

Cognitive Transmissions under a Primary ARQ process via Backward Interference Cancellation

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Abstract—In this work, we address the problem of the coexistence of Primary and Secondary Users (PU and SU, respectively) in a wireless network, where the PU employs a retransmission based error control technique (ARQ). This mechanism offers the SU a non trivial opportunity: by decoding the Primary Message (PM), the Secondary Receiver (SR) can perform interference cancellation during the whole primary ARQ window, thus enhancing its own outage performance. In particular, we investigate a *Backward Interference Cancellation (BIC)* mechanism: the SR buffers the secondary transmissions that underwent outage due to primary interference, and attempts to recover them once the knowledge about the PM becomes available due to decoding operation in a future instant. We present analytical results for the scenario where the primary ARQ process is limited to one retransmission, and show by numerical results the throughput benefit of BIC, over other techniques investigated in the literature.

Index Terms—Cognitive radios, dynamic resource allocation, Markov decision processes, ARQ, backward interference cancellation

I. INTRODUCTION

Cognitive Radio (CR) [1] is emerging as a compelling technology for next generation wireless networks. By sensing the environment and collecting side-information about the activity of existing users in the network, CRs are able to adapt their operation to the current state of the system. In particular, this feature makes CR a promising technology for Primary (PU, licensed) and Secondary Users (SU, unlicensed with CR capabilities) coexistence. In fact, by sensing PUs' activity, SUs are able to adapt their operation, so as to limit harmful interference which

would excessively impair PUs' performance. This makes it possible to significantly improve the spectral efficiency of today's wireless networks [2]–[4].

The potential of CRs to improve network performance has been extensively researched in the literature. For a survey on cognitive networks, dynamic spectrum access and the related research challenges, we refer the interested reader to [5]–[7].

In this work, we consider a scenario where the PU uses ARQ to improve system performance. In [8], a cognitive spectrum access scheme based on spectrum sensing and overhearing the acknowledgement information from the PU was designed. Herein, as in [9], [10], we follow a different approach and investigate techniques for exploiting the structure of the primary transmission process, as induced by the use of ARQ. In particular, the ARQ mechanism introduces redundancy in the system, in the form of copies of the same message transmitted in subsequent time slots. The idea of exploiting this redundancy was first investigated in [11], [12], for a scenario where the ARQ mechanism is limited to one retransmission. In [11], several protocols are proposed, where the secondary transmitter collects side-information about the Primary Message (PM) in the first primary transmission, which is exploited to relay the PM, if a retransmission occurs.

In [10], we investigated a scenario where, once the knowledge about the PM becomes available at the Secondary Receiver (SR), it is exploited in the next ARQ rounds to perform interference cancellation, and we characterized the optimal access strategy of the SU under a constraint on the performance loss induced at the PU. In this paper, we use a similar setting. However, we introduce an important new mechanism: the SR buffers the secondary transmissions that underwent outage due to primary interference, and attempts to recover them once the knowledge about the PM becomes available due to decoding operation in a future instant. This new mechanism is termed *Backward Interference Cancellation (BIC)*. Moreover, we characterize the optimal

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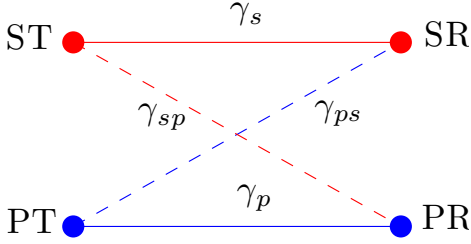


Figure 1: System model

secondary access strategy for the case where the primary ARQ mechanism is limited to one retransmission. This simple case gives us some fundamental insight on the general solution, which is confirmed by simulation.

The paper is organized as follows. Section II presents the system model. Section III introduces the performance metrics and the problem statement. Section IV presents the main results, for the case where the primary ARQ scheme is limited to one retransmission. Section V presents the numerical results. Section VI discusses some future work, and Section VII concludes the paper.

II. SYSTEM MODEL

As in [10], we consider a two-user interference network, depicted in Fig. 1. A Primary and a Secondary Transmitters (PT and ST, respectively) communicate to their respective Receivers (PR and SR) over the channels with instantaneous gains γ_p and γ_s , and generate mutual interference over the channels with gains γ_{ps} and γ_{sp} , respectively. The channel gains are modeled as quasi-static, i.i.d. processes over time. No Channel State Information (CSI) is available at the transmitter sides, hence transmissions may suffer from outage when the selected rate is not supported by the current channel quality.

A. PU operation

In order to improve reliability, the PU uses a retransmission-based error control technique (ARQ), with a maximum number of transmissions of the same packet equal to $T \geq 1$. We define the *ARQ state* $t \in \mathbb{N}(1, T)^1$ as the number of ARQ transmissions performed on the current PM (e.g., $t=1$ for a new PU transmission, $t=2$ for the first retransmission, and so on).

The SU activity affects the PU outage performance by interfering over the channel γ_{sp} . When the ST is silent, the PU outage probability is given by

$$\rho_{p0}(R_p) = \Pr \left(R_p > C(\gamma_p P_p) \right), \quad (1)$$

¹We define $\mathbb{N}(n_0, n_1) = \{t \in \mathbb{N}, n_0 \leq t \leq n_1\}$ for $n_0 \leq n_1 \in \mathbb{N}$

where R_p denotes the primary transmission rate, measured in bits/s/Hz, $C(x) = \log_2(1+x)$ represents the capacity of the Gaussian channel with SNR x at the receiver, P_p is the primary transmission power, and we assume unit variance Gaussian noise at the receivers.

On the other hand, when the ST transmits, the outage probability is given by

$$\rho_{p1}(R_p) = \Pr \left(R_p > C \left(\frac{\gamma_p P_p}{1 + \gamma_{sp} P_s} \right) \right) > \rho_{p0}(R_p), \quad (2)$$

where P_s is the secondary transmission power, and we assume that the PU is oblivious to the SU and treats secondary transmissions as noise.

B. SU operation

As in [10], we assume that the SU does not employ ARQ to recover from transmission failure. However, the SR, which is assumed to have perfect knowledge of the PU parameters, e.g., T and the codebook used, attempts to decode the PM. Diverse opportunities to exploit this knowledge arise. In fact, due to the inherent redundancy in the primary ARQ process, knowledge of the PM can be exploited in the subsequent primary ARQ rounds to achieve a larger secondary throughput via primary interference cancellation, as in [10]. Moreover, unlike [10] where the Secondary Message (SM) is dropped in case of transmission failure, we explore a mechanism where the SR buffers the secondary transmissions that underwent outage due to primary interference. These may then be recovered via BIC, should the PM become available at the SR in the following ARQ rounds.

In the following, we let $\phi \in \{0, 1\}$ be the *SR state variable*, where $\phi = 1$ if the SR knows the PM, and $\phi = 0$ otherwise, and $b \in \mathbb{N}(0, t-1)$ be the *buffer state variable*, which represents the number of secondary transmissions buffered at the t th ARQ round. For simplicity, we assume that the SU can perfectly track the current values of t , ϕ and b . We now analyze the SU outage performance for $\phi \in \{0, 1\}$.

1) PM unknown to the SR ($\phi = 0$):

When $\phi = 0$, transmissions are performed with power P_s and rate R_{s0} (bits/s/Hz). We define $\alpha_{s0}(R_p)$ and $\alpha_{s1}(R_{s0}, R_p) < \alpha_{s0}(R_p)$ as the probability of successfully decoding the PM at the SR, when the ST is silent and transmits, respectively. Although a control policy which regulates the decodability of the PM at the SR by continuously varying the rate R_{s0} and power P_s might be devised, in this work we use a binary control strategy, i.e., either a fixed secondary rate/power pair R_{s0}, P_s is employed, or no transmission is made at all.

We let $\rho_{s0}(R_{s0}, R_p)$ be the secondary outage probability under primary interference. The accrued throughput is given by $T_{s0}(R_{s0}, R_p) = R_{s0}(1 - \rho_{s0}(R_{s0}, R_p))$.

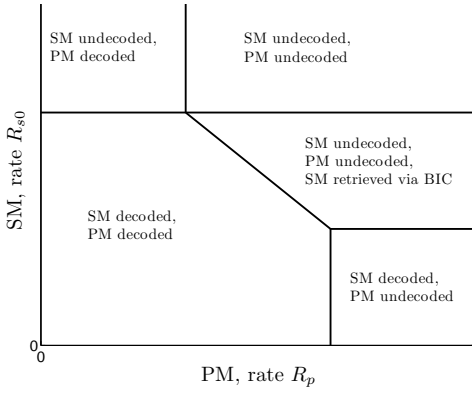


Figure 2: Capacity region of MAC channel, and decodability regions for PM (rate R_p) and SM (rate R_{s0})

The probability $\rho_{s0}(R_{s0}, R_p)$ ($\alpha_{s1}(R_{s0}, R_p)$) is calculated by treating the received signal at the SR as coming through a two-user Multiple Access Channel (MAC) [13], where the SR is interested only in decoding the SM (PM), by exploiting the codebook structure of the PM (SM) [14]. Let $\text{MAC}_s(R_{s0}, R_p)$ and $\text{MAC}_p(R_{s0}, R_p)$ be the set of channel gain pairs (γ_s, γ_{ps}) supporting the rate pair (R_{s0}, R_p) of the two-user MAC channel, when we are interested in decoding only the SM or PM, respectively. We have

$$\begin{cases} \rho_{s0}(R_{s0}, R_p) &= \Pr[(\gamma_s, \gamma_{ps}) \notin \text{MAC}_s(R_{s0}, R_p)] \\ \alpha_{s1}(R_{s0}, R_p) &= \Pr[(\gamma_s, \gamma_{ps}) \in \text{MAC}_p(R_{s0}, R_p)]. \end{cases}$$

Notice that the SR can acquire CSI whenever the links PT-SR and ST-SR are active. Assuming this can be performed without estimation errors (we refer to Section VI for a discussion on the imperfect CSI case), the SR can predict whether a failed transmission can be recovered via BIC: if both the SM and the PM undergo outage at the SR, i.e., $(\gamma_s, \gamma_{ps}) \notin \text{MAC}_s(R_{s0}, R_p) \cup \text{MAC}_p(R_{s0}, R_p)$, but the SM can be decoded after cancelling interference from the PM, i.e., $(\gamma_s, \gamma_{ps}) \in \text{MAC}_s(R_{s0}, 0)$, then the SR buffers the secondary transmission. In fact, since $(\gamma_s, \gamma_{ps}) \in \text{MAC}_s(R_{s0}, 0)$, its recovery via BIC is guaranteed, if the PM becomes available in a future slot. We define the probability of buffering the secondary transmission as

$$\begin{aligned} \omega_s(R_{s0}, R_p) &= \Pr[(\gamma_s, \gamma_{ps}) \in \text{MAC}_s(R_{s0}, 0) \cap \\ &\quad \text{MAC}_s(R_{s0}, R_p)^c \cap \text{MAC}_p(R_{s0}, R_p)^c] \\ &= \rho_{s0}(R_{s0}, R_p) - \rho_{s0}(R_{s0}, 0) > 0, \end{aligned} \quad (3)$$

where \mathcal{Q}^c is the complementary set of \mathcal{Q} . The decodability regions for the PM/SM are depicted in Fig. 2.

2) PM known to the SR ($\phi = 1$):

When $\phi = 1$, secondary transmissions are performed with power P_s and rate R_{s1} . The accrued average throughput is given by $T_{s1}(R_{s1}) = R_{s1}(1 - \rho_{s1}(R_{s1}))$, where $\rho_{s1}(R_{s1}) = \Pr(R_{s1} > C(\gamma_s))$ is the outage probability. Since the choice of R_{s1} does not affect the outage behavior at the PR (2) and the evolution of the ARQ process, we assume that the transmission rate R_{s1} maximizes $T_{s1}(R_{s1})$, and therefore

$$T_{s1}(R_{s1}) \geq T_{s1}(R_{s0}) > T_{s0}(R_{s0}, R_p). \quad (4)$$

It can also be shown that

$$T_{s1}(R_{s0}) = T_{s0}(R_{s0}, R_p) + \omega_s(R_{s0}, R_p)R_{s0}. \quad (5)$$

Notice that the same argument cannot be applied to R_{s0} , since its value reflects a trade-off between helping the SR to decode the PM, maximizing the throughput $T_{s0}(R_{s0}, R_p)$, and maximizing the probability of recovering a failed secondary transmission via BIC.

III. PERFORMANCE METRICS AND PROBLEM STATEMENT

The ST follows a generic past-dependent policy μ , taking actions in the set $\mathcal{A} = \{0, 1\}$, which correspond to the ST staying silent (0) or transmitting (1), respectively.

We define the long-term secondary throughput induced by policy μ as

$$\begin{aligned} \mathcal{T}_s(\mu) &= \liminf_{N \rightarrow +\infty} \frac{1}{N} \mathbb{E} \left[\sum_{n=0}^{N-1} \mathbf{1}(\Psi_S^n(\mu)) R_{s\phi_n} \right. \\ &\quad \left. + \sum_{n=0}^{N-1} \mathbf{1}(\Lambda_S^n(\mu)) b_n R_{s0} \right], \end{aligned} \quad (6)$$

where $\mathbf{1}(\cdot)$ is the indicator function, $\Psi_S^n(\mu)$ and $\Lambda_S^n(\mu)$ are the events corresponding to the SR successfully decoding the SM and the PM, respectively, ϕ_n is the SR variable and b_n is the *buffer state* variable in time slot n , as defined in Section II-B. A similar expression holds for the long-term primary throughput $\mathcal{T}_p(\mu)$ and secondary power $\mathcal{P}_s(\mu)$, for a proper choice of the events.

In this work, we study the problem

$$\mu^* \left(\mathcal{T}_p^{(th)}, \mathcal{P}_s^{(th)} \right) = \arg \max_{\mu} \mathcal{T}_s(\mu) \quad (7)$$

$$\text{subject to } \begin{cases} \mathcal{T}_p(\mu) \geq \mathcal{T}_p^{(th)} \\ \mathcal{P}_s(\mu) \leq \mathcal{P}_s^{(th)}, \end{cases}$$

where $\mathcal{T}_p^{(th)} \in [R_p(1 - \rho_{p1}), R_p(1 - \rho_{p0})]$ and $\mathcal{P}_s^{(th)} \in [0, P_s]$ represent the primary throughput and the secondary power constraints, respectively.

We remark that, although the setup is very similar to [10], the different assumption on the SU behavior

(BIC mechanism) results in a richer interaction between SU and PU than in [10], and in a different solution to (7). In fact, on the one hand, the SU is incentivized to transmit, not only to accrue secondary throughput but also to optimize the buffer occupancy and enable BIC at the SR. On the other hand, secondary transmissions diminish the allowed margin on the interference to the PU, and on the secondary power budget, and preclude the ability of the SR to decode the PM and exploit it to perform interference cancellation (unlike [10], where this knowledge was exploited only in the subsequent ARQ rounds, here we have the additional BIC mechanism).

We define the state space of the network as

$$\mathcal{S} \equiv \{(t, b, \phi), t \in \mathbb{N}(1, T), b \in \mathbb{N}(0, t-1), \phi = 0\} \cup \{(t, b, \phi), t \in \mathbb{N}(2, T), b = 0, \phi = 1\}. \quad (8)$$

The network is in state (t, b, ϕ) when the PU is in ARQ state t , the SR in state ϕ and the SR buffer in state b .

In this work, we consider only the class of randomized stationary policies $\mathcal{U} = \{\mu : \mathcal{S} \mapsto [0, 1]\}$, since they are optimal for (7) [15]. Therefore, $\mu(\mathbf{s})$ represents the transmission probability in state $\mathbf{s} \in \mathcal{S}$. Under $\mu \in \mathcal{U}$, the state of the network is modeled as a Homogeneous Markov Process $\{\mathbf{s}_n, n = 0, \dots, +\infty, \mathbf{s}_n \in \mathcal{S}\}$.

For the sake of notational convenience, we omit the dependence of the parameters defined above on the rates R_p, R_{s0}, R_{s1} , as long as this does not lead to confusion.

IV. MAIN RESULTS

Let $\pi_\mu : \mathcal{S} \mapsto [0, 1]$ be the steady state distribution [16] of the network under a stationary policy $\mu \in \mathcal{U}$, *i.e.*, $\pi_\mu(\mathbf{s})$ represents the long term fraction of the time slots that the network spends in state \mathbf{s} . The average long-term cost/reward under μ , $\mathcal{C}(\mu)$, is then given by $\mathcal{C}(\mu) = \sum_{\mathbf{s} \in \mathcal{S}} \pi_\mu(\mathbf{s}) c(\mathbf{s})$, *i.e.*, by weighting the cost/reward associated to each state, $c(\mathbf{s})$, by its steady state probability $\pi_\mu(\mathbf{s})$. From (6), $\mathcal{T}_s(\mu)$ is then given by

$$\mathcal{T}_s(\mu) = \sum_{\mathbf{s}=(t,b,\phi) \in \mathcal{S}} \pi_\mu(\mathbf{s}) \mu(\mathbf{s}) T_{s\phi} + \sum_{\mathbf{s}=(t,b,0) \in \mathcal{S}} \pi_\mu(\mathbf{s}) [\mu(\mathbf{s}) \alpha_{s1} + (1 - \mu(\mathbf{s})) \alpha_{s0}] b R_{s0}. \quad (9)$$

Notice that the expected secondary throughput reward in state $\mathbf{s} = (t, b, 0)$ accounts for the throughput accrued from both the current transmission ($\mu(\mathbf{s}) T_{s0}$), and the b buffered transmissions, recovered via BIC with probability $\mu(\mathbf{s}) \alpha_{s1} + (1 - \mu(\mathbf{s})) \alpha_{s0}$.

We define the *secondary access rate* under μ as

$$\mathcal{W}_s(\mu) = \sum_{\mathbf{s} \in \mathcal{S}} \pi_\mu(\mathbf{s}) \mu(\mathbf{s}). \quad (10)$$

This represents the long term number of secondary accesses per time slot. We have the following result [10].

Lemma 1. The problem (7) is equivalent to

$$\mu^{*(\epsilon)} = \arg \max_{\mu \in \mathcal{U}} \mathcal{T}_s(\mu) \quad (11)$$

$$\text{s.t. } \mathcal{W}_s(\mu) \leq \min \left\{ \frac{R_p(1 - \rho_{p0}) - \mathcal{T}_p^{(th)}}{R_p(\rho_{p1} - \rho_{p0})}, \frac{\mathcal{P}_s^{(th)}}{P_s} \right\} \equiv \epsilon.$$

Then, (7) is equivalent to the secondary throughput maximization, under a secondary access rate constraint.

Unfortunately, an explicit characterization of the performance metrics and of the optimal policy for the general case $T \geq 2$ is far too complex, due to the quadratic increase of the number of states with T , and on the non trivial interaction between the states of the system. Therefore, we consider the case $T = 2$, which can be treated analytically. Namely, we prove that, under a condition on the secondary throughputs T_{s0}, T_{s1} , the optimal policy has a unique structure. Yet, this analysis provides valuable insight on the structure of the optimal SU operation and on the interaction between PU and SU in the general case, which is confirmed by simulation.

A. Optimal policy for the case $T = 2$

For the sake of a more intuitive readability, we label the four states of the system as:

$$\begin{aligned} \mathbf{s}_{new} &= (1, 0, 0) \text{ new primary transmission,} \\ \mathbf{s}_{nobuf} &= (2, 0, 0) \text{ no buffered transmission available,} \\ \mathbf{s}_{buf} &= (2, 1, 0) \text{ buffered transmission available,} \\ \mathbf{s}_{noint} &= (2, 0, 1) \text{ no primary interference.} \end{aligned}$$

Before proceeding with the proof, we briefly describe the structure of the optimal policy, depicted in Fig. 3. The secondary transmissions are allocated starting from the leftmost non-fully allocated state, until either the constraint on $\mathcal{W}_s(\mu) \leq \epsilon$ in (11) is attained with equality, or $\mathcal{T}_s(\mu)$ starts decreasing.

In particular, transmissions are prioritized in \mathbf{s}_{noint} , where the SR knows the PM, since primary interference

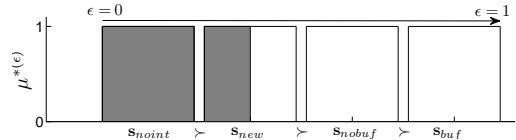


Figure 3: Optimal policy $\mu^{*(\epsilon)}$ for $T = 2$ under the hypothesis of Theorem 1. The area of the shaded region in each bin gives the transmission probability in the corresponding state. The transmissions are allocated starting from the leftmost non-fully allocated state, until the constraint $\mathcal{W}_s(\mu) \leq \epsilon$ in (11) is attained, or the secondary throughput starts decreasing.

can be cancelled, thus accruing a throughput gain $T_{s_1} - T_{s_0} > 0$ over the other states where the PM is unknown.

Once transmissions in this state occur with probability one (we say that the state is *filled*), further secondary transmissions may be allocated to s_{new} . In fact, if a transmission in this state undergoes outage due to primary interference, its recovery may be possible in the next ARQ round via BIC. If no transmission is performed in s_{new} , then the BIC capability is not enabled at the SR. This opportunity is not available in s_{buf} and s_{nobuf} .

Once s_{new} has been filled, further secondary transmissions are privileged in s_{nobuf} over s_{buf} . In fact, the ST is incentivized to stay silent in s_{buf} , so as to help the SR to decode the PM, which enables recovery of the buffered secondary transmission via BIC. This incentive is not available in s_{nobuf} , where the buffer is empty.

Finally, once s_{nobuf} has been filled, further secondary transmissions may be allocated to s_{buf} . However, this occurs only if $R_{s_0}(1 - \rho_{s_0} + \alpha_{s_1}) > R_{s_0}\alpha_{s_0}$, *i.e.*, if the expected reward in s_{buf} when the ST transmits (current transmission and BIC recovery) exceeds the expected reward when the ST stays silent (BIC recovery).

In order to prove the optimal policy, we make the following definitions.

Definition 1. We say that policy $\tilde{\mu}$ *dominates* μ (we write $\tilde{\mu} \supseteq \mu$) if $\mathcal{W}_s(\tilde{\mu}) = \mathcal{W}_s(\mu)$ and $\mathcal{T}_s(\tilde{\mu}) \geq \mathcal{T}_s(\mu)$.

If the latter inequality is strict, we say that $\tilde{\mu}$ *strictly dominates* μ (we write $\tilde{\mu} \supset \mu$).

A policy $\tilde{\mu} \supset \mu$ attains a better operational point than μ , from the perspective of (11). Hence, μ is sub-optimal.

Definition 2. We say that policy $\mu \in \mathcal{U}$ gives priority to state s_1 over s_2 if either $\mu(s_1) = 1$ or $\mu(s_2) = 0$. We define the set of policies with this property as ²

$$\mathcal{U}(s_1 \succ s_2) = \{\mu \in \mathcal{U} : \mu(s_1) = 1\} \cup \{\mu \in \mathcal{U} : \mu(s_2) = 0\}.$$

In general, we define the set of policies giving priority to s_1 over s_2 , to s_2 over s_3 , and so on, to s_{n-1} over s_n as

$$\mathcal{U}(s_1 \succ s_2 \succ \dots \succ s_n) = \bigcap_{i=1}^{n-1} \mathcal{U}(s_i \succ s_{i+1}). \quad (12)$$

According to policy $\mu \in \mathcal{U}(s_1 \succ s_2)$, the ST transmits more frequently when in state s_1 than when in state s_2 (except when $\mu(s_1) = \mu(s_2) \in \{0, 1\}$). In fact, the condition $\mu(s_1) = 1$ or $\mu(s_2) = 0$ implies $\mu(s_1) \geq \mu(s_2)$.

²Notice that the priority relation is not strict. In fact,

$$\mathcal{U}(s_1 \succ s_2) \cap \mathcal{U}(s_2 \succ s_1) \equiv \{\mu \in \mathcal{U} : \mu(s_1) = \mu(s_2) = 0\} \cup \{\mu \in \mathcal{U} : \mu(s_1) = \mu(s_2) = 1\} \neq \emptyset.$$

However, this feature is of no importance in the following treatment.

Definition 3. State $s_1 \in \mathcal{S}$ has *priority* over $s_2 \in \mathcal{S} \setminus \{s_1\}$ (we write $s_1 \succ s_2$) if, $\forall \mu \notin \mathcal{U}(s_1 \succ s_2)$, $\exists \tilde{\mu} \in \mathcal{U}(s_1 \succ s_2)$ such that $\tilde{\mu} \supset \mu$.

Lemma 2 (Characterization of the optimal policy). Assume $s_1 \succ s_2$, and let $\mu \notin \mathcal{U}(s_1 \succ s_2)$. Then, $\exists \tilde{\mu} \in \mathcal{U}(s_1 \succ s_2)$ such that $\tilde{\mu} \supset \mu$, *i.e.*, attaining a better operational point than μ , from the perspective of (11). Equivalently, μ is strictly sub-optimal for (11).

Alternatively, any policy μ which allocates transmissions to state s_2 *before* state s_1 is fully allocated with transmissions (thus not obeying the priority of s_1 over s_2) is strictly sub-optimal. Necessarily, if $s_1 \succ s_2$, the optimal policy $\mu^{*(\epsilon)} \in \mathcal{U}(s_1 \succ s_2)$. We conclude that we can characterize the optimal policy by the priority of a state over another, which determines the order according to which the network states are filled.

Definition 4. We define the *transmission efficiency* under policy μ in state s , such that $\frac{d\mathcal{W}_s(\mu)}{d\mu(s)} \neq 0$, as

$$\eta(\mu, s) = \frac{\frac{d\mathcal{T}_s(\mu)}{d\mu(s)}}{\frac{d\mathcal{W}_s(\mu)}{d\mu(s)}}. \quad (13)$$

$\eta(\mu, s)$ gives the rate increase (or decrease when negative) of the secondary throughput, per unit increase of the secondary access rate, due to an increase of the transmission probability in state s .

The following Lemma can be proved.

Lemma 3. Let $\mu \notin \mathcal{U}(s_1 \succ s_2)$. If $\eta(\mu, s_1) > \eta(\mu, s_2)$, then $\exists \tilde{\mu} \in \mathcal{U}(s_1 \succ s_2)$ such that $\tilde{\mu} \supset \mu$.

Moreover, assume that $\forall \mu \notin \mathcal{U}^*$, $\exists \tilde{\mu} \in \mathcal{U}^*$ such that $\tilde{\mu} \supset \mu$, where $\mathcal{U}^* \subseteq \mathcal{U}$ is a given set of policies. If $\eta(\mu, s_1) > \eta(\mu, s_2)$, $\forall \mu \notin \mathcal{U}(s_1 \succ s_2) \cap \mathcal{U}^*$, then $s_1 \succ s_2$.

This is a consequence of the fact that, if $\eta(\mu, s_1) > \eta(\mu, s_2)$, transmissions in state s_1 are *more efficient* than those in state s_2 , and a new policy $\tilde{\mu}$ may be devised, which prioritizes transmissions in state s_1 over state s_2 . One such $\tilde{\mu}$ has the following structure:

$$\begin{cases} \tilde{\mu}(s) = \mu(s) & s \in \mathcal{S} \setminus \{s_1, s_2\} \\ \tilde{\mu}(s_1) = \mu(s_1) + \nu_1 & 0 < \nu_1 \leq 1 - \mu(s_1) \\ \tilde{\mu}(s_2) = \mu(s_2) - \nu_2 & 0 < \nu_2 \leq \mu(s_2), \end{cases} \quad (14)$$

where (ν_1, ν_2) is the unique solution of $\tilde{\mu} \in \mathcal{U}(s_1 \succ s_2)$ and $\mathcal{W}_s(\tilde{\mu}) = \mathcal{W}_s(\mu)$. Compared to μ , policy $\tilde{\mu}$ moves as many transmissions as possible from s_2 to s_1 , until either $\tilde{\mu}(s_1) = 1$ (s_1 is filled) or $\tilde{\mu}(s_2) = 0$ (s_2 is emptied).

Notice that the hypothesis of the Lemma $\forall \mu \notin \mathcal{U}^*$, $\exists \tilde{\mu} \in \mathcal{U}^*$, $\tilde{\mu} \supset \mu$, where $\mathcal{U}^* \subseteq \mathcal{U}$ is a given set of policies, is important in the proof of the optimal policy, which

follows. In fact, under this hypothesis the optimal policy satisfies $\mu^{*(\epsilon)} \in \mathcal{U}^*$, hence we can restrict the search of the optimal policy within the set \mathcal{U}^* , rather than over the whole set \mathcal{U} , since any $\mu \notin \mathcal{U}^*$ is strictly sub-optimal.

The structure of the optimal policy is stated in the following Theorem. An intuitive, non rigorous argument follows; a detailed proof is given in the Appendix.

Theorem 1. *If*

$$\delta_s \equiv \frac{T_{s1} - T_{s0} - \omega_s R_{s0}}{R_{s0}} < \frac{\alpha_{s1}}{\alpha_{s0} - \alpha_{s1}} \omega_s, \quad (15)$$

the optimal policy obeys the following priority:

$$\mathbf{s}_{noint} \succ \mathbf{s}_{new} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf}. \quad (16)$$

Moreover, transmissions are allocated to state \mathbf{s}_{buf} only if $1 - \rho_{s0} > \alpha_{s0} - \alpha_{s1}$.

Proof: We prove the Theorem by contradiction. Let $\mu \notin \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{new} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$ be a policy violating the priority of the states.

We now define a sequence of policies $\{\mu^{(i)}, i \geq 0\}$ with $\mu^{(0)} = \mu$ such that $\mu^{(i+1)} \supset \mu^{(i)} \forall i$, characterized by attaining a better operational point, from the perspective of solving (11), thus proving the sub-optimality of μ .

Step 1: If $\mu^{(0)} \in \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf})$, we let $\mu^{(1)} \equiv \mu^{(0)}$. Otherwise ($\mu^{(0)} \notin \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf})$), we show that $\eta(\mu^{(0)}, \mathbf{s}_{noint}) > \eta(\mu^{(0)}, \mathbf{s}_{nobuf})$, *i.e.*, transmissions in state \mathbf{s}_{noint} are more efficient than in state \mathbf{s}_{nobuf} , from the perspective of solving (11). From Lemma 3, $\exists \mu^{(1)} \in \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf})$ such that $\mu^{(1)} \supset \mu^{(0)}$. This is defined as in (14), *i.e.*, it is obtained by moving transmissions from state \mathbf{s}_{nobuf} to \mathbf{s}_{noint} .

Step 2: If $\mu^{(1)} \in \mathcal{U}(\mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$, we let $\mu^{(2)} \equiv \mu^{(1)}$. Otherwise ($\mu^{(1)} \notin \mathcal{U}(\mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$), we show that $\eta(\mu^{(1)}, \mathbf{s}_{nobuf}) > \eta(\mu^{(1)}, \mathbf{s}_{buf})$, *i.e.*, transmissions in state \mathbf{s}_{nobuf} are more efficient than those in state \mathbf{s}_{buf} , from the perspective of solving (11). Then, from Lemma 3, we can define, using (14), $\mu^{(2)} \in \mathcal{U}(\mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$, by moving transmissions from state \mathbf{s}_{buf} to \mathbf{s}_{nobuf} , so that $\mu^{(2)} \supset \mu^{(1)}$. Notice that at this point we may have $\mu^{(2)} \notin \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf})$. It may then be necessary to repeat Steps 1 and 2 to obey both priority constraints. Notice that the transfer of transmissions from \mathbf{s}_{buf} to \mathbf{s}_{nobuf} , and finally to \mathbf{s}_{noint} is unidirectional, hence in a finite number of iterations of Steps 1 and 2 we obtain $\mu^{(2)} \in \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$.

Step 3: If $\mu^{(2)} \in \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{new})$, we let $\mu^{(3)} \equiv \mu^{(2)}$. Otherwise ($\mu^{(2)} \notin \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{new})$), we show that $\eta(\mu^{(2)}, \mathbf{s}_{noint}) > \eta(\mu^{(2)}, \mathbf{s}_{new})$. With the same approach used in the previous steps, we can thus define, using (14), $\mu^{(3)} \in \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{new})$, with $\mu^{(3)} \supset \mu^{(2)}$.

Notice that we have also $\mu^{(3)} \in \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{new})$, hence $\mu^{(3)} \in \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf}) \cap \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{new})$.

Step 4: If $\mu^{(3)} \in \mathcal{U}(\mathbf{s}_{new} \succ \mathbf{s}_{buf})$, we let $\mu^{(4)} \equiv \mu^{(3)}$. Otherwise ($\mu^{(3)} \notin \mathcal{U}(\mathbf{s}_{new} \succ \mathbf{s}_{buf})$), we show that $\eta(\mu^{(3)}, \mathbf{s}_{new}) > \eta(\mu^{(3)}, \mathbf{s}_{buf})$. We can thus define, using (14), $\mu^{(4)} \in \mathcal{U}(\mathbf{s}_{new} \succ \mathbf{s}_{buf})$, with $\mu^{(4)} \supset \mu^{(3)}$. Notice that we have also $\mu^{(4)} \in \mathcal{U}(\mathbf{s}_{new} \succ \mathbf{s}_{nobuf})$, so that $\mu^{(4)} \in \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf}) \cap \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{new} \succ \mathbf{s}_{buf})$.

Step 5: If $\mu^{(4)} \in \mathcal{U}(\mathbf{s}_{new} \succ \mathbf{s}_{nobuf})$, we let $\mu^{(5)} \equiv \mu^{(4)}$. Otherwise ($\mu^{(4)} \notin \mathcal{U}(\mathbf{s}_{new} \succ \mathbf{s}_{nobuf})$), we show that $\eta(\mu^{(4)}, \mathbf{s}_{new}) > \eta(\mu^{(4)}, \mathbf{s}_{nobuf})$. We can thus define, using (14), $\mu^{(5)} \in \mathcal{U}(\mathbf{s}_{new} \succ \mathbf{s}_{nobuf})$, with $\mu^{(5)} \supset \mu^{(4)}$. Then, since $\mu^{(4)} \in \mathcal{U}(\mathbf{s}_{new} \succ \mathbf{s}_{nobuf})$, we have also $\mu^{(5)} \in \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{new} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$.

Starting from a generic policy $\mu \notin \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{new} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$, we have thus defined a sequence of policies $\{\mu^{(i)}, i = 0, \dots, 5\}$ such that $\mu^{(5)} \supset \mu^{(4)} \supset \dots \supset \mu$. This proves the sub-optimality of μ . ■

B. Discussion

Notice that $T_{s1} - T_{s0} - \omega_s R_{s0}$ represents the gap between the throughput accrued when the SR knows the PM, and the sum of the throughput accrued when it does not know it, plus the "throughput" recovered via BIC. Therefore, the hypothesis (15) simply sets an upper bound to this gap. If the upper bound is exceeded, *i.e.*, the throughput accrued when the SR knows the PM is much larger than the combined instantaneous and BIC throughputs when it does not know it, then transmissions in state \mathbf{s}_{new} , by impairing the ability of the SR to decode the PM, may induce the system to visit the states \mathbf{s}_{buf} and \mathbf{s}_{nobuf} more frequently than \mathbf{s}_{noint} , where the reward is much larger, thus causing a secondary throughput degradation. In this case, priority of \mathbf{s}_{new} over \mathbf{s}_{nobuf} and \mathbf{s}_{buf} is not guaranteed.

V. NUMERICAL RESULTS

In this section, we discuss numerical results for the case $T = 2$, which demonstrate the performance improvement achievable by the BIC mechanism (*BIC* policy) over the transmission strategy investigated in [10], which does not make use of BIC (*no BIC* policy). We also compare them with an *opportunistic BIC* policy. Namely, the SR does not inform the ST about the buffer state. As a consequence, the ST obviously allocates transmissions without being able to take advantage of such knowledge, whereas the SR, whenever the PM is decoded, performs BIC on the buffered transmission.

Each channel is modeled as i.i.d. Rayleigh fading with common power $\Gamma \equiv \Gamma_s = \Gamma_p = \Gamma_{ps} = \Gamma_{sp}$, and zero mean-unit variance circular Gaussian noise at each receiver. Moreover, we let $P_p = P_s = 1$,

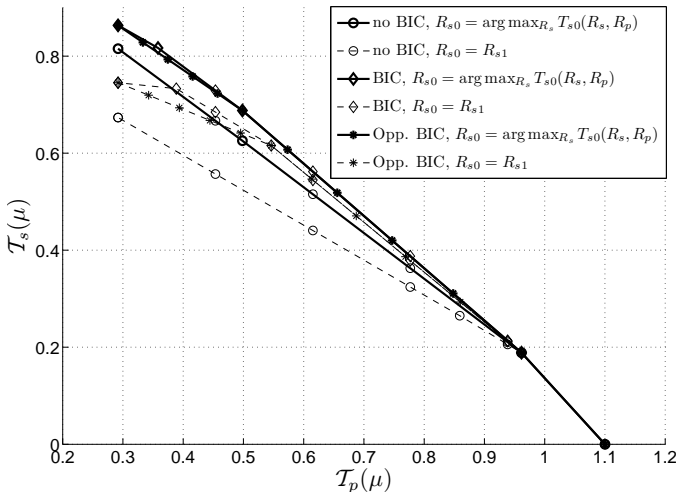


Figure 4: PU vs SU throughputs, $T = 2$, $\Gamma = 5$

$R_{s1} = \arg \max_{R_s} T_{s1}(R_{s1})$, and R_p maximizes the primary throughput when the ST is silent.

Fig. 4 compares the performance of the three policies in terms of the primary and secondary throughputs for the case $\Gamma = 5$ (see Table I for the corresponding system parameters), obtained by varying the secondary access rate constraint $\epsilon \in [0, 1]$. Notice that each policy is evaluated for two different values of the rate R_{s0} , namely $R_{s0} = \arg \max_{R_s} T_{s0}(R_s, R_p)$ (solid line) and $R_{s0} = R_{s1}$ (dashed line).

As expected, for a specific value of the rate R_{s0} the *no BIC* strategy performs the worst. In fact, *no BIC* drops the failed transmissions in the first ARQ round, whereas *opportunistic BIC* and *BIC* buffer part of them, since they may be recovered in the next ARQ round via BIC, thus getting a net throughput benefit. Moreover, *BIC* outperforms *opportunistic BIC*. In fact, from the operational perspective, with the *BIC* strategy the ST takes advantage of the knowledge of the buffer state to make better decisions, e.g., by staying silent when a secondary transmission is buffered, so as to help the SR to decode the PM, thus favoring the BIC mechanism. This additional degree of freedom brings a net benefit over the *opportunistic BIC* strategy. Notice also that the two policies perform the same in the low interference range (high T_p). In fact in this regime the same transmission probability is allocated to states s_{new} and s_{noint} , whereas states s_{buf} and s_{nobuf} are kept silent.

We now compare the choice of the rate R_{s0} . We observe that, for a specific policy, using $R_{s0} = \arg \max_{R_s} T_{s0}(R_s, R_p)$ outperforms $R_{s0} = R_{s1}$. This is expected for the *no BIC* policy, since using $R_{s0} = R_{s1}$, compared to $R_{s0} = \arg \max_{R_s} T_{s0}(R_s, R_p)$, both decreases the accrued throughput T_{s0} , and impairs the

Primary System		
$R_p = 1.9141$	$\rho_{p0} = 0.4252$	$\rho_{p1} = 0.8475$
Secondary System, $R_{s0} = \arg \max_{R_s} T_{s0}(R_s, R_p)$		
$R_{s0} = 1.3214$	$\rho_{s0} = 0.4380$	$T_{s0} = 0.7427$
$\alpha_{s0} = 0.5748$	$\alpha_{s1} = 0.4431$	$\omega_s = 0.1789$
$R_{s1} = 1.9141$	$\rho_{s1} = 0.4252$	$T_{s1} = 1.1002$
Secondary System, $R_{s0} = R_{s1}$		
$R_{s0} = 1.9141$	$\rho_{s0} = 0.6857$	$T_{s0} = 0.6015$
$\alpha_{s0} = 0.5748$	$\alpha_{s1} = 0.3143$	$\omega_s = 0.2605$
$R_{s1} = 1.9141$	$\rho_{s1} = 0.4252$	$T_{s1} = 1.1002$

Table I: System parameters, SNR $\Gamma = 5$. In the first row we list the PU parameters, in the second and third rows we list the SU parameters, for $R_{s0} = \arg \max_{R_s} T_{s0}(R_s, R_p)$ and $R_{s0} = R_{s1}$.

ability of the SR to decode the PM (Table I). However, this is non trivial for the other two policies. In fact, using $R_{s0} = R_{s1}$ rather than $R_{s0} = \arg \max_{R_s} T_{s0}(R_s, R_p)$ has opposing effects on the system: on the one hand, it reduces both the accrued secondary throughput T_{s0} and the probability that the SR successfully decodes the PM, α_{s1} ; on the other hand, it increases the probability that a secondary transmission is buffered, ω_s (Table I). In this case, the gain in terms of buffering is smaller than the loss due to a decrease of both T_{s0} and α_{s1} .

VI. DISCUSSION AND FUTURE WORK

In the case $T > 2$, the analysis becomes far more complicated than $T = 2$, due to the complex interaction among the states of the system. For this case, we have conducted extensive simulations, showing evidence that the optimal policy has less structure than in the case $T = 2$, since there is not a well defined priority of the states. However, we observe that, as in the case $T = 2$, the optimal policy prioritizes transmissions in the states where the PM is known at the SR, and in the initial ARQ rounds. This behavior can be explained by noticing that such a policy optimizes the buffer occupancy, thus maximizing the expected reward accrued via BIC.

So far, we have assumed perfect CSI at the SR. However, this is rarely the case in a real system, where channel estimation errors result in a performance degradation [17]–[19]. Of particular relevance in this work is the fact that the SR, once the PM is successfully decoded, cannot exactly remove its interference from the received signal, due to the mismatch between the true channel and its estimate, resulting in a residual error term which degrades the performance of interference cancellation techniques [18]. This can be circumvented by defining a worst case model where the channel is replaced by its estimate, and the residual interference from channel mismatch is modeled as additive Gaussian noise [19].

VII. CONCLUDING REMARKS

In this work, following [10]–[12], we have further investigated the idea of exploiting the primary ARQ process to enhance the performance of cognitive networks, and to enable primary and secondary users coexistence. In particular, we have proposed and analyzed a *Backward Interference Cancellation* scheme, according to which the SR buffers the secondary transmissions that undergo outage and attempts to recover them once the knowledge about the PM becomes available in a future instant.

Following [9], [10], we have used a control based approach to optimize the secondary access strategy, under a constraint on the PU's degradation. We have characterized the optimal policy for the case where the primary ARQ scheme is limited to one retransmission, proving that, under certain conditions on the secondary throughput terms, the policy admits a unique structure. Finally, we have shown numerically the throughput benefit of this scheme, over other techniques that either do not use BIC, or use it in an opportunistic fashion.

APPENDIX PROOF OF THEOREM 1

Proof: By solving the stationary equation for a specific policy $\mu \in \mathcal{U}$, we have

$$\begin{cases} \pi_\mu(\mathbf{s}_{new}) = \frac{1}{1+\mu(\mathbf{s}_{new})\rho_{p1}+(1-\mu(\mathbf{s}_{new}))\rho_{p0}} \\ \pi_\mu(\mathbf{s}_{nobuf}) = \frac{\mu(\mathbf{s}_{new})\rho_{p1}(1-\alpha_{s1}-\omega_s)+(1-\mu(\mathbf{s}_{new}))\rho_{p0}(1-\alpha_{s0})}{1+\mu(\mathbf{s}_{new})\rho_{p1}+(1-\mu(\mathbf{s}_{new}))\rho_{p0}} \\ \pi_\mu(\mathbf{s}_{buf}) = \frac{\mu(\mathbf{s}_{new})\rho_{p1}\omega_s}{1+\mu(\mathbf{s}_{new})\rho_{p1}+(1-\mu(\mathbf{s}_{new}))\rho_{p0}} \\ \pi_\mu(\mathbf{s}_{noint}) = \frac{\mu(\mathbf{s}_{new})\rho_{p1}\alpha_{s1}+(1-\mu(\mathbf{s}_{new}))\rho_{p0}\alpha_{s0}}{1+\mu(\mathbf{s}_{new})\rho_{p1}+(1-\mu(\mathbf{s}_{new}))\rho_{p0}}, \end{cases}$$

where $\pi_\mu(\cdot)$ is the stationary distribution under policy μ , and the system parameters are defined in Section II. We now prove, in order, $\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf}$, $\mathbf{s}_{noint} \succ \mathbf{s}_{new}$, $\mathbf{s}_{new} \succ \mathbf{s}_{buf}$ and $\mathbf{s}_{new} \succ \mathbf{s}_{nobuf}$.

1) *Proof of $\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf}$:*

Let $\mu \notin \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf})$. Notice that

$$\eta(\mu, \mathbf{s}_{noint}) = T_{s1} > \eta(\mu, \mathbf{s}_{nobuf}) = T_{s0}. \quad (17)$$

Therefore, from Lemma 3 with $\mathcal{U}^* \equiv \mathcal{U}$, $\exists \tilde{\mu} \in \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf})$ such that $\tilde{\mu} \supset \mu$. Since this property holds $\forall \mu \notin \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf})$, the result follows.

2) *Proof of $\mathbf{s}_{nobuf} \succ \mathbf{s}_{buf}$:*

Let $\mu \notin \mathcal{U}(\mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$. Assume $\mu(\mathbf{s}_{new}) = 0$ first. Then, \mathbf{s}_{buf} is not accessible (since no message can be buffered if the SU stays silent in the first ARQ round), and therefore we can trivially define $\tilde{\mu} \in \mathcal{U}(\mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$ as $\tilde{\mu}(\mathbf{s}) = \mu(\mathbf{s}), \forall \mathbf{s} \neq \mathbf{s}_{buf}, \tilde{\mu}(\mathbf{s}_{buf}) = 0$, such that $\mathcal{W}_s(\tilde{\mu}) = \mathcal{W}_s(\mu)$ and $\mathcal{T}_s(\tilde{\mu}) = \mathcal{T}_s(\mu)$. Hence, $\tilde{\mu} \supseteq \mu$.

Otherwise ($\mu(\mathbf{s}_{new}) > 0$), we have

$$\begin{aligned} \eta(\mu, \mathbf{s}_{buf}) &= T_{s0} + (\alpha_{s1} - \alpha_{s0}) R_{s0} \\ &< T_{s0} = \eta(\mu, \mathbf{s}_{nobuf}), \end{aligned} \quad (18)$$

where we have used the fact that $\alpha_{s1} < \alpha_{s0}$. From Lemma 3 with $\mathcal{U}^* \equiv \mathcal{U}$, this proves the existence of $\tilde{\mu} \in \mathcal{U}(\mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$ such that $\tilde{\mu} \supset \mu$.

In both cases, we have shown that $\exists \tilde{\mu} \in \mathcal{U}(\mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$ such that $\tilde{\mu} \supseteq \mu$. Therefore, $\mathbf{s}_{nobuf} \succ \mathbf{s}_{buf}$.

3) *Proof of $\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf}$:*

By combining the two cases above, we then obtain $\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf}$. Notice that, from Lemma 3, any policy $\mu \notin \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$ is sub-optimal. In the following, we then assume $\mu \in \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$, and thus restrict the search of the optimal policy within this set.

4) *Proof of $\mathbf{s}_{noint} \succ \mathbf{s}_{new}$:*

Let $\mu \notin \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{new})$, $\mu \in \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$. The first assumption implies $\mu(\mathbf{s}_{noint}) < 1$. Then, from the second, we have $\mu(\mathbf{s}_{buf}) = \mu(\mathbf{s}_{nobuf}) = 0$.

We prove $\eta(\mu, \mathbf{s}_{new}) < \eta(\mu, \mathbf{s}_{noint})$, or equivalently

$$T_{s0} - T_{s1} + \rho_{p1}\omega_s\alpha_{s0}R_{s0} < 0. \quad (19)$$

This inequality is trivially verified, by using (4), (5), and $\rho_{p1}\alpha_{s0} < 1$. Therefore, from Lemma 3 with $\mathcal{U}^* \equiv \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$, $\exists \tilde{\mu} \in \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{new})$ such that $\tilde{\mu} \supset \mu$, which proves $\mathbf{s}_{noint} \succ \mathbf{s}_{new}$.

5) *Proof of $\mathbf{s}_{new} \succ \mathbf{s}_{buf}$:*

Let $\mu \notin \mathcal{U}(\mathbf{s}_{new} \succ \mathbf{s}_{buf})$, $\mu \in \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$. The first assumption implies $\mu(\mathbf{s}_{buf}) > 0$. Hence, from the second, we have $\mu(\mathbf{s}_{noint}) = \mu(\mathbf{s}_{nobuf}) = 1$.

If $\mu(\mathbf{s}_{new}) = 0$, \mathbf{s}_{buf} is not accessible, and therefore we can trivially define $\tilde{\mu} \in \mathcal{U}(\mathbf{s}_{new} \succ \mathbf{s}_{buf})$ as $\tilde{\mu}(\mathbf{s}) = \mu(\mathbf{s}), \forall \mathbf{s} \neq \mathbf{s}_{buf}, \tilde{\mu}(\mathbf{s}_{buf}) = 1$. Clearly, $\mathcal{W}_s(\tilde{\mu}) = \mathcal{W}_s(\mu)$ and $\mathcal{T}_s(\tilde{\mu}) = \mathcal{T}_s(\mu)$, hence $\tilde{\mu} \supseteq \mu$.

Otherwise ($\mu(\mathbf{s}_{new}) > 0$), we prove that $\eta(\mu, \mathbf{s}_{new}) > \eta(\mu, \mathbf{s}_{buf}) = T_{s0} - R_{s0}(\alpha_{s0} - \alpha_{s1})$, or equivalently

$$g(\rho_{p0}, \rho_{p1}) = (\alpha_{s0} - \alpha_{s1})(1 + \rho_{p1}) + \rho_{p1}\omega_s\alpha_{s1}(1 + \rho_{p0}) + (\omega_s + \delta_s)(\rho_{p1}\alpha_{s1} - \rho_{p0}\alpha_{s0} + \rho_{p1}\rho_{p0}(\alpha_{s1} - \alpha_{s0})) > 0,$$

Notice that $g(\rho_{p0}, \rho_{p1})$ is a linear function of ρ_{p0} . Since $\rho_{p0} \in [0, \rho_{p1}]$, we then have $g(\rho_{p0}, \rho_{p1}) \geq \min\{g(0, \rho_{p1}), g(\rho_{p1}, \rho_{p1})\}$, where

$$\begin{cases} g(0, \rho_{p1}) = (\alpha_{s0} - \alpha_{s1})(1 + \rho_{p1}) + (2\omega_s + \delta_s)\rho_{p1}\alpha_{s1} \\ g(\rho_{p1}, \rho_{p1}) = (\alpha_{s0} - \alpha_{s1})(1 + \rho_{p1}) + \rho_{p1}\omega_s\alpha_{s1}(1 + \rho_{p1}) \\ \quad + (\omega_s + \delta_s)\rho_{p1}(1 + \rho_{p1})(\alpha_{s1} - \alpha_{s0}). \end{cases}$$

Moreover, $\frac{g(\rho_{p1}, \rho_{p1})}{1 + \rho_{p1}}$ is a linear function of ρ_{p1} , hence

$$g(\rho_{p1}, \rho_{p1}) \geq (1 + \rho_{p1}) \min \left\{ g(0, 0), \frac{g(1, 1)}{2} \right\},$$

where

$$\begin{cases} g(0, 0) = (\alpha_{s0} - \alpha_{s1}) \\ \frac{1}{2}g(1, 1) = (\alpha_{s0} - \alpha_{s1}) + \omega_s \alpha_{s1} \\ \quad + (\omega_s + \delta_s)(\alpha_{s1} - \alpha_{s0}). \end{cases} \quad (20)$$

By combining the above inequalities, we obtain

$$\frac{g(\rho_{p0}, \rho_{p1})}{1 + \rho_{p1}} \geq \min \left\{ \frac{g(0, \rho_{p1})}{1 + \rho_{p1}}, g(0, 0), \frac{1}{2}g(1, 1) \right\}.$$

Clearly, $g(0, \rho_{p1}) > 0$, $g(0, 0) > 0$. If $g(1, 1) > 0$, i.e.,

$$\delta_s < 1 - \omega_s + \omega_s \frac{\alpha_{s1}}{\alpha_{s0} - \alpha_{s1}} \quad (21)$$

(this is implied by the hypothesis of the Theorem), then $g(\rho_{p0}, \rho_{p1}) > 0$. Therefore, $\exists \tilde{\mu} \in \mathcal{U}(\mathbf{s}_{new} \succ \mathbf{s}_{buf})$ such that $\tilde{\mu} \supset \mu$, which proves $\mathbf{s}_{new} \succ \mathbf{s}_{buf}$.

6) *Proof of $\mathbf{s}_{new} \succ \mathbf{s}_{nobuf}$:*

Let $\mu \notin \mathcal{U}(\mathbf{s}_{new} \succ \mathbf{s}_{nobuf})$, $\mu \in \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf})$. Since we have proved $\mathbf{s}_{noint} \succ \mathbf{s}_{new} \succ \mathbf{s}_{buf}$, we also assume $\mu \in \mathcal{U}(\mathbf{s}_{noint} \succ \mathbf{s}_{new} \succ \mathbf{s}_{buf})$. Then we have $\mu(\mathbf{s}_{nobuf}) > 0$, $\mu(\mathbf{s}_{buf}) = 0$ and $\mu(\mathbf{s}_{noint}) = 1$.

We prove that $\eta(\mu, \mathbf{s}_{new}) > \eta(\mu, \mathbf{s}_{nobuf}) = T_{s0}$, i.e.,

$$\begin{aligned} h(\rho_{p0}, \rho_{p1}) &= \omega_s [(\rho_{p1} - \rho_{p0})\alpha_{s0} + \rho_{p1}\alpha_{s1} (1 + \rho_{p0})] \\ &+ \delta_s [(1 + \rho_{p0})\rho_{p1}\alpha_{s1} - (1 + \rho_{p1})\rho_{p0}\alpha_{s0}] > 0. \end{aligned} \quad (22)$$

We prove the inequality by lower bounding $h(\rho_{p0}, \rho_{p1})$ with a non-negative function. $h(\rho_{p0}, \rho_{p1})$ is a linear function of $\rho_{p0} \in [0, \rho_{p1}]$, therefore $h(\rho_{p0}, \rho_{p1}) \geq \min\{h(0, \rho_{p1}), h(\rho_{p1}, \rho_{p1})\}$, where

$$\begin{cases} h(0, \rho_{p1}) = \omega_s \rho_{p1} (\alpha_{s0} + \alpha_{s1}) + \delta_s \rho_{p1} \alpha_{s1} \\ h(\rho_{p1}, \rho_{p1}) = \rho_{p1} (1 + \rho_{p1}) [\omega_s \alpha_{s1} - \delta_s (\alpha_{s0} - \alpha_{s1})]. \end{cases}$$

Clearly, $h(0, \rho_{p1}) > 0$. If $h(\rho_{p1}, \rho_{p1}) > 0$, or equivalently

$$\delta_s < \omega_s \frac{\alpha_{s1}}{\alpha_{s0} - \alpha_{s1}} \quad (23)$$

(from the hypothesis of the Theorem), then $h(\rho_{p0}, \rho_{p1}) > 0$.

Therefore, $\exists \tilde{\mu} \in \mathcal{U}(\mathbf{s}_{new} \succ \mathbf{s}_{nobuf})$ such that $\tilde{\mu} \supset \mu$.

This proves $\mathbf{s}_{new} \succ \mathbf{s}_{nobuf}$.

7) *Proof of $\mathbf{s}_{noint} \succ \mathbf{s}_{new} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf}$:*

By combining the above results, we have proved that

$$\mathbf{s}_{noint} \succ \mathbf{s}_{new} \succ \mathbf{s}_{nobuf} \succ \mathbf{s}_{buf}. \quad (24)$$

Finally, if $\eta(\mu, \mathbf{s}_{buf}) \leq 0$ (equivalently, $1 - \rho_{s0} \leq \alpha_{s0} - \alpha_{s1}$), then transmissions in \mathbf{s}_{buf} induce a secondary throughput degradation, hence they should be avoided. ■

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