Comparison between radiative transfer and ray tracing for indoor propagation

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Abstract — **In this paper the authors report how - under specific and limited condition- radiative transfer proved to be an useful tool for power fluctuation predictions experienced by an indoor wireless propagation channel. The analysis is carried out by comparing the radiative transfer results with the second order statistics of the ray tracing computed signal in a two-dimensional geometry comprising randomly placed scatterers.**

1 INTRODUCTION

A versatile technique for studying propagation in indoor wireless communications systems is ray tracing [1], through which, a number of paths, stemming from the transmitter, are traced along their way to the receiver, accounting for reflection over the obstacles within the scenario. Other mechanisms of interaction between the wave and the environment, such as diffraction, can be accommodated in ray tracing procedures by appropriate generalization of the basic theory [2]. While this method is purely deterministic, in actual environments with many randomly placed scatterers of size comparable to the wavelength (Mie scattering), statistical characterization of the multipath channel [3] may be the only - though computationally cumbersome - viable approach in order to have an accurate model of the propagation [4].

The purpose of this paper is to report the validity of Radiative Transfer (RT) results for the evaluation of power fluctuations in an indoor environment described as a homogeneous medium filled with arbitrarily placed scatterers. The radiative transfer outcome is compared with ray tracing to assess its limits of applicability. It should be clear that while the ray tracing approach can be in principle used for any geometry and provide information about the phase of the wave, RT is in practice only applicable to simple geometries and can only yield information about the second order statistics of the wave.

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Here, the experiment of interest consists in a monochromatic plane wave impinging upon the layered parallel plane medium in fig. 1, where each layer is modelled as random medium containing N randomly placed scatterers per square meter, and received by an antenna array. This modelization is a first approximation of an indoor environment including people, benches or, for instance, rows of chairs in an auditorium. The scatterers number density is chosen so as interference and interaction between scatterers could be neglected. RT reliability under this condition has been thoroughly investigated [5]; here, we show how it can be used as a simple tool for predicting the angular distribution of the received power and the spatial correlation of the received signal for indoor propagation.

2 MODELING

2.1 Radiative transfer approach

For the two-dimensional geometry of interest (see fig. 1), the RT equation [6] states the conservation of energy within a differential element dy :

$$\sin\phi \frac{\partial I(y,\phi)}{\partial y} = = -k_{ext}(y) I(y,\phi) + \int_0^{2\pi} p(y,\phi',\phi) I(y,\phi') d\phi', \quad (1)$$

where: (i) the *specific intensity* $I(y,\phi)$ is the power flux density within a unit angle centered at a given azimuth ϕ (and within a unit frequency band centered at a given frequency f) corresponding to a certain polarization (either vertical or horizontal); (ii) the extinction coefficient within the random medium depends on both the scatterers density N and the extinction cross-width σ_{ext} of each particle as $k_{ext}(y) = N\sigma_{ext}$. The extinction cross-width of each particle σ_{ext} can be computed from the forward scattering theorem in terms of the scattering function $F(\phi',\phi)$ as [7]:

$$\sigma_{ext} = -4\sqrt{\frac{\pi}{2k_0}}\,\mathrm{Re}(F(\phi,\phi)). \quad (2)$$

Notice that outside the random medium the extinction coefficient reads $k_{ext}(y) = 0$; (iii) the phase

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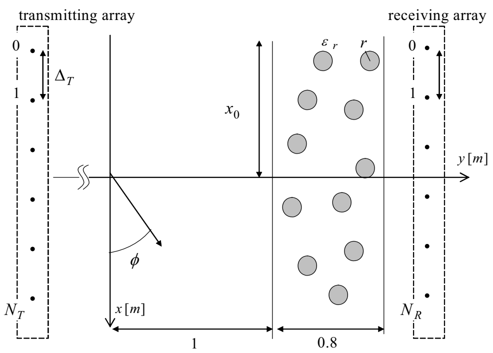

Figure 1: Geometry of the experiment studied by means of ray tracing and radiative transfer.

function $p(y, \phi', \phi)$ within the random medium depends on both the scatterers density N and the scattering function as:

$$p(y, \phi', \phi) = N|F(\phi',\phi)|^2. \qquad (3)$$

Outside the random medium we have $p(y, \phi', \phi) = 0$. The analytical expression of the scattering function $F(\phi',\phi)$ for infinite-length circular cylinders can be found in [7]. In order to ease the enforcement of boundary conditions, (1) is generally converted into two coupled integro-differential equations by introducing the progressive intensity I^+, that corresponds to propagating directions $0 \le \phi < \pi$, and the regressive intensity I^-, that accounts for the propagating directions $\pi \le \phi < 2\pi$. The solution of (1) can be obtained through numerical quadrature techniques that discretize the continuum of azimuth directions ϕ into a finite number of directions [6].

In a communication link, it is of great interest to assess the degree of correlation between the signals received by different antennas as a function of their inter-spacing Δ $[m]$. Such an indicator, defined as $r_y(\Delta)$, can be computed from the specific intensity. In fact, by interpreting (after appropriate scaling) the power received over a certain direction ϕ as a measure of the probability $p_y(\phi)$ that a signal is received through direction ϕ, we obtain:

$$r_y(\Delta) = \int_0^{\pi} p_y(\phi)\, e^{\text{-jk}_0 \Delta \cos\phi} d\phi. \qquad (4)$$

2.2 Ray tracing approach

The code used for this works is based on a geometric pre-computation of a tree-like topological representation of the reflections in the environment (beam tree) through spatial subdivision techniques models [8]. The method finds all paths combining transmission and specular reflection up to user-specified termination criteria (e.g., all paths arriving after a certain number of interactions) without sampling or exhaustive enumeration. Developed for prediction in acoustic problems [8], this method was adapted to electromagnetic problems.

The ray trajectories post-processing models the propagation in the environment characterized by the scatterers and, if any, delimiting walls, with known or inferred electromagnetic parameters. Hence, at each location along the array the received signal $E_i(y)$ is calculated (up to the chosen termination criteria) as:

$$E_i = \sum_{h=1}^{N_R} \frac{1}{L_h} \left(\prod_{j=1}^{M} \rho_{h,j} \right) e^{j\omega\tau_h} e^{j\alpha_h} \qquad (5)$$

where $e^{j\alpha_h}$ is the phase contribution due to the interaction (reflection or transmission) and $e^{j\omega\tau_h}$ is the phase delay contribution at the frequency of operation ω for delay τ_h; L_h is the path length associated to the hth ray. $\prod_{j=1}^{M} \rho_{h,j}$ accounts for the intensity reduction due to the jth reflection and/or transmission (up to M) experienced by the hth ray, where each $\rho_{h,j}$ is the appropriate Fresnel coefficient for the considered interaction and polarization.

3 EXPERIMENT DESCRIPTION

According to fig. 1, the experiment of interest consists in a plane wave impinging with azimuth ϕ_i on a slab (uniform in the $\mathbf{x}$ direction) of random medium. The latter is characterized by randomly placed circular cylinders. For radiative transfer, this situation is studied by enforcing the initial condition $I^+(0, \phi) = \delta(\phi - \phi_i)$ (and also $I^-(D, \phi) = 0$ where D is any value of y larger than the rightmost bound of the random medium) on the solution of (1). On the other hand, for ray tracing, some approximations have to be made, as: (i) according to the Huygens principle, a plane wave has been approximated by building a linear antenna array of N_T elements with inter-element spacing Δ_T lying parallel to the $\mathbf{y}$ axis. In order to work in the far field regime, the transmitting array has been placed at a distance $R > (N\Delta_T)^2/\lambda$ [6] from the random medium; (ii) the circular scatterers were approximated by polygons of N_P sides and the random slab ranges in the $\mathbf{x}$-direction within $-x_0 \le x \le x_0$; (iii) the signal is received at the desired points along the $\mathbf{y}$ axis by a linear antenna array of N_R elements with inter-element spacing Δ_R lying parallel to the $\mathbf{y}$ axis.

In order to compare the outcome of radiative transfer theory and ray tracing, it is necessary to estimate the specific intensity within the ray tracing experiment from the signal received over the receiving array. Let us define the N_R complex values of the electric field received by the array as the $N_R \times 1$ vector $\mathbf{E}(y) = [E_0(y) \cdots E_{N_R-1}(y)]^T$, where $E_i(y)$ is the electric field measured at y by the ith antenna element as given by (5). The specific intensity can be estimated as follows [9]:

$$\hat{I}(y,\phi) = \frac{1}{N_R} \left| \sum_{i=0}^{N_R-1} E_i(y) e^{(-jk_0 \Delta_R \cos\phi \cdot i)} \right|^2 . \quad (6)$$

Due to the practical limitations on the length of the receiving array, the angular distribution (6) is computed within a given angular resolution; i.e., the estimated specific intensity $\hat{I}(y,\phi)$ is related to $I(y,\phi)$ by a convolution with a sinc squared function with main lobe of width $\Delta\phi = 2\lambda_0/N_R\Delta_R$. [9]. On the other hand, the spatial correlation can be computed as:

$$\hat{r}_y(m\Delta_R) = \frac{1}{N - |m|} \sum_{i=0}^{N_R-1} E_i(y) E^*_{i+m}(y). \quad (7)$$

Since ray tracing is a deterministic approach, in order to compare the specific intensity (6) and the spatial correlation (7) with the corresponding quantities from transfer theory, both (6) and (7) have to be averaged over multiple realizations N_I of the random medium.

4 NUMERICAL RESULTS

The geometry in fig. 1 was considered as a benchmark to compare the results from RT and ray tracing. Parameters are selected as follows: $N_T = 100$, $\Delta_T = \lambda/2$, $x_0 = 5\ m$, $N_P = 40$, $N_R = 80$, $\Delta_R = \lambda/4$, $r = 6\ cm$, $N = 10\ m^{-2}$, and, according to the WLAN standard, we consider a carrier frequency of $f = 5.2\ GHz$. The random medium ranges within $1 \leq y \leq 1.8$ $[m]$ and we measure the quantities of interest at $y = 1.9m$. The relative dielectric constant of the scatterers is $\varepsilon_{r,1}{=}\varepsilon'_{r,1}{-}i \cdot \varepsilon''_{r,1}$, where $\varepsilon'_{r,1} = 4$ and $\varepsilon''_{r,1} = \sigma_1/(2\pi f \cdot \varepsilon_0)$ with conductivity $\sigma_1 = 10^{-3}\ S/m$. Let $A_{ext} = \pi(\sigma_{ext}/2)^2$ be the effective area occupied by each particle, then the fraction of effective area that is occupied by the scatterers is $\eta = NA_{ext}$.

The progressive specific intensity $I^+(y,\phi)$ computed according to RT is compared with $\hat{I}^+(y,\phi)$ from the ray tracing procedure in fig. 2 for vertical polarization. The impinging wave has azimuth $\phi_i = 90$ deg and intensity $I_o^+ = 1\ W\ m^{-2}\ Hz^{-1}$

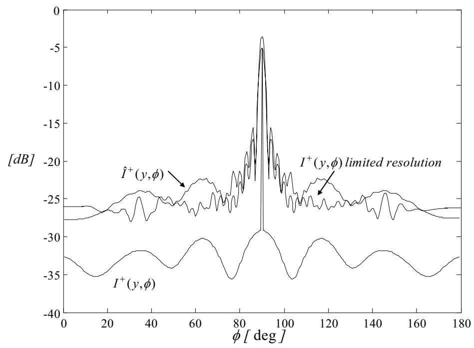

Figure 2: RT and ray tracing computed specific intensities, $I^+(y,\phi)$ and $\hat{I}^+(y,\phi)$, for $\varepsilon'_{r,1} = 4$, $\sigma_1 = 10^{-3}\ S/m$, $y = 1.9\ m$ (on the right of the random medium), $N = 10\ m^{-2}$ and vertical polarization.

rad^{-1}. In order to compare the results from the two approaches, the effect of limited angular resolution is accounted for by processing the outcome of the RT $I^+(y,\phi)$ (curve labeled as "$I^+(y,\phi)$ limited resolution") and the number of averaging iterations for ray tracing is $N_I = 100$. The specific intensity is normalized with respect to the incident intensity I_o^+ and thus shown in dB. Both the line of sight component (LOS, i.e., for $\phi = 90$ deg) and the diffuse part computed by means of ray tracing match with the radiative transfer specific intensity with limited resolution. In particular, the LOS component is slightly underestimated (about 1 dB) by RT.

By increasing the density N, and consequently the fraction η of effective area occupied by the scatterers, the difference between the prediction of ray tracing and RT increases beyond 1 dB. To elaborate on this, in fig. 3, the attenuation constant $\alpha = 10\log_{10}(I^+(y,\phi)/I^+(0,\phi)) \cdot 1/y$ $[dB/m]$ on the LOS component predicted by ray tracing and RT is plotted versus η. It can be concluded that for the example at hand whenever $\eta \leq 15\%$, the RT approach yields fairly accurate results, i.e. within 1 dB on the peak specific intensity if the slab has width $0.8\ m$. This result has been enforced by letting η vary for scatterers with different radius and electromagnetic characteristics. Fig. 3 shows the same quantities described above for scatterers with radius $a = 6\ cm$ and dielectric constant given by $\varepsilon'_{r,2} = 3.4$ and $\sigma_2 = 5.9 \cdot 10^{-3}\ S/m$ ($\sigma_{ext.2} \simeq 0.3372$ m). The agreement between radiative transfer and ray tracing discussed above in terms of specific intensity is confirmed in fig. 4 by comparing the spatial correlation $r_y(\Delta)$ computed by RT and the same quantity $\hat{r}_y(\Delta)$ from ray tracing versus Δ/λ

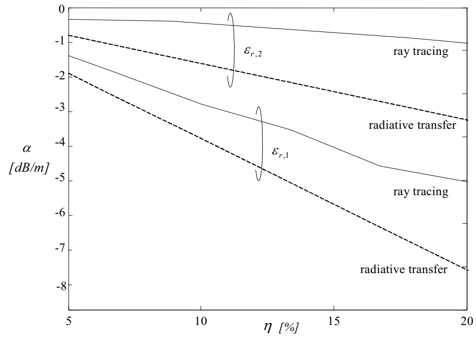

Figure 3: RT and ray tracing computed specific intensity, $I^{+}(y, \phi)$ and $\hat{I}^{+}(y, \phi)$, versus the effective scattering area η for $y = 1.9\ m$ (just on the right of the random medium), $\phi = 90$ deg and vertical polarization ($\varepsilon'_{r,1} = 4$, $\sigma_1 = 10^{-3}\ S/m$, $\varepsilon'_{r,2} = 3.4$ and $\sigma_2 = 5.9 \cdot 10^{-3}\ S/m$).

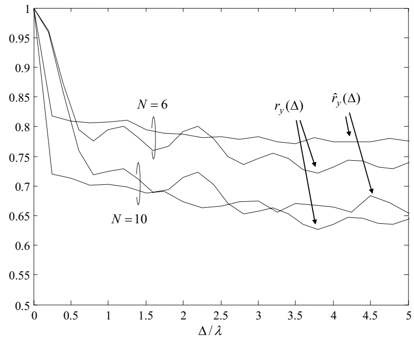

Figure 4: RT and ray tracing computed spatial correlation, $r_y(\Delta)$ and $\hat{r}_y(\Delta)$, versus Δ/λ for $y = 1.9$ m (on the right of the random medium) and different values of the density $N = 6, 10\ m^{-2}$ (vertical polarization and $\varepsilon'_{r,1} = 4$, $\sigma_1 = 10^{-3}\ S/m$).

for $y = 1.9\ m$ (on the right of the table) and two values of the density $N = 6, 10$.

5 CONCLUSION

In this paper, we investigated the limits of applicability of radiative transfer as a mean to study power fluctuations of an indoor propagation channel through numerical comparison with a ray tracing numerical calculation. We concluded that for scatterers with relatively low extinction cross-width or low density, radiative transfer provides a solution that closely matches with the results of second order statistics from ray tracing.

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