On the Sum-Rate of Broadcast Channels with Outdated 1-Bit Feedback

Bo Niu, Osvaldo Simeone, Oren Somekh, and Alexander M. Haimovich
Center for Wireless Communications and Signal Processing Research
Department of Electrical and Computer Engineering
New Jersey Institute of Technology
Newark, NJ 07102, USA
Email: {bo.niu, osvaldo.simeone, oren.s.somekh, alexander.m.haimovich}@njit.edu

***Abstract*— In this paper, a single-input single-output (SISO) downlink channel with K users is analyzed in the presence of Rayleigh flat fading. A limited channel state information (CSI) feedback scheme is considered, where only an *outdated* 1-bit feedback per user is available at the base station for each fading block. A closed-form expression for the achievable ergodic sum-rate of the 1-bit feedback scheme is presented for any number of users, as a function of the fading temporal correlation coefficient, the threshold of the 1-bit CSI quantizer and the SNR. It is proved that the sum-rate scales with increasing number of users as $\log\log K$, which is the same scaling law achieved by the optimal non-delayed full CSI feedback scheme. In addition, the sum-rate degradation due to outdated CSI is evaluated in the asymptotic regimes of either large K or low SNR.**

I. Introduction

Multiuser diversity capitalizes on independent fading channels across different users in order to enhance the throughput in the downlink/uplink of a cellular system. Serving the user with the best instantaneous channel quality, has been proved to be optimal in terms of ergodic sum-rate for both the uplink [1] and for the downlink [2]. However, it requires all users to feed back their instantaneous channel state information (CSI) to the transmitter. In [3], it is shown that, given this optimal scheduling, the ergodic sum-rate capacity of the downlink Rayleigh fading channel scales as $\log\log K$ with the number of users K.

There are two problems inherent in the optimal scheduling discussed above: 1) the large amount of required feedback, and 2) the feedback delay that may cause the CSI fed back to the base station to be outdated. In order to reduce the feedback load, various schemes have been proposed. A common approach prescribes feedback of a quantized version of the CSI [4]. Recently, a 1-bit feedback scheme was proposed (without considering feedback delay) in [5], and was further analyzed in [6]. According to this scheme, for each fading block, users with channel power exceeding a given predetermined threshold feed back the bit “1”, otherwise they indicate “0” to the base station. The base station randomly chooses one among the users with feedback bit “1” for data transmission with power P. When there is no user signaling a channel gain larger than the threshold, the transmitter keeps silent for one block period. It has been proved that the 1-bit feedback scheme suffers from a negligible loss of multiuser diversity gain as compared to the full CSI feedback scheme. In particular, the optimal scaling law of $\log\log K$ is preserved [6].

In a realistic situation, it is impossible for the scheduler at the base station to access the instantaneous (and possibly quantized) CSI of each user. In fact, channel feedback information may become outdated if the fading channel is changing rapidly. This leads to a degradation of the system sum-rate. For instance, in case of full CSI feedback, the “best” user may no longer be the “best” after a feedback delay. In [7], the impact of outdated CSI is studied for a selective feedback scheme where only the users with the best channel conditions (i.e. above a given threshold) feed back their full CSI. A closed-form expression for the sum-rate as a function of the fading temporal correlation ρ is therein derived.

In this paper, we focus on the study of the achievable ergodic sum-rate of broadcast channels with outdated 1-bit feedback per user per fading block. By outdated, we mean that a delay occurs between the time of the measurement of the channel at the user side and that of the scheduling at the base station. The system setup is similar to [5] and [6], except that we account for the CSI feedback delay. Previous results of [5], [6] are obtained as a special case of the current work, when the temporal channel correlation coefficient is $\rho = 1$. We first derive the achievable ergodic sum-rate for the 1-bit feedback scheme with delay (Sec. III). Next, by exploiting a lower bound of the rate, we show that with outdated 1-bit feedback per fading block for each user, the achievable sum-rate demonstrates the same growth rate of $\log\log K$ as the full CSI feedback scheme, with a carefully selected threshold. We also quantify the sum-rate degradation due to outdated CSI in the asymptotic regimes of either large K or small SNR [8].

II. System Model

We consider a single antenna base station that transmits to K single-antenna receivers in a broadcast channel. Users are assumed to be homogeneous and experience independent “block” Rayleigh flat fading. Accordingly, the fading processes are independent among different users, and the block duration is sufficiently small so as to guarantee that the fading gains remain constant during one block and change from block to

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block. A discrete time baseband representation of the channel is mathematically described as

$$y_k(t) = h_k(t)x(t) + n_k(t), \qquad k = 1, \ldots, K \tag{1}$$

where $h_k(t) \sim \mathcal{CN}(0,1)$ is the channel fading coefficient of user k, $n_k(t) \sim \mathcal{CN}(0,1)$ is complex Gaussian noise with unit variance and assumed statistically independent among different users.

We assume that each user is aware of its own fading power level $v_k^2(t) = |h_k(t)|^2$ based on a perfect channel estimation at time t, and compares it with a prescribed threshold α. If the received power $v_k^2(t)$ is larger than the threshold α, the user feeds back a single bit of "1" through a reliable uplink channel for the current fading block. Otherwise it feeds back a single bit of "0" for the current fading block. At time $t+\tau$, the base station receives all the feedback bits and randomly chooses one of the users with feedback bit "1" for transmission with power P. In case no user has fed back a "1" bit, the base station keeps silent for the current block. With the above scheduling mechanism, the transmitted signal x is a zero mean complex Gaussian random variable with power P ($E[|x|^2] = P$), when at least one user has fading power level larger than the threshold α; it reads $x = 0$ when all the feedback bits are zeros.

During the delay τ between perfect channel estimation and scheduling decision, the state of the channel chosen for transmission is subject to change. We denote as ρ the temporal channel correlation coefficient between the channels at time t and $t+\tau$. As an example, the temporal channel correlation ρ can be related to the delay τ through Jake's model [9] as $\rho = J_0(2\pi f_D|\tau|)$, where J_0 is the zero-order Bessel function of the first kind, and f_D is the Doppler Spread. In this paper, we assume that the transmitter has knowledge about ρ, through, e.g., estimation of the Doppler spread.

III. Achievable Sum-Rate of the 1-Bit Feedback Scheme

The main goal of this section is to derive the achievable ergodic sum-rate of the 1-bit feedback scheme described in Sec. II in presence of feedback delay.

Proposition 1 *The achievable ergodic sum-rate of the 1-bit feedback scheme with power P, K users, temporal channel correlation coefficient ρ, and arbitrary threshold α, is given by*[1]

$$\begin{aligned} R(\alpha,\rho,P) &= \left(1-\left(1-e^{-\alpha}\right)^K\right)\int_0^\infty \log(1+z^2P)2z \\ &\quad \times e^{-z^2+\alpha}Q_1\left(\frac{\sqrt{2}\rho}{\sqrt{1-\rho^2}}z, \frac{\sqrt{2\alpha}}{\sqrt{1-\rho^2}}\right)dz, \end{aligned} \tag{2}$$

where $Q_1(a,b) = \int_b^\infty x\exp\left(-\frac{x^2+a^2}{2}\right)I_0(ax)dx$ is the first-order Marcum-Q function.

[1]A natural logarithmic base is used throughout this paper.

Proof: According to [6], at any fading block, either one or no user is selected for transmission, and long codewords (spanning multiple fading blocks) are chosen from a Gaussian code book. The achievable ergodic sum-rate is the product of two terms: (*i*) the probability that at least one user is qualified to be chosen for transmission and (*ii*) the ergodic sum-rate for the chosen users over the fading blocks selected for transmission:

$$R(\alpha,\rho,P) = \Pr(N>0)E[\log(1+v_\tau^2P)|v^2>\alpha], \tag{3}$$

where we have dropped the subscript k due to the statistical equivalence of different users and denoted $v = v(t)$ and $v_\tau = v(t+\tau)$ (i.e., channel envelopes at the channel estimate and scheduling decision time instants, respectively).

The probability that at least one user is qualified to be chosen for transmission is

$$\begin{aligned} \Pr(N>0) &= 1 - Pr(N=0) = 1 - \left(Pr\left(v^2<\alpha\right)\right)^K \\ &= 1-\left(1-e^{-\alpha}\right)^K, \end{aligned} \tag{4}$$

where N is the number of users with channel power gain v^2 above the threshold α.

In order to calculate the ergodic sum-rate for the chosen user over the fading blocks, we need the probability density function (pdf) of v_τ given the condition $v^2 \geq \alpha$. We start from the cumulative distribution function (cdf) of v_τ given the condition $v^2 \geq \alpha$,

$$\begin{aligned} F_{v_\tau}(z|v^2\geq\alpha) &= \frac{Pr(v_\tau<z, v\geq\sqrt{\alpha})}{Pr(v\geq\sqrt{\alpha})} \\ &= \frac{\int_0^z dv_\tau\int_{\sqrt{\alpha}}^\infty f(v_\tau,v)dv}{\int_{\sqrt{\alpha}}^\infty 2ve^{-v^2}dv}, \end{aligned} \tag{5}$$

where $f(v_\tau,v) = \frac{4v_\tau v}{1-\rho^2}e^{-(v_\tau^2+v^2)/(1-\rho^2)}I_0(\frac{2\rho v_\tau v}{1-\rho^2})$ is the joint pdf of two correlated Rayleigh random variables [10]. By taking the derivative of (5) with respect to z, we achieve the conditional pdf as

$$\begin{aligned} f_{v_\tau}\left(z|v^2\geq\alpha\right) &= \frac{\int_{\sqrt{\alpha}}^\infty \frac{4zv}{1-\rho^2}e^{-(z^2+v^2)/(1-\rho^2)}I_0(\frac{2\rho zv}{1-\rho^2})dv}{e^{-\alpha}} \\ &= 2ze^{-z^2+\alpha}Q_1\left(\frac{\sqrt{2}\rho}{\sqrt{1-\rho^2}}z, \frac{\sqrt{2\alpha}}{\sqrt{1-\rho^2}}\right). \end{aligned} \tag{6}$$

Substituting (4) and (6) into (2) completes the proof ■

IV. Asymptotic Analysis

To gain insight into the impact of delay on the achievable ergodic sum-rate of the 1-bit feedback scheme, it is convenient to derive upper and lower bounds on (2). An upper bound of the rate is directly derived by using Jensen's inequality on (2),

$$R_{\text{up}}(\alpha,\rho,P) = \left(1-\left(1-e^{-\alpha}\right)^K\right)\log\left(1+P\left(1+\rho^2\alpha\right)\right). \tag{7}$$

On the other hand, a lower bound is obtained as (see Appendix A for derivation)

$$R_{\text{low}}(\alpha,\rho,P) = \left(1-\left(1-e^{-\alpha}\right)^K\right)\log\left(1+\alpha P\right) \left\{1+Q_1\left(\frac{\rho\sqrt{2\alpha}}{\sqrt{1-\rho^2}},\frac{\sqrt{2\alpha}}{\sqrt{1-\rho^2}}\right) -Q_1\left(\frac{\sqrt{2\alpha}}{\sqrt{1-\rho^2}},\frac{\rho\sqrt{2\alpha}}{\sqrt{1-\rho^2}}\right)\right\}. \quad (8)$$

Exploiting the lower bound (8), the scaling law of the 1-bit feedback scheme with respect to the number of users K can be found as follows.

Proposition 2 *For any finite power P and positive channel correlation coefficient $0<\rho\le 1$, with increasing number of users K, the 1-bit feedback scheme achieves the same growth rate as the full CSI feedback scheme*

$$\lim_{K\to\infty}\frac{R(\alpha_o(K),\rho,P)}{\log\log K}=1, \quad (9)$$

where $\alpha_o(K)$ is the optimal threshold that maximizes $R(\alpha,\rho,P)$ for given K.

Proof: The lower bound (8) suggests that, in order to get a multiuser diversity gain of $\mathcal{O}(\log K)$ and to make the pre-log term close to 1, a "good" choice of the threshold is $\alpha_{\text{so}}(K)=\log K-\delta$, where δ is a positive constant smaller than $\log K$. In fact, with this choice of threshold (we term it sub-optimal threshold), we have

$$\lim_{\substack{K\to\infty\\ \alpha=\alpha_{\text{so}}(K)}}\left(1-\left(1-e^{-\alpha}\right)^K\right)=1-e^{-e^{\delta}}, \quad (10)$$

and, as proved in Appendix B,

$$\lim_{\substack{K\to\infty\\ \alpha=\alpha_{\text{so}}(K)}} 1+Q_1\left(\frac{\rho\sqrt{2\alpha}}{\sqrt{1-\rho^2}},\frac{\sqrt{2\alpha}}{\sqrt{1-\rho^2}}\right) -Q_1\left(\frac{\sqrt{2\alpha}}{\sqrt{1-\rho^2}},\frac{\rho\sqrt{2\alpha}}{\sqrt{1-\rho^2}}\right)=1. \quad (11)$$

From (8), (10) and (11), it follows that

$$\lim_{K\to\infty}\frac{R_{\text{low}}(\alpha_{\text{so}}(K),\rho,P)}{\log\log K}=1-e^{-e^{\delta}}. \quad (12)$$

Since δ can be chosen any arbitrary large number (after taking K to infinity), the ratio in (12) goes to 1. Therefore, since a sub-optimal threshold preserves the scaling law of $\log\log K$, the 1-bit feedback scheme guarantees the same growth rate as the full CSI feedback scheme with an optimal threshold $\alpha_{\text{o}}(K)$, thus completing the proof. ∎

Another interesting asymptotic result comes from the upper bound on the achievable ergodic sum-rate of the 1-bit feedback scheme (7). With large number of users K and optimal threshold $\alpha_{\text{o}}(K)$, we have

$$\lim_{K\to\infty}R_{\text{up}}(\alpha_{\text{o}}(K),\rho,P) = \lim_{K\to\infty}\left(1-\left(1-e^{-\alpha_{\text{o}}(K)}\right)^K\right) \times\log\left(1+P\left(1+\rho^2\alpha_{\text{o}}(K)\right)\right). \quad (13)$$

Since it has been shown in the proof of Proposition 1 that there exists a suboptimal threshold $\alpha_{\text{so}}(K)$ such that $\lim_{K\to\infty}\left(1-\left(1-e^{-\alpha_{\text{so}}(K)}\right)^K\right)=1$, it is apparent that the same condition holds with an optimal threshold $\alpha_{\text{o}}(K)$. Therefore,

$$\lim_{K\to\infty}R_{\text{up}}(\alpha_{\text{o}}(K),\rho,P) \approx \log P\alpha_{\text{o}}(K)+2\log\rho. \quad (14)$$

The approximation comes from the fact that in order to get a multiuser diversity of $\log\log K$, the optimal threshold $\alpha_{\text{o}}(K)$ is of $\mathcal{O}(\log K)$, and $\log(1+x)\approx\log x$, for $x\gg 1$.

The first term in (14), $\log P\alpha_{\text{o}}(K)$, is the optimized asymptotic rate with large number of users for the 1-bit feedback scheme without delay [6]. Therefore, the second term $2\log\rho$ provides a bound on the sum-rate degradation due to feedback delay in the asymptotic regime of large K. In Sec. VI, it will be shown via numerical results that this bound is in fact an accurate prediction of the real sum-rate degradation for $K\gg 1$.

V. Low-SNR Characterization

In this section we study the sum-rate of broadcast channels with outdated 1-bit feedback and operating in a power-limited (or wideband) regime. This regime is characterized by low SNR and low spectral efficiency. In [8] it is shown that in order to characterize the spectral efficiency in the low SNR regime, two parameters should be considered: (*i*) the minimum signal energy-per-information bit $\frac{E_b}{N_0}_{\text{min}}$ required for reliable communication; (*ii*) the spectral efficiency slope S_0, also referred to as wideband slope, as a function of $\frac{E_b}{N_0}$, at $\frac{E_b}{N_0}_{\text{min}}$. Thus a linear approximation of the spectral efficiency versus $\frac{E_b}{N_0}$ in this regime is[2]

$$\mathrm{R}(\alpha,\rho,\frac{E_b}{N_0})\cong\frac{S_0}{3\text{dB}}\left(\left.\frac{E_b}{N_0}\right|_{\text{dB}}-\left.\frac{E_b}{N_0}_{\text{min}}\right|_{\text{dB}}\right). \quad (15)$$

In this section, we study the performance of the 1-bit feedback scheme in the wideband limit by deriving closed-form expressions for both $\frac{E_b}{N_0}_{\text{min}}$ and S_0. An asymptotic analysis of the two parameters with large number of users K and a sub-optimal threshold α_{so} is also presented.

From [8], the minimum energy per bit required for reliable communication $\frac{E_b}{N_0}_{\text{min}}$ depends on the first order derivative of the sum-rate with respect to the SNR P, evaluated at

[2]Following [8], the notation R is introduced so as to denote the sum-rate as a function of $\frac{E_b}{N_0}$. This notion is related to the sum-rate of R as a function of the SNR P, through $\mathrm{R}(\frac{E_b}{N_0})=R(P)$ and $P=R(P)\frac{E_b}{N_0}$.

$P = 0$. Using (2), and with the help of integration from [10, Eq.(B.28)], we have

$$\begin{aligned} \frac{E_b}{N_0}_{\min} &= \left. \frac{\log 2}{\frac{\partial R(\alpha,\rho,P)}{\partial P}} \right|_{P=0} \\ &= \frac{\log 2}{\left(1-(1-e^{-\alpha})^K\right)(1+\rho^2\alpha)}. \end{aligned} \tag{16}$$

The spectral efficiency slope S_0 is a function of both the first order and the second order derivatives of the sum-rate:

$$\begin{aligned} S_0 &= \left. \frac{2\left(\frac{\partial R(\alpha,\rho,P)}{\partial P}\right)^2}{-\frac{\partial^2 R(\alpha,\rho,P)}{\partial P^2}} \right|_{P=0} \\ &= \frac{\left(1-(1-e^{-\alpha})^K\right)(1+\rho^2\alpha)^2}{1+2\alpha\rho^2-\alpha\rho^4+\frac{1}{2}\alpha^2\rho^4}. \end{aligned} \tag{17}$$

Using the results in (16) (17), we can quantify the sum-rate degradation due to the feedback delay in the asymptotic regime of low SNR for any number of users by means of (15).

To further analyze the above results (16) (17), we consider the asymptotic scenario with large number of users K and a sub-optimal threshold $\alpha_{\text{so}}(K) = \log K - \delta$ (this sub-optimal threshold guarantees the asymptotic optimality of the scaling law, as proved in Proposition 2). From (16) and (17), we have for $\rho > 0$ and $K \gg 1$

$$\frac{E_b}{N_0}_{\min} \rightarrow \frac{\log 2}{\rho^2 \log K}, \tag{18}$$

and

$$S_0 \rightarrow 2. \tag{19}$$

Substituting these results in (15), we obtain a linear approximation of the sum-rate versus $\frac{E_b}{N_0}$ in the wideband regime with large number of users

$$\mathrm{R}(\alpha,\rho,\frac{E_b}{N_0}) \cong \frac{2}{3\text{dB}}\left(\left.\frac{E_b}{N_0}\right|_{\text{dB}} - 10\log_{10}\frac{\log 2}{\rho^2 \log K}\right). \tag{20}$$

It is known that $\frac{E_b}{N_0}_{\min}$ for reliable communication over a fading channel with no CSI at the transmit side is $\log 2 = -1.59$dB. With 1-bit CSI feedback, a large number of users K and a temporal channel correlation coefficient $\rho > 0$, the denominator in (18) shows a multiuser diversity gain of $\rho^2 \log K$. With increasing ρ, this leads to a decreasing required $\frac{E_b}{N_0}_{\min}$ for reliable communication. Regarding the spectral efficiency slope S_0, it can be obtained from (17) that it equals 1 when the temporal channel correlation coefficient is $\rho = 0$. This result coincides with the case of perfect receiver side information but with no CSI at the transmitter described in [8]. When $\rho > 0$, the asymptotic spectral efficiency slope (19) equals 2 as for an AWGN channel [8].

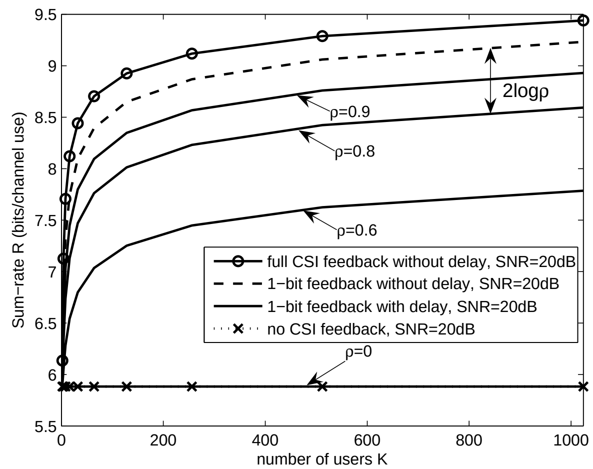

Fig. 1. Sum-rate R versus the number of users K for the 1-bit feedback scheme with different channel temporal correlation ρ and optimal threshold $\alpha_{\text{o}}(K)$. Sum-rate with non-delayed full CSI, no CSI, and 1-bit feedback without delay, are also shown for reference ($P = 20$dB).

VI. Numerical Results

Figure 1 shows the achievable ergodic sum-rate of the 1-bit feedback scheme versus the number of users K for different values of the channel temporal correlation coefficient ρ, with optimal threshold $\alpha_{\text{o}}(K)$ and SNR $P = 20$ dB. The ergodic sum-rate with non-delayed full CSI, no CSI, and 1-bit feedback without delay, are also shown for reference. It can be seen that the 1-bit feedback scheme shows the same scaling law of the sum-rate with large number of users, for different channel correlation coefficients, but suffers a rate degradation that is well quantified by $2\log\rho$ as derived in Sec. IV.

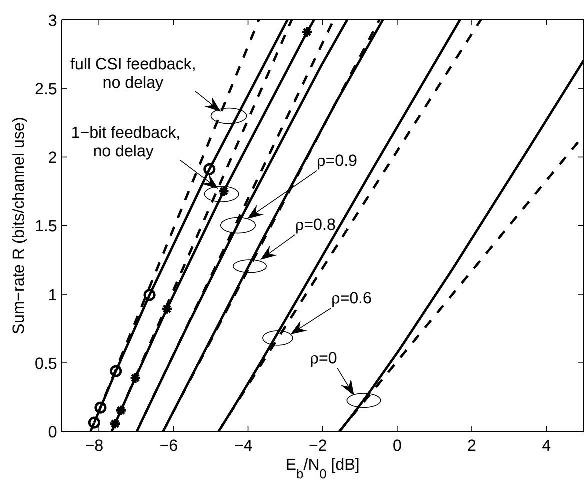

Fig. 2. Sum-rate of the 1-bit feedback scheme in low SNR regime (solid line) and its wideband approximation (15) for different channel temporal correlation ρ (dashed line). Sum-rate with non-delayed full CSI, no CSI, and 1-bit feedback without delay, are also shown for reference ($K = 100$).

Figure 2 shows the sum-rate of the 1-bit feedback scheme as a function of E_b/N_0, and its wideband approximation (15) according to (16) and (17), for different channel temporal correlation coefficients ρ, with optimal threshold $\alpha_o(K)$ and finite number of users $K = 100$. Spectral efficiencies and their linear approximations with non-delayed full CSI, no CSI, and 1-bit feedback without delay at low SNR regime, are also shown for reference. It is seen that there is a multiuser diversity gain in terms of $E_b/N_{0_{\min}}$ between the 1-bit feedback scheme with different temporal channel correlation coefficients and the no CSI feedback scheme. This can be quantified as $10\log_{10}(\rho^2 \log K)$dB when the number of users K is large (see (18)). The spectral efficiency slope increases from 1, which corresponds to the no CSI feedback case, to $S_0 = 2$, which equals to the spectral efficiency slope of a Gaussian channel [8].

VII. Concluding Remarks

In this work we have investigated the sum-rate of a SISO broadcast channel with 1-bit feedback in presence of feedback delays. A closed-form expression of the achievable ergodic sum-rate, which holds for any number of users, temporal channel correlation coefficient and threshold, has been derived, along with simple upper and lower bounds. We have also shown that, not only reducing the CSI feedback to 1 bit, but also considering the feedback delay, does not affect the scaling law of the sum-rate. Finally, the feedback delay yields a sum-rate degradation which has been quantified for both cases of large number of users and low SNR.

Appendix

A. Derivation of (8):

Starting from (2), since the integrand is positive, we replace the lower limit of integration with $\sqrt{\alpha}$, obtaining the following lower bound

$$\begin{aligned} R(\alpha,\rho,P) \;\geq\; & \left(1-(1-e^{-\alpha})^K\right)\int_{\sqrt{\alpha}}^{\infty}\log(1+z^2P)2z \\ & \times e^{-z^2+\alpha}Q_1\left(\frac{\sqrt{2}\rho}{\sqrt{1-\rho^2}}z, \frac{\sqrt{2\alpha}}{\sqrt{1-\rho^2}}\right)dz. \end{aligned} \tag{21}$$

Then, the integration variable z in the increasing function of log is replaced by the lower limit of the integration in (21), yielding the strict lower bound

$$\begin{aligned} R(\alpha,\rho,P) \;>\; & \left(1-(1-e^{-\alpha})^K\right)\log(1+\alpha P)\int_{\sqrt{\alpha}}^{\infty}2z \\ & \times e^{-z^2+\alpha}Q_1\left(\frac{\sqrt{2}\rho}{\sqrt{1-\rho^2}}z, \frac{\sqrt{2\alpha}}{\sqrt{1-\rho^2}}\right)dz \\ = \; & \left(1-\left(1-e^{-\alpha}\right)^K\right)\log\left(1+\alpha P\right) \\ & \left\{1+Q_1\left(\frac{\rho\sqrt{2\alpha}}{\sqrt{1-\rho^2}}, \frac{\sqrt{2\alpha}}{\sqrt{1-\rho^2}}\right)\right. \\ & \left. -Q_1\left(\frac{\sqrt{2\alpha}}{\sqrt{1-\rho^2}}, \frac{\rho\sqrt{2\alpha}}{\sqrt{1-\rho^2}}\right)\right\} \\ = \; & R_{\text{low}}(\alpha,\rho,P). \end{aligned} \tag{22}$$

The first equality in (22) follows from Eq.(B.18) of [10].

B. Proof of (11):

As $b \to \infty$, using the asymptotic form of the zero-order modified Bessel function of the first kind, $Q_1(a,b)$ can be approximated as [11, Eq.(A-27)]

$$\begin{aligned} Q_1(a,b) \;\cong\; & \int_b^{\infty} x\exp\left(-\frac{x^2+a^2}{2}\right)\frac{\exp(ax)}{\sqrt{2\pi a x}}dx \\ \cong\; & \sqrt{\frac{b}{a}}\frac{1}{\sqrt{2\pi}}\int_b^{\infty}\exp\left(-\frac{(x-a)^2}{2}\right)dx \\ =\; & \sqrt{\frac{b}{a}}\Phi(b-a) \\ \cong\; & (2\pi ab)^{-1/2}\exp\left(-\frac{(b-a)^2}{2}\right), \end{aligned} \tag{23}$$

where $\Phi(t) \equiv \int_{-\infty}^{t} dx(2\pi)^{-1/2}\exp(-x^2/2)$.

Therefore, plugging (23) in (11), we easily obtain the limit we set out to prove.

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