Low-SNR analysis of cellular systems with cooperative base stations and mobiles

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***Abstract*— In this paper, joint (cooperative) decoding at the base stations combined with collaborative transmission at the mobile terminals is investigated as a means to improve the uplink throughput of current cellular systems over fading channels. Intra-cell orthogonal medium access control and Decode-and-Forward collaborative transmission among terminals are assumed. Moreover, the cellular system is modelled according to a simplified framework introduced by Wyner. The focus of this work is on low-power transmission (or equivalently on the wideband regime), where the ergodic per-cell throughput can be described by the minimum energy per bit required for reliable communication and the slope of the spectral efficiency at low SNR. These two parameters are derived for different system configurations and, capitalizing on the analysis, the relative merits of both cooperation among base stations and among terminals are assessed.**

I. INTRODUCTION

In cellular mobile communications, achieving satisfactory coverage and quality of service through low power transmissions is a primary requirement on the uplink, due to the battery-powered transceivers employed by typical mobile terminals (MTs). Two solutions seem to be among the most viable and promising: 1) *collaborative transmission between MTs*: multihop transmission was proposed in [1] in the context of cellular systems so as to increase coverage and quality of service. Information theoretical analyses of the throughput of such hybrid networks [2] have recently been proposed in the limit of a large number of nodes, following the framework of [3]. More complex forms of node cooperation have been investigated extensively in a single-link or ad hoc scenario [4] [5] building on the classical relay channel [6]; 2) *cooperative (joint) decoding at the base stations* (BSs): allowing the BSs to jointly decode the received signals equivalently creates a distributed receiving antenna array [7]. Performance gain of this technology within a simplified cellular model was first studied in [8] [9], and then extended to fading channels by [10], under the assumption that BSs are connected by a backbone with high capacity and low latency. Practical decoding algorithm based on message-passing techniques that only assume local interactions between BS have been studied in, e.g., [11].

In this paper, we focus on assessing the relative merits of the two aforementioned technologies in the low-SNR regime. The scenarios where either of the two techniques is deployed and the case where a combination of both is in place are considered. We limit the analysis to the uplink of a cellular system that employs intracell orthogonal medium access control (i.e., TDMA, FDMA or orthogonal CDMA). Moreover, the cellular system is modelled according to the framework introduced in [8] and later adopted in a relevant number of publications [9]-[11]. Following to the linear variant of this model, as shown in fig. 1-(a), cells are arranged in a linear geometry and only adjacent cells interfere with each other. Moreover, intercell interference is described by a single parameter $\alpha \in [0, 1]$, that defines the gain experienced by signals travelling to interfered cells. Notwithstanding its simplicity, this model is able to capture the essential structure of a cellular system and it allows to get insight into the system performance. Finally, we constraint the scope of our work to a specific form of collaboration between terminals, namely the Decode-and-Forward (DF) protocol described in [4].

Performance comparison between different collaborative schemes is herein carried out by evaluating the per-cell achievable sum-rate (throughput) in the low-SNR regime. Accordingly, the throughput R of a given scheme is characterized by the minimum energy per bit required for reliable communication (normalized to the background noise level) $E_b/N_0|_{\min}$ and by the slope $\mathcal{S}_0$ at $E_b/N_0|_{\min}$ (measured in $bit/s/Hz/(3dB)$), following the low-SNR affine approximation [12]:

$$R \simeq \frac{\mathcal{S}_0}{3}\left(\frac{E_b}{N_0}[dB] - \left.\frac{E_b}{N_0}\right|_{\min}[dB]\right). \tag{1}$$

Throughout the paper, the low-SNR parameters $E_b/N_0|_{\min}$ and $\mathcal{S}_0$ are evaluated for different cooperative scenarios and, based on the analysis, the relative merits of both collaboration among base stations and among terminals are assessed. A similar analysis limited to a single-link relay channel has been recently reported in [13].

II. SYSTEM MODEL AND MAIN ASSUMPTIONS

The system layout is illustrated in fig. 1, where the upper part (a) refers to the scenario where no cooperation between MTs is allowed, and the lower part (b) sketches the case where transmission between an active MT and its BS takes place through DF cooperation by a relay MT. In each of the M cells, deployed according to a linear geometry, there is

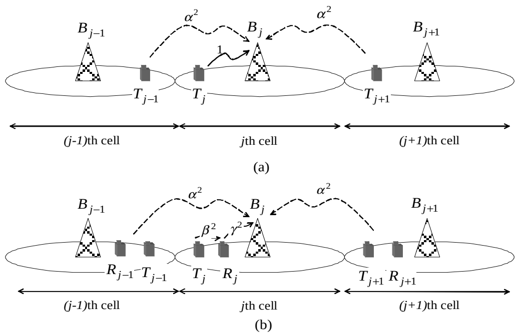

Fig. 1. (a) Linear variant of the Wyner's model of a cellular system [8]; (b) extended Wyner's model with cooperative transmission between MTs. B_j, T_j and R_j represent the BS, the active MT and the relay MT within the jth cell.

only one active source MT at each time, due to the intracell TDMA protocol considered in this paper. The BSs are denoted as $\{B_j\}_{j=1}^M$, the source MTs, one for each cell, as $\{T_j\}_{j=1}^M$, and the MT acting as relays are referred to as $\{R_j\}_{j=1}^M$. It is assumed that each active terminal T_j has available a relay terminal R_j for cooperation.

Fading gains are identified by their subscripts, e.g., $h_{T_jB_i}$ is the the channel between terminal T_j and BS B_i. These gains are assumed to be ergodic complex circularly symmetric Gaussian processes (Rayleigh fading). The average power received on different link is illustrated in fig. 1. In particular, the channel between active source MT T_j and the corresponding BS B_j has average power 1; the average channel gain power between source MT T_j and relay MT R_j is β^2 and between relay MT R_j and BS B_j is γ^2; the channel gains relative to the signal received by adjacent BS, B_{j-1} and B_{j+1}, from source MT T_j and relay MT R_j equal the Wyner's intercell factor α^2. Notice that it is assumed that a relay R_j receives with negligible power the signal transmitted by MTs T_{j+i}, $i=\pm 1$, belonging to adjacent cells. This assumption is reasonable if the relays are MTs, but it may be questionable if the relays are fixed wireless stations with antennas placed at heights comparable to the BSs. A more reasonable assumption in this case would be that of setting the average power of the channels between MTs T_{j+i}, $i=\pm 1$, and R_j equal to the intercell factor α^2. The analysis under this setting can be easily derived from the treatment presented below and it will not be further illustrated here for the sake of simplicity. Perfect channel state information is considered to be available at the receiver side, as detailed for different scenarios in the following Sections.

III. Non-cooperative scenario

As a reference, here we consider the scenario in fig. 1-(a), where direct transmission between MTs and BSs takes place and each BS independently processes the received signal (i.e., no collaboration between BSs is employed). The discrete-time baseband signal received in each time instant by the BS B_j ($j=1,...,M$) can be written as (discrete-time dependence is omitted for simplicity of notation)

$$y_j = h_{T_jB_j}x_j + w_j + n_j \tag{2}$$

with x_j denoting the signal transmitted by the MT T_j, that is assumed to be taken from a Gaussian codebook with $\mathrm{E}[|x_j|^2] = E_s$. The additive Gaussian thermal noise has power $\mathrm{E}[|n_j|^2] = N_0$. The remaining term $w_j = \alpha(h_{T_{j-1}B_j}x_{j-1} + h_{T_{j+1}B_j}x_{j+1})$ accounts for *intercell interference*. In single-cell processing, the interference w_j is regarded at the BS B_j as additive Gaussian noise with power: $\mathrm{E}[|w_j|^2] = \alpha^2 E_s(|h_{T_{j-1}B_j}|^2 + |h_{T_{j+1}B_j}|^2)$. Therefore, the compound additive Gaussian noise $w_j + n_j$ has power $\mathrm{E}[|w_j|^2] + N_0$. Since the BS is assumed to have knowledge of the channel gains $h_{T_{j+i}B_j}$ with $i=-1,0,1$, the ergodic per-cell achievable sum-rate (throughput) measured in $bit/s/Hz$ reads

$$R_{NC}(\mathrm{SNR},\alpha) = \mathrm{E}_h\left[\log_2\left(1 + \mathrm{SNR}\frac{|h_{T_jB_j}|^2}{1+W_j(\mathrm{SNR},\alpha)}\right)\right], \tag{3}$$

with $\mathrm{E}_h[\cdot]$ denoting the ensemble average with respect to the fading distribution, $\mathrm{SNR} = E_s/N_0$ the signal to noise ratio and

$$W_j(\mathrm{SNR},\alpha) = \frac{\mathrm{E}[|w_j|^2]}{N_0} = \alpha^2\mathrm{SNR}(|h_{T_{j-1}B_j}|^2 + |h_{T_{j+1}B_j}|^2), \tag{4}$$

where $\mathrm{E}[\cdot]$ denotes the average with respect to noise for fixed channel realization. Notice that (3) assumes that the channel coherence time is small enough so that the transmitted codeword spans a large (theoretically infinite) number of channel states (i.e., for delay tolerant applications or fast-varying channels).

A. Low-SNR analysis

Here we derive the two key performance measures in the low-power regime, namely the minimum energy per bit $E_b/N_0|_{\min}$ required for reliable communication and the slope $\mathcal{S}_0$ of the spectral efficiency at point $E_b/N_0|_{\min}$ (measured in $bit/s/Hz/(3dB)$). For reference, in case of a single-link Rayleigh fading channel, we have [12]:

$$\left.\frac{E_b}{N_0}\right|_{\min} = \log 2 = -1.59dB \tag{5a}$$

$$\mathcal{S}_0 = 1. \tag{5b}$$

In the case of no collaboration between either BSs or MTs, the low-SNR performance characterization is easily found to be:

$$\left.\frac{E_b}{N_0}\right|_{\min,NC} = \log 2 = -1.59dB \tag{6a}$$

$$\mathcal{S}_{0,NC} = \frac{1}{1+2\alpha^2}. \tag{6b}$$

Fig. 3 shows the exact per-cell achievable rate with no cooperation (3) and the affine low-SNR approximation obtained from the minimum energy per bit and slope (1). The intercell factor α is selected as $\alpha^2 = 0.5 = -3dB$. It is seen that the low-SNR approximation yields a fairly accurate prediction of the actual rate for spectral efficiencies less than $0.3bit/s/Hz$. Moreover, comparing (6) to the low-SNR performance of a single link fading channel (5), it can be concluded that intercell interference does not modify $E_b/N_0|_{\min}$ but only affects the slope. In particular, the slope $S_{0,NC}$ can be as low as $1/3$ when the intercell interference is maximum, i.e., for $\alpha = 1$. The performance with no collaboration (6) will be used in the next Section as a reference in order to assess the effects of cooperation.

IV. Cooperative decoding at the BSs and no collaboration between MTs

In this Section, we address again the scenario in fig. 1-(a) where the terminals do not employ cooperative transmission. However, differently form Sec. III, here the BSs are assumed to jointly decode the signals $\{x_j\}_{j=1}^M$ transmitted by all active terminals. Therefore, the contribution from the other cells to the signal received by each base station (2), accounted for by the term w_j, is now considered as useful signal instead of as an additional nuisance. Accordingly, by gathering the signals received by all M BSs (2) into the $M \times 1$ vector $\mathbf{y} = [y_1 \cdots y_M]^T$, the signal model becomes

$$\mathbf{y} = \mathbf{H}_{TB}\mathbf{x} + \mathbf{n}, \tag{7}$$

where the $M \times M$ channel matrix is

$$\mathbf{H}_{TB} = \begin{bmatrix} h_{T_1B_1} & \alpha h_{T_2B_1} & 0 & \cdots \\ \alpha h_{T_1B_2} & h_{T_2B_2} & \ddots & 0 \\ 0 & \ddots & \ddots & \alpha h_{T_{M-1}B_2} \\ \vdots & 0 & \alpha h_{T_{M-1}B_M} & h_{T_MB_M} \end{bmatrix}, \tag{8}$$

whereas the transmitted vector is $\mathbf{x} = [x_1 \cdots x_M]^T$ and the additive noise $\mathbf{n} = [x_1 \cdots x_M]^T$. Assuming the the hyper-receiver that performs joint decoding is aware of the realization of the channel matrix $\mathbf{H}_{TB}$, the per-cell achievable throughput of BS collaboration (BS) is then [10]

$$R_{BS}(\mathrm{SNR}, \alpha) = \frac{1}{M}\mathrm{E}_h\left[\log_2\left|\mathbf{I} + \mathrm{SNR}\mathbf{H}_{TB}\mathbf{H}_{TB}^H\right|\right]. \tag{9}$$

A. Low-SNR analysis

As proved in [14], for a sufficiently large number of BSs M the low-SNR characterization of the per-cell throughput of BS collaboration reads:

$$\left.\frac{E_b}{N_0}\right|_{\min,BS} = \frac{\log 2}{1+2\alpha^2} \tag{10a}$$

$$\mathcal{S}_{0,BS} = 1. \tag{10b}$$

The proof is omitted for lack of space and can be found in [14]. Fig. 3 includes the exact throughput (9) and the affine low-SNR approximation (1) for $\alpha^2 = -3dB$ and $M = 20$. It is seen that the approximation is fairly accurate for relatively large spectral efficiencies even for M as small as 20. Moreover, comparing (10) to the performance of no-cooperation (6), we can conclude that collaborative reception at the BSs is able to reduce the minimum energy per bit required for reliable communication by $1 + 2\alpha^2 \leq 3$, where the maximum gain of $3 = 4.77dB$ is achieved for $\alpha = 1$. This performance advantage can be interpreted as an array gain due to collaborative decoding at the BSs and is limited by the linear geometry of the Wyner's model. In the example in fig. 3, we have $E_b/N_0|_{\min,BS} = -4.59dB$, showing the expected gain of $3dB$ with respect to the non-cooperative case. Notice that BS cooperation also improves the slope by a factor of $1+2\alpha^2$ (that equals 2 in the example of fig. 3).

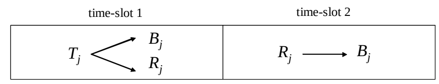

Fig. 2. Time-slot structure of the DF protocol.

V. Non-cooperative decoding at the BSs and DF collaboration between MTs

In this Section, the scenario in fig. 1-(b) is investigated where each active terminal (T_j) cooperates with a given relay terminal R_j in order to communicate with the BS B_j. Moreover, it is assumed, as in Sec. III, that decoding at each BS is independent, i.e., no collaboration among BSs occurs. Cooperation between terminals T_j and R_j is assumed to follow the DF protocol, that is illustrated in fig. 2. In the first time-slot, each active terminal T_j broadcasts to both relay MT R_j and BS B_j. The signal received by B_j is given by (2), whereas the relay nodes R_j receives $y_{R_j} = \beta h_{T_jR_j}x_j + n_{R_j}$, where the noise term n_{R_j} has power $\mathrm{E}[|n_{R_j}|^2] = N_0$. According to the DF protocol, the codeword transmitted by the source in the first slot must be decoded by the relay. Therefore, assuming that the relay R_j is aware of the realization of the channel gain $h_{T_jR_j}$, the rate is limited by

$$\begin{aligned} R_{MT}(\mathrm{SNR}, \alpha, \beta, \gamma) &\leq R_{relay}(\mathrm{SNR}, \beta) = \\ &= \frac{1}{2}\mathrm{E}_h[\log_2\left(1 + \mathrm{SNR}\beta^2|h_{T_jR_j}|^2\right)] \end{aligned} \tag{11}$$

The signal received by the BS B_j in the second time-slot is

$$y'_j = \gamma h_{R_jB_j}x_j + w'_j + n'_j, \tag{12}$$

with n'_j denoting thermal noise at B_j, assumed to be independent of the noise in the first time-slot and with power $\mathrm{E}[|n'_j|^2] = N_0$. The remaining term $w'_j = \alpha(h_{R_{j-1}B_j}x_{j-1} + h_{R_{j+1}B_j}x_{j+1})$ accounts for *intercell interference*. In single-cell processing, the interference w_j is regarded at the BS B_j as additive Gaussian noise with power $\mathrm{E}[|w'_j|^2]$:

$$W'_j(\mathrm{SNR}, \alpha) = \mathrm{E}[|w'_j|^2]/N_0 = \alpha^2\mathrm{SNR}(|h_{R_{j-1}B_j}|^2 + |h_{R_{j+1}B_j}|^2) \tag{13}$$

For given realization of the channels, the equivalent additive Gaussian noise at the BS in the two slots is correlated as (recall (2) and (12))

$$\begin{aligned} \rho(\mathrm{SNR}, \alpha) =& \mathrm{E}[(w_j + n_j)(w'_j + n'_j)^*]/N_0 = \\ =& \alpha^2\mathrm{SNR}(h_{T_{j-1}B_j}h^*_{R_{j-1}B_j} + h_{T_{j+1}B_j}h^*_{R_{j+1}B_j}). \end{aligned} \tag{14}$$

Since the BS B_j has full channel state information (i.e., knowledge of channel gains $h_{T_{j+i}B_j}$ and $h_{R_{j+i}B_j}$ for $i = -1, 0, 1$) and decodes the signal x_j based on both the received signal in the first y_j and in the second time slot y'_j, it follows from (2) and (12) that the resulting ergodic per-cell achievable rate is limited by the inequality $R_{MT}(\mathrm{SNR}, \alpha, \beta, \gamma) \leq R_d(\mathrm{SNR}, \alpha, \gamma)$, where

$$R_d(\mathrm{SNR}, \alpha, \gamma) = \frac{1}{2}\mathrm{E}_h \left[\log_2 \left(1 + \mathrm{SNR} \left[\begin{array}{cc} h^*_{T_jB_j} & \gamma h^*_{R_jB_j} \end{array}\right] \cdot \cdot \mathbf{Q}(\mathrm{SNR},\alpha)^{-1} \left[\begin{array}{c} h_{T_jB_j} \\ \gamma h_{R_jB_j} \end{array}\right]\right)\right], \quad (15)$$

with

$$\mathbf{Q}(\mathrm{SNR},\alpha) = \left[\begin{array}{cc} 1 + W_j(\mathrm{SNR}, \alpha) & \rho(\mathrm{SNR}, \alpha) \\ \rho^*(\mathrm{SNR}, \alpha) & 1 + W'_j(\mathrm{SNR}, \alpha) \end{array}\right]. \quad (16)$$

From (11) and (15), we finally get the ergodic per-cell achievable sum-rate:

$$R_{MT}(\mathrm{SNR}, \alpha, \beta, \gamma) = \min\{R_{relay}(\mathrm{SNR}, \beta), R_d(\mathrm{SNR}, \alpha, \gamma)\}. \quad (17)$$

A. Low-SNR analysis

As proved in [14], for the case at hand where the terminals transmit with the aid of a relay through DF and the BSs do not cooperate, the low-SNR parameters read

$$\left.\frac{E_b}{N_0}\right|_{\min, MT} = \max\left\{\frac{2\log 2}{\beta^2}, \frac{2\log 2}{1+\gamma^2}\right\} \quad (18)$$

$$\mathcal{S}_{0,MT} = \frac{1}{2}\min\left\{1, \frac{1 + 2\gamma^2 + \gamma^4}{2 + \gamma^2 + \gamma^4 + 6\alpha^2(1+\gamma^2)}\right\}. \quad (19)$$

In fig. 3 the low-SNR approximation (1) is again compared with the exact throughput (17) for $\alpha^2 = -3dB$, $\beta^2 = 20dB$, $\gamma^2 = 10dB$, showing that the approximation holds for spectral efficiencies as large as $0.4bit/s/Hz$. From inspection of (18), it is clear that, if the average channel gains between relay R_j and both active terminal T_j and BS B_j are larger than the average channel gain of the direct link between T_j and B_j, or more precisely if $\beta^2 > 2$ and $\gamma^2 > 1$, then relevant gains in terms of minimum energy per bit can be obtained. On the other hand if $\beta^2 \leq 2$ or $\gamma^2 \leq 1$, cooperation between terminals yields a power loss as compared to the non-cooperative case. For instance, the example in fig. 3 shows a gain of $\min(\beta^2/2, (1+\gamma^2)/2) = 5.5 = 7.4dB$ over the non cooperative case, i.e., $E_b/N_0|_{\min,MT} = -9dB$. On the other hand, the slope $\mathcal{S}_{0,MT}$ is at most $1/2$ (for the example $\mathcal{S}_{0,MT} = 0.4172$). This reduction in the low-SNR slope is immaterial if $E_b/N_0|_{\min,MT}$ is sufficiently small as for the case in fig. 3. In Sec. VII, further comments on (17) are provided based on a simple distance-based geometric model for the channel gains β^2 and γ^2.

VI. Cooperative decoding at the BSs and DF collaboration between MTs

Here we focus again on the scenario in fig. 1-(b), where each terminal employs DF collaboration with a given in-cell relay in order to communicate with its BS. However, differently from the previous Section, the BSs are herein assumed to be able to jointly decode the received signals in order to detect the transmitted vector $\mathbf{x} = [x_1 \cdots x_M]^T$. Therefore, both collaboration between BSs and MTs is considered in this Section. Due to the DF protocol, the per-cell achievable sum-rate is limited by the maximum rate at which the relay is able to correctly decode the transmitted signal, i.e., (recall (11))

$$\begin{aligned} R_{BS+MT}(\mathrm{SNR}, \alpha, \beta, \gamma) &\leq R_{relay}(\mathrm{SNR}, \beta) \quad (20) \\ &= \frac{1}{2}\mathrm{E}_h[\log_2\left(1 + \mathrm{SNR}\beta^2|h_{T_jR_j}|^2\right)]. \end{aligned}$$

In the second time-slot, the signal received by the BS is (12), that, similarly to (7) can be expressed according to a matricial formulation by defining the $M \times 1$ vector $\mathbf{y}' = [y'_1 \cdots y'_M]^T$:

$$\mathbf{y}' = \mathbf{H}_{RB}\mathbf{x} + \mathbf{n}', \quad (21)$$

where the $M \times M$ tridiagonal channel matrix reads

$$\mathbf{H}_{RB} = \left[\begin{array}{cccc} \gamma h_{R_1B_1} & \alpha h_{R_2B_1} & 0 & \cdots \\ \alpha h_{R_1B_2} & \gamma h_{R_2B_2} & \ddots & 0 \\ 0 & \ddots & \ddots & \alpha h_{R_MB_{M-1}} \\ \vdots & 0 & \alpha h_{R_{M-1}B_M} & \gamma h_{R_MB_M} \end{array}\right], \quad (22)$$

and $\mathbf{n}' = [n_1 \cdots n_M]^T$. Recalling that the BSs jointly decode the transmitted signal vector $\mathbf{x}$ based on both the signal received in the first (7) and in the second (21) time-slot and that full channel state information is assumed at the hyper-receiver (i.e., knowledge of matrices $\mathbf{H}_{TB}$ and $\mathbf{H}_{RB}$), the achievable per-cell throughput has to satisfy the inequality $R_{BS+MT}(\mathrm{SNR}, \alpha, \beta, \gamma) \leq R_m(\mathrm{SNR}, \alpha, \gamma)$

$$\begin{aligned} R_m(\mathrm{SNR}, \alpha, \gamma) &= \frac{1}{2M}\mathrm{E}_h[\log_2|\mathbf{I} + \mathrm{SNR}(\mathbf{H}_{TB}\mathbf{H}_{TB}^H \\ &\quad + \mathbf{H}_{RB}\mathbf{H}_{RB}^H)|] \quad (23) \end{aligned}$$

Then, combining (20) and (23), we finally get

$$\begin{aligned} R_{BS+MT}(\mathrm{SNR}, \alpha, \beta, \gamma) &= \min\{R_{relay}(\mathrm{SNR}, \beta), \\ &\quad R_m(\mathrm{SNR}, \alpha, \gamma)\} \quad (24) \end{aligned}$$

A. Low-SNR analysis

The low-SNR characterization of cooperation between both BSs and MTs reads for M large enough (see [14] for proof):

$$\left.\frac{E_b}{N_0}\right|_{\min, BS+MT} = \max\left\{\frac{2\log 2}{\beta^2}, \frac{2\log 2}{1+\gamma^2+4\alpha^2}\right\} \quad (25)$$

$$\mathcal{S}_0 = \frac{1}{2}\min\left\{1, \frac{(1+4\alpha^2+\gamma^2)^2}{2(8\alpha^4 + 4\alpha^2(1+\gamma^2) + 1 + \gamma^4)}\right\}. \quad (26)$$

Comparison between the actual throughput (24) and the affine low-SNR approximation is shown in fig. 3 for $\alpha^2 = -3dB$,

$\beta^2 = 20dB$, $\gamma^2 = 10dB$ and $M = 20$. The affine approximation is valid for spectral efficiencies smaller than $0.1bit/s/Hz$ and for M as small as 20. From (25) and (18), BS collaboration prove to be beneficial in a system that employs DF cooperation at the terminals only if $\beta^2 > 1+\gamma^2$ and in this case the energy gain is easily quantified as $\min\{(1+\gamma^2+4\alpha^2)/(1+\gamma^2), \beta^2/(1+\gamma^2)\}$ (equal to $0.72dB$ in the example). We remark that this problem could be alleviated by implementing the selective DF protocol proposed in [4], wherein if the channel gain between active terminal and relay falls between a given threshold then direct transmission is employed In Sec. VII, further comments on (25)-(26) are provided using a simple distance-based geometric model for the channel gains β^2 and γ^2.

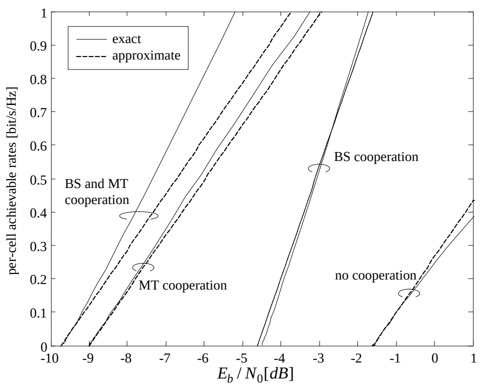

Fig. 3. Exact per-cell achievable rates and low-SNR approximations (1) of different schemes with or without cooperation between either BSs or MTs versus E_b/N_0 ($\alpha^2 = -3dB$, $\beta^2 = 20dB$, $\gamma^2 = 10dB$).

VII. Performance comparison with a simple geometric model

In order to get a better insight into the performance of scenarios where collaboration between MTs is allowed, here we specialize the results of the previous Sections to a simple geometric model. The relay station R_j is assumed to be on a line that connects the active MT T_j to the BS B_j at a normalized distance from T_j equal to $0 \leq d \leq 1$, where $1-d$ is the normalized distance of R_j to the BS B_j. The average channel gains between active terminals T_j and corresponding relays R_j, namely β^2, and between relay terminals R_j and relative BSs B_j, namely γ^2, are defined by d and by the path loss exponent P (integer $P > 1$) as $\beta^2 = 1/d^P$ and $\gamma^2 = 1/(1-d)^P$. Fig. 4 shows the minimum energy per bit $E_b/N_0|_{\min}$ for $P = 4$ and $\alpha^2 = -3dB$. The set of distances where MT collaboration is advantageous over the non-cooperative scenario excludes only the cases where the relay is close to the BS. Moreover, as stated in Sec. VI-A, the gains from adding BS cooperation on top of MT collaboration are limited to scenarios where the channel gain from the active terminal to the relay is good enough, i.e., to small d. Further analysis on this geometric model, including optimal placement of relay MTs, can be found in [14].

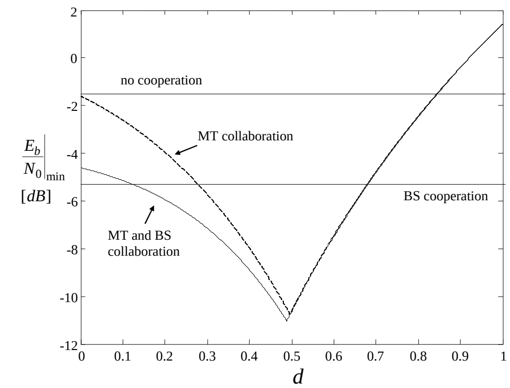

Fig. 4. Minimum energy per bit $E_b/N0|_{\min}$ versus distance d for path loss exponent $P = 2, 4$ ($\alpha^2 = -3dB$).

VIII. Conclusion

In this paper, base station and mobile cooperation have been investigated as means to improve the uplink per-cell throughput of low-power cellular systems over fading channels.

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