results for multiple access channels with common messages [10] (see Appendix for a simplification of the general result of [10] to our scenario).

Remark 2: If $X_2 = \phi$, rate (2) reduces to the one derived in [2]. Moreover, the achievable rate at hand can also be seen as a special case of the more general achievable regions in [11] and, for the Gaussian case to be presented below, in [12][13]. The specialization of the results [11][12][13] is made here to investigate the achievable performance in the presence of a fixed (i.e., not subject to design) interferer, and to show the connection with [2] and [8].

Remark 3: If $R_3 \leq \min_{P_{X_2}P_{X_1|X_2}} I(X_3;Y)$, then the achievable rate (2) equals the upper bound set by the performance of a system where the interference is known at the receiver, i.e., $R_1 = \max_{P_{X_2}P_{X_1|X_2X_3}} I(X_1X_2;Y|X_3)$, and is thus optimal. A more general upper bound on R_1 can be found by following [2, Sec. III-C].

For the additive Gaussian-noise channel (1), and similarly for the binary symmetric channel, in the setting of [2] (i.e., without transmitter 1) there is no benefit to be accrued by exploiting the signal structure, and thus by interference forwarding, since the upper bound set by the performance of a system with no interference can immediately be achieved via interference precoding (DPC in the Gaussian case). For the setting at hand, instead, interference precoding does not achieve such upper bound [8] and therefore exploiting the interference structure may help. We will show this with an example, after giving the following result.

Proposition 2: The rate (3) is achievable for the Gaussian model (1) of Fig. 2 for an interferer rate R_3 with

$$R_u = \max_{\substack{0 \leq \rho_2 \leq 1,\ -1 \leq \rho_3 \leq 0 \\ \rho_2^2 + \rho_3^2 \leq 1}} \frac{1}{2}\log_2\left(1 + P_1(1-\rho_2^2-\rho_3^2)\right) + \frac{1}{2}\log_2\left(1 + \frac{(\rho_2\sqrt{P_1}+\sqrt{P_2})^2}{1+P_1(1-\rho_2^2-\rho_3^2)+(\sqrt{P_3}+\rho_3\sqrt{P_1})^2}\right), \tag{6}$$

and

$$R_s = \max_{\substack{0 \leq \rho_i \leq 1, \\ \rho_2^2+\rho_3^2 \leq 1}} \min \left\{ \begin{array}{l} \frac{1}{2}\log_2\left(\begin{array}{c} 1 + P_1(1-\rho_3^2) + P_2 \\ +2\rho_2\sqrt{P_1P_2} \end{array} \right), \\ \frac{1}{2}\log_2\left(1 + \sum_{i=1}^{3} P_i + 2\sum_{i=2}^{3}\rho_i\sqrt{P_1P_i}\right) \\ -R_3 \end{array} \right. \tag{7}$$

Proof: The rate (6) follows from (4) by choosing Gaussian inputs and similarly (7) follows from (5) as explained in [8, Theorem 7].

Remark 4: In (6) and (7), correlation coefficients ρ_i, $i = 2, 3$, represent the correlation of the signal of transmitter 1 with respect to transmitter 2 and the interferer, respectively. It is noted that in (6), we limit the interval of interest to $-1 \leq \rho_3 \leq 0$ since, when treating the interference as unstructured,

[8]This requires $R_1 \geq H(S)/b$.

[9]It is noted that the considered problem also draws a connection with [9], where an unstructured state sequence, known non-causally at the encoder, is required to be known with reduced uncertainty at the decoder.

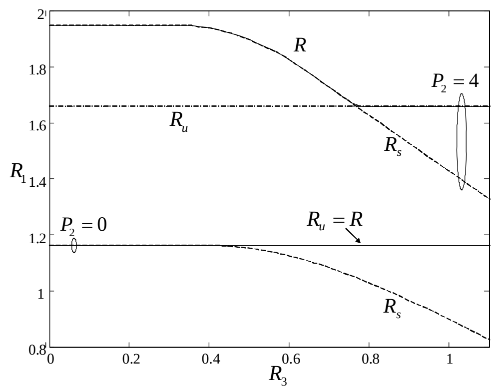

Fig. 3. Achievable rate (3) versus the interference rate R_3 for $P_1 = P_3 = 4$ and $P_2 = 0$ or $P_2 = 4$. For comparison, we also show the rate achievable rate R_s (7), obtained by performing interference relaying, and R_u (6), obtained by performing interference precoding and cancellation.

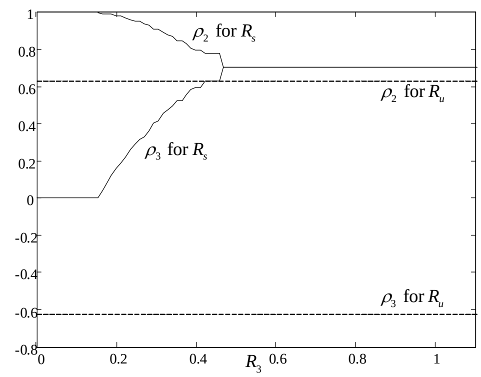

Fig. 4. Optimal values of the correlation coefficients ρ_2 (correlation with transmitter 2) and ρ_3 (correlation with the interference) versus the rate R_3 for $P_1 = P_3 = 4$ and $P_2 = 4$ for the achievable rates R_s (7) and R_u (6).

interference relaying (which would entail $0 \leq \rho_3 \leq 1$) is not useful, whereas interference cancellation (which requires a negative ρ_3) may be beneficial. The opposite holds true for (7).

A. An Example

Fig. 3 shows the achievable rate (3) versus the interference rate R_3 for $P_1 = P_3 = 4$ and $P_2 = 0$ or $P_2 = 4$. For comparison, we also show the rate achievable rate R_s (7) and R_u (6). It can be seen that while if transmitter 2 is absent ($P_2 = 0$), as in [2], treating the interference as unstructured is optimal, this is not the case if $P_2 > 0$ and the interference rate R_3 is not too large. Optimal values of the correlation coefficients ρ_i for R_s and R_u are shown in Fig. 4, demonstrating that: (*i*) For low interference rate R_3, when treating the signal as structured is advantageous, a combination of signal and interference relaying is needed (i.e., $\rho_2, \rho_3 > 0$); (*ii*) For large R_3, where it is optimal to treat interference as unstructured, a combination of signal relaying and interference neutralization is optimal. This suggests that exploiting the structure of the interference becomes more and more relevant in complex network scenarios with interacting terminals.

B. A Note on a Parallel Channel Model

In this section, we consider a special case of the model considered in this section, where $Y = (Y_1, Y_2)$ and

$$P_{Y|X_1X_2X_3} = P_{Y_1|X_1} \cdot P_{Y_2|X_2X_3}. \tag{8}$$

This corresponds to a setting in which transmitter 1 operates over an orthogonal channel or bandwidth as compared to transmitter 2 and interferer. In other words, the interference only affects the channel Y_2, but does not affect the channel Y_1 where the transmitter 1, which is aware of the interference sequence, is active. The Gaussian version of this model prescribes received signals

$$Y_1 = X_1 + Z_1 \tag{9a}$$

$$\text{and } Y_2 = X_2 + X_3 + Z_2 \tag{9b}$$

with independent unit-power Gaussian noises Z_1, Z_2.

Reference [8] shows that, if the interference is i.i.d., then there is no loss in optimality if transmitter 1 ignores the available side information about the interference. If the interference is instead structured, as considered in this paper, it is not clear a priori whether transmitter 1 should or not neglect the side information. The next Propositions answers this question for discrete and Gaussian models, respectively.

Proposition 3: For the discrete memoryless parallel model (8), the maximum achievable rate is obtained by neglecting the side information at transmitter 1 about the interference and is given by

$$\max_{P_{X_1}} I(X_1; Y_1) + R_2, \tag{10}$$

where R_2 is the maximum rate achievable by encoding X_2^n on channel $P_{Y_2|X_2X_3}$ for the given interferer (R_3, P_{X_3}).

Proof: From Fano inequality and standard inequalities, we have

$$\begin{aligned} nR_1 &\leq I(W; Y_1^n Y_2^n) + n\epsilon_n \\ &\leq I(W; Y_2^n) + I(W; Y_1^n | Y_2^n) + n\epsilon_n \\ &\leq I(X_2^n; Y_2^n) + h(Y_1^n) - h(Y_1^n | Y_2^n W) + n\epsilon_n \\ &\leq I(X_2^n; Y_2^n) + h(Y_1^n) - h(Y_1^n | X_1^n Y_2^n W) + n\epsilon_n \\ &\leq I(X_2^n; Y_2^n) + I(X_1^n; Y_1^n) + n\epsilon_n, \end{aligned}$$

which can be achieved by transmitting independent information over the two channels Y_1 and Y_2.

Proposition 4: For the Gaussian parallel model (9), the maximum achievable rate is given by

$$R_1 = \frac{1}{2}\log_2(1 + P_1) + \min \left\{ \begin{array}{l} \frac{1}{2}\log_2(1 + P_2), \\ \max \left\{ \begin{array}{l} \frac{1}{2}\log_2\left(1 + \frac{P_2}{1+P_3}\right), \\ \frac{1}{2}\log_2(1 + P_2 + P_3) \\ -R_3 \end{array} \right. \end{array} \right. , \tag{11}$$

and is achieved by neglecting the side information at transmitter 1 about the interference. Moreover, it is obtained by either treating the interference as noise at the receiver, for

$R_3 \geq 1/2\log_2(1+P_3)$, or by decoding the interference in full otherwise.

Proof: Following the proof of Proposition 3, we only need to characterize R_2 in (10), since $\max_{P_{X_1}} I(X_1;Y_1) = 1/2\log_2(1+P_1)$ (first term in (11)). To do so, fix the interferer rate R_3 and consider a Z-interference channel (ZIC) given by received signals (9b) and $Y_3 = aX_3 + Z_3$, with unit-power noise Z_3 and a selected such that

$$R_3 = \frac{1}{2}\log_2(1+aP_3),$$

i.e., the interferer is transmitting at capacity on channel Y_3 (with a Gaussian codebook). We can equivalently model this ZIC as

$$\begin{aligned} Y_2 &= X_2 + \frac{1}{a}X_3' + Z_2 \\ Y_3 &= X_3' + Z_3, \end{aligned}$$

with codebook X_3' generated according to $\mathcal{N}(0, aP_3)$. Notice that the receiver measuring Y_i requires decoding of X_i, for $i = 2,3$, in the Z-channel at hand. It is known that the sum-capacity of this ZIC (when one can optimize both X_i, is given by [15]

$$R_{sum} = R_3 + \begin{cases} \frac{1}{2}\log_2(1+\frac{P_2}{1+P_3}), \text{ if } a \geq 1 \\ \min\left\{ \begin{array}{l} \frac{1}{2}\log_2(1+P_2+P_3) - R_3 \\ \frac{1}{2}\log_2(1+P_2) \end{array} \right\}, \\ \text{if } a < 1 \end{cases}. \tag{12}$$

Since R_3 is fixed, this implies that the maximum rate R_2 is given by the second term in (12), which concludes the proof.

Remark 6: The rate in Proposition 4 is a special case of Proposition 1 obtained by setting Gaussian input distributions.

IV. Correlated Sources

We now turn to the analysis of the system in Fig. 2 for the case in which sources (S^m, T^m) are correlated. In this case, the structure of the interference sequence encompasses not only the fact that such sequence belongs to a given codebook, as in the previous Section, but also the fact that it is correlated with the signal sequence S^m. The following proposition derives an achievable bandwidth ratio for this scenario.

Proposition 5: The following bandwidth ratio is achievable for both the discrete memoryless and Gaussian models of Fig. 2:

$$b = \min(b_u, b_s) \tag{13}$$

with

$$b_u = \frac{H(S)}{R_u} \tag{14}$$

and

$$b_s = \max\left(\frac{H(S|T)}{R_s}, \frac{H(ST)}{R_3 + R_s}\right), \tag{15}$$

where R_u and R_s are defined as in (4)-(5) and (6)-(7) for the discrete and Gaussian models, respectively.

Remark 6 (sketch of proof): The bandwidth ratio b_u is achievable by treating the interference as unstructured and uncorrelated to the source S^m via generalized interference precoding as in [8], while ratio b_s is achieved by exploiting the structure and correlation of the interference via interference relaying. Thus, as for Propositions 1 and 2, the scheme of Proposition 4 switches between interference precoding and relaying. More specifically, we use a source-channel separation-based coding scheme. When treating the interference as unstructured, we first compress source S^m at both transmitter 1 and 2 and then encode it via the channel coding strategy of Proposition 1 and 2 that achieves rate R_u. The condition

$$H(S) \leq bR_u \tag{16}$$

thus guarantees achievability. Instead, when treating the interference as structured, we perform Slepian-Wolf random binning at both transmitter 1 and 2; The bin index is then transmitted following the coding scheme used in Propositions 1 and 2 that achieves R_s. Therefore, it can be seen that the condition

$$\max(H(S,T) - bR_3,\ H(S|T)) \leq bR_s, \tag{17}$$

guarantees achievability. Notice that in calculating (17) we have excluded the error event in decoding the bin indices from transmitters and interferer where T^m is not correctly decoded but S^m is, similar to the calculation of R_s (see Appendix).

A. An Example

Consider the Gaussian model (1) and assume that there is no transmitter 2. As discussed above, if the sources were uncorrelated, interference precoding via DPC would be optimal. Now, assume instead that the source are binary and correlated according to $T = S \oplus Z$ with Z independent and $\Pr[Z=1] = p$ with $0 \leq p \leq 1/2$. Set $P_2 = 0$, $P_1 = 1$, $P_3 = 8$. Fig. 5 shows the achievable bandwidth ratio b_u (14) for interference precoding, which does not depend on p since interference precoding does not exploit source correlation, and for interference relaying, b_s (15), with different values of p (i.e., of source correlation). It can be seen that, while if $p = 1/2$ (uncorrelated sources) interference precoding (DPC) is always optimal, this is not the case if the sources are dependent, i.e., $0 \leq p < 1/2$. In particular, interference precoding is advantageous only if the interference rate is large enough so that the codeword X_3^m becomes undecodable, or small enough so that, decoding X_3^m is possible, but recovering the corresponding sequence T^m from the bin indices sent by the interferer and transmitter becomes impossible. With regard to the latter scenario, we notice that here, unlike the previous section, the entropy of the interference is $H(T) = 1$ irrespective of R_3.

V. Conclusions

Interference management is a critical task in many communication systems. Effective interference management requires to properly account for the structure of the interference. The results of this paper lend evidence to this conclusion by considering a simple scenario in which not all transmitters may be informed a priori about the interfering signal and/

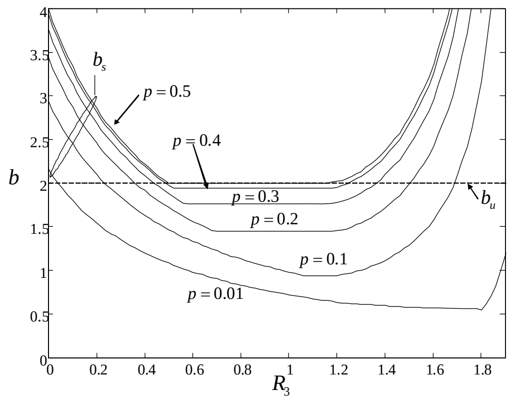

Fig. 5. Achievable bandwidth ratio for intererence precoding (b_u (14)) and for interference relaying (b_s (15)) with different values of p ($T = S \oplus Z$ with Z independent and $\Pr[Z = 1] = p$, $P_2 = 0$, $P_1 = 1$, $P_3 = 8$).

or the signal and interference sources are correlated. Many open problems remain to be addressed, including the impact of imperfect channel state information and the derivation of tight upper bounds on the achievable performance.

VI. APPENDIX

Consider the scenario of Fig. 1-(b), with the message structure in Fig. 2, and assume independent sources so that we can focus on achievable rates R_1 and R_3. When treating the interference as structured, this channel can be seen as a multiple access channel with two messages, one, of rate R_1, known at terminals 1 and 2, and the other, of rate R_2, known at terminals 2 and 3. The capacity region of a general multiple access channel with common messages was found in [10][16]. It can be shown that the capacity region therein boils down to the following conditions for the considered special case:

$$\begin{aligned} R_1 &\le I(X_1X_2;Y|X_3) \\ R_3 &\le I(X_1X_3;Y|X_2) \\ R_1 + R_3 &\le I(X_1X_2X_3;Y), \end{aligned}$$

where union and convex hull operations should be applied with respect to the joint input distribution $P_{X_2}P_{X_3}P_{X_1|X_2X_3}$. Moreover, in our model, the second bound can be waived since it corresponds to an error event where only the interference is not decoded correctly.

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