Uplink Sum-Rate Analysis of a Multicell System With Feedback

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***Abstract*—The capacity region of a Multiple Access Channel can be increased by feedback to the sources, since feedback enables cooperative transmission. Focusing on a linear cellular system (as for a highway or a corridor), a novel transmission strategy is proposed that exploits feedback from the neighboring mobile stations (MSs). The strategy enables cooperative communications via "analog network coding" (i.e., broadcasting and interference cancellation via side information) to exchange signalling information among MSs. Numerical results show that the proposed technique provides gains over non-cooperative strategies in the low-signal-to-noise-ratio regime.**

I. Introduction

Feedback plays a number of roles in communication systems, such as predicting and correcting noise, enabling source cooperation, decreasing computational complexity and reducing delay [1]. In particular, in the context of Multiple Access Channels (MACs), feedback enlarges the capacity region by enabling cooperation at the sources, here referred to as mobile stations (MSs) [2]. Different types of feedback signals can be available, ranging from output-feedback, where the destination output is directly obtained by the MSs [2]–[4] or "generalized" feedback where each source observes different channel outputs [5]–[7].

In this paper, we extend the transmission strategies of [5] –[7] to the uplink of a multicell scenario with generalized feedback, as depicted in Fig. 1 (see also [8]). The cellular scenario at hand follows the linear Wyner model [9], where cells are arranged in a linear geometry with one active user per cell at any given time as for intra-cell time-division multiple-access (TDMA). Moreover, decoding is carried out at a central processor which is connected to every base station (BS) via ideal backhaul links (i.e. multicell joint decoding, see [10] for a review). We extend the basic linear Wyner model by allowing for generalized feedback. Specifically, due to the broadcast nature of the wireless medium, each MS is assumed to receive signals from the MSs in adjacent cells. Similarly to [5]–[7], these signals constitute generalized feedback that can be used to communicate among neighboring MSs and set up cooperative transmission strategies.

Unlike [5]–[7], the presence of multiple MSs and the specific geometry call for sophisticated techniques for the exchange of the cooperation-enabling information (also referred to as signalling in the following) among MSs that exploit local communications. For instance, in Fig. 1 the mth MS might want to cooperate with the $(m+2)$th MS, thus requiring the signalling information to be propagated along two hops. The proposed technique recognizes that such transmissions can benefit from "*analog network coding*" techniques [11]–[16], that take advantage of the broadcast channels between adjacent MSs and the side information available at the MSs: when decoding the signalling information broadcast by the neighboring MSs, each MS can cancel the contribution due to the messages that were originated at the MS itself or previously relayed through it.

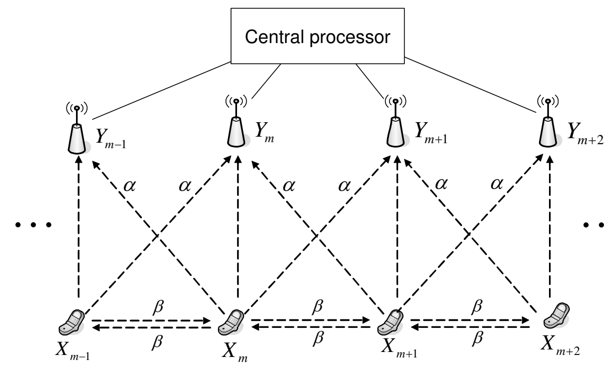

Fig. 1. Uplink of a multi-cell system with generalized feedback at the MSs.

This paper is organized as follows. In Sec. II, we discuss basic background material covering Gaussian MACs with feedback and analog network coding. Then, in Secs. III and IV, an achievable per-cell rate for the system in Fig. 1 is derived. This rate is evaluated numerically in Sec. V.

Notation: $X \sim \mathcal{CN}(\mu, \sigma^2)$ denotes a complex Gaussian circularly symmetric random variable with mean μ and variance σ^2.

II. Background and Related Work

In this section, we present background material and discuss related work. In particular, Sec. II-A focuses on a two-user Gaussian MAC with feedback and reviews known results as applied to a symmetric system. Sec. II-B then briefly discusses the basics of analog network coding and previous work in the area. The techniques and concepts recalled in this section will be instrumental in the analysis of the system of Fig. 1 in later sections.

A. Gaussian Multiple Access Channel With Feedback

Consider a symmetric two-user Gaussian MAC with signal received by the destination at time instant i given by:

$$Y_i = X_{1,i} + X_{2,i} + Z_i,\ i = 1, 2, ..., n, \tag{1}$$

where the noise sequence $\{Z_i\}_{i=1}^n$ is such that $Z_i \sim \mathcal{CN}(0, 1)$, independent and identically distributed (i.i.d.) over $i = 1, ..., n$, and we enforce a per-symbol power constraint $E[|X_{m,i}|^2] \leq P$, $m = 1, 2$. A fairly general model for feedback prescribes the signal received by the two MSs at the ith symbol to be given by (see Fig. 2-(a)):

$$\mathcal{Y}_{1,i} = \beta X_{2,i} + \mathcal{Z}_{1,i} \tag{2a}$$

$$\mathcal{Y}_{2,i} = \beta X_{1,i} + \mathcal{Z}_{2,i}, \tag{2b}$$

with noise sequences $\{\mathcal{Z}_{m,i}\}_{i=1}^n$ such that $\mathcal{Z}_{m,i} \sim \mathcal{CN}(0, 1)$, i.i.d. over $i = 1, ..., n$ $(m = 1, 2)$. Notice that model (2) assumes perfect echo cancelation. The parameter $\beta \geq 0$ measures the quality of the inter-MS channels. The noise samples $\{\mathcal{Z}_{m,i}\}_{i=1}^n$ are generally correlated with $\{Z_i\}_{i=1}^n$ with given correlation coefficient ρ $(|\rho| \leq 1)$; i.e.,

$$\mathcal{Z}_{m,i} = \rho Z_i + \sqrt{1-\rho^2}\tilde{\mathcal{Z}}_{m,i}, \tag{3}$$

with $\tilde{\mathcal{Z}}_{m,i}$ being an independent i.i.d. noise process with $\tilde{\mathcal{Z}}_{m,i} \sim \mathcal{CN}(0, 1)$. For instance, if $\beta = 1$ and $\rho = 1$, this model corresponds to output-feedback [17]. Note that the model above entails full-duplex operation at the MSs. Moreover, we remark that the model at hand is related to relay channels where the relay also has a message to deliver to the destination [18].

Focusing on a symmetric scenario, each user has a message W_m in the range $\{1, 2, ..., 2^{nR}\}$, $m = 1, 2$, of *equal rate* R to deliver to the BS. Using standard definitions, encoding is carried out at each mth MS by mapping the message W_m and the previously received samples $\mathcal{Y}_m^{i-1} = [\mathcal{Y}_{m,1} \cdots \mathcal{Y}_{m,i-1}]$ into the ith channel input $X_{m,i}$ for $i = 1, 2, ..., n$. Rate R is said to be achievable if the average probability of decoding error $\Pr[(\hat{W}_1, \hat{W}_2) \neq (W_1, W_2)]$, where $\hat{W}_1, \hat{W}_2$ are the decoded messages, can be made to tend to zero as $n \to \infty$.

For any $\beta \geq 0$ and any $|\rho| \leq 1$, an achievable equal rate can be found by specializing the results of [5] [6] [7] to the scenario at hand, obtaining (see Appendix-A for a brief derivation)

$$R = \min \left\{ \begin{array}{l} \frac{1}{2}\log\left(1 + 2P_p\right) + R_c, \\ \frac{1}{2}\log\left(1 + 2P + 2\nu^2 P_c\right) \end{array} \right\}, \tag{4}$$

with

$$R_c = \log\left(1 + \frac{\beta^2(1-\nu^2)P_c}{1 + \beta^2 P_p}\right), \tag{5}$$

power allocation $P = P_p + P_c$ and $0 \leq \nu \leq 1$. We remark that the notation used here is different from previous references and is tailored to facilitate the discussion in the following sections. The rate (4)-(5) is achieved by a block-Markov encoding scheme, where each MS splits its rate and powers between a *private* and a *common* part. Specifically, rate R is split as $R = R_p + R_c$ (subscripts denote private "p" and common "c" parts) and the power as $P = P_p + P_c$. The private part, of rate R_p, is sent to the BS with power P_p without any cooperation from the other MS, while transmission of the common part, of rate R_c, benefits from the cooperation with the other MS. In order to enable cooperative transmission, a fraction $(1 - \nu^2)$ of the common power P_c is devoted to *signalling* the local message to the other MS. The remaining power $\nu^2 P_c$ is then employed for cooperative transmission to the BS. Condition (5) enables each MS to decode the signalling message (of rate R_c) from the other MS and (4) is sufficient for correct decoding at the BS. We refer the reader to [5] - [8] and Appendix-A for details.

Remark 1: The achievable rate (4)-(5) is not the capacity of the system at hand. To see this, consider the case $\beta = 1$ and $\rho = 1$, which corresponds to output-feedback [17]. With this choice, the entire capacity region has been derived in [3], which, when specialized to our symmetric case, leads to the equal-rate capacity (i.e., maximum achievable equal rate) $C_f = 1/2\log(1 + 2P(1 + \theta^*))$ with θ^* being the solution of the equation $1 + 2P(1 + \theta) = (1 + P(1 - \theta^2))^2$ satisfying $0 \leq \theta \leq 1$. The equal-rate capacity C_f and can be shown to be larger than (4)-(5). The main reason for the suboptimality of (4)-(5) is that the transmission scheme leading to (4)-(5) limits the correlation structure between the channel codewords (via a specific Markov chain condition, see Appendix-A) and thus does not enable full exploitation of the coherent addition of the two codewords at the receiver (see also [8] and [19]).

Remark 2: A simple upper bound on the equal rate is provided by $R_{full-coop} = 1/2\log(1+4P)$, which corresponds to full cooperation between the two transmitters (that is, to a multiple-input-single-output system). As explained in the previous remark, this rate cannot be achieved with output-feedback, but, as discussed below, it can be attained with arbitrary precision if the quality of the MS measurement (2) is sufficiently good (i.e., if β is large enough). To elaborate, suppose that we are interested in achieving $R_{full-coop} - \epsilon$ with $\epsilon > 0$, then, defining as ν^* the value $0 \leq \nu^* \leq 1$ such that $\frac{1}{2}\log\left(1 + 2(1 + \nu^{*2})P\right) = R_{full-coop} - \epsilon$, it is easy to see that by setting $P_c = P$ (and $P_p = 0$), if $\beta > (2^{R_{full-coop}-\epsilon} - 1)/((1 - \nu^{*2})P)$, we have $R = R_{full-coop} - \epsilon$ in (4)-(5). In other words, if β is large enough, it is optimal to invest all the power in the common message and dedicate a vanishingly small portion $(1 - \nu^{*2})$ of such power to signalling, since this leads to a rate that approaches the upper bound set by the full-cooperation scenario.

B. Analog Network Coding

In the cellular network of Fig. 1, the MSs can exchange signals among neighbors in order to enable *cooperation*. As will be shown in the next section, such signalling can benefit from a transmission strategy sometimes referred to as *analog network coding*, due to the broadcast nature of the transmission of each MS. The basic idea behind analog network coding is to broadcast a mix of signals rather than of bits, as in

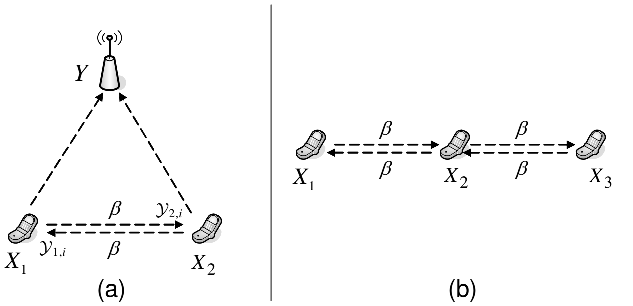

Fig. 2. (a) Two-user Gaussian MAC with generalized feedback; (b) Two-way relay network.

conventional network coding, and to cancel interfering signals using side information available at the decoder. This idea is illustrated by the example in Fig. 2-(b), which represents the so called two-way relay network. In this channel, studied in a large number of recent works, see, e.g., [11]-[16], nodes 1 and 3 have data to communicate to each other via relay 2: upon reception of the signal transmitted by nodes 1 and 3, relay node 2 can broadcast a mix of the two signals relying on the fact that both nodes 1 and 3 can cancel their own signal, as they clearly have side information about it. Different techniques have been proposed to perform transmission from nodes 1 and 3 to 2 first and then from 2 to 1 and 2, according to amplify-and-forward, decode-and-forward or denoise-and-forward techniques (see [11]). The extension to linear networks with more than three nodes, which is of interest for our scenario, has also been treated in different works (see, e.g., [16]).

III. System model

The system model we consider provides a simple abstraction for the uplink of a cellular system with generalized feedback. The setup generalizes the linear Wyner model studied in a large number of works (see [10] for a review). We assume that only one user is active at any transmission block (e.g., intra-cell TDMA) and that we have perfect synchronization. Using notation similar to that in Sec. II-A, and referring to Fig. 1, the signal received by the mth BS at the ith symbol is given by $(0 \leq \alpha \leq 1)$

$$Y_{m,i} = X_{m,i} + \alpha(X_{m-1,i} + X_{m+1,i}) + Z_{m,i}, \tag{6}$$

with i.i.d. (over both time i and BS index m) Gaussian noise $Z_{m,i} \sim \mathcal{CN}(0,1)$ and per-symbol power constraint $E[|X_{m,i}|^2] \leq P$. Generalized feedback at the MSs amounts to having the mth MS receive at the ith symbol

$$\mathcal{Y}_{m,i} = \beta(X_{m-1,i} + X_{m+1,i}) + \mathcal{Z}_{m,i}, \tag{7}$$

with i.i.d. (over both time i and BS index m) Gaussian noise $\mathcal{Z}_{m,i} \sim \mathcal{CN}(0,1)$, and $\beta \geq 0$. The noise processes $\{\mathcal{Z}_{m,i}\}_{i=1}^n$ are generally correlated with $\{Z_{m,i}\}_{i=1}^n$ similarly to (4) as $(|\rho| \leq 1)$

$$\mathcal{Z}_{m,i} = \rho Z_{m,i} + \sqrt{1-\rho^2}\tilde{\mathcal{Z}}_{m,i}. \tag{8}$$

Encoding is defined similarly to Sec. II-A and so is the *equal rate* R, which can be interpreted here as a *per-cell rate*. Decoding is performed via *multicell processing*, i.e., the central processor decides on the transmitted messages $\{W_m\}_{m=1}^M$ based on the signals received by all the BSs. The achievability of a per-cell rate R is defined based on the error probability $\Pr[\{\hat{W}_m\}_{m=1}^M \neq \{W_m\}_{m=1}^M]$ at the central processor.

Finally, we recall the expression of the per-cell capacity of the system at hand in the absence of cooperation (i.e., for $\beta = 0$ and $\rho = 0$), which is given by (see [10])

$$R_{no-coop} = \int_0^1 \log(1 + PH(f)^2)df, \tag{9}$$

where we have defined the transfer function $H(f) = 1 + 2\alpha\cos(2\pi f)$ with $0 \leq f \leq 1$. Rate (9) clearly sets a lower bound on the maximum achievable per-cell rate R. An upper bound can be instead obtained by considering full cooperation at the MSs, which leads to [20]

$$R_{full-coop} = \int_0^1 \log\left(1 + P\cdot H(f)^2 S(f)\right)df, \tag{10}$$

with $S(f)$ corresponding to the waterfilling power spectral density:

$$S(f) = \left(\mu - \frac{1}{PH(f)^2}\right)^+ \tag{11a}$$

$$\text{s.t.} \int_0^1 S(f)df = 1. \tag{11b}$$

IV. An Achievable Per-Cell Rate

The following proposition is the main contribution of this paper and describes an achievable per-cell rate for the system in Fig. 1.

Proposition 1: Fix an integer $K > 0$, a $(2K+1)\times 1$ complex unit-norm symmetric vector $\mathbf{g} = [g_K \;\; g_{K-1}\cdots g_0 \cdots g_{K-1} \;\; g_K]^T$ $(\sum_{k=-K}^K |g_k|^2 = 1)$ with Fourier transform $G(f)$, and constants $0 \leq \nu_i \leq 1$, $i = 1, 2$, the following rate is achievable for any $\beta \geq 0$ and $|\rho| \leq 1$:

$$R = \min\left\{\begin{array}{l} \int_0^1 \log(1 + P_p H(f)^2)df + R_c, \\ \int_0^1 \log(1 + P_p H(f)^2 + \nu_1^2 P_c |G(f)|^2 H(f)^2)df \end{array}\right\} \tag{12}$$

with

$$R_c = \min\left\{\begin{array}{l} \frac{1}{2}\log\left(1 + \frac{\beta^2 2P_c(1-\nu_1^2)(1-\nu_2^2)}{1+\beta^2(2|g_K|^2 P_c\nu_1^2 + 2P_p)}\right), \\ \frac{1}{2(K-1)}\log\left(1 + \frac{\beta^2 P_c(1-\nu_1^2)\nu_2^2}{1+\beta^2(2|g_K|^2 P_c\nu_1^2 + 2P_p)}\right), \\ \frac{1}{2K}\log\left(1 + \frac{\beta^2 P_c(1-\nu_1^2)(2-\nu_2^2)}{1+\beta^2(2|g_K|^2 P_c\nu_1^2 + 2P_p)}\right), \\ \frac{1}{2K-1}\log\left(1 + \frac{\beta^2 P_c(1-\nu_1^2)}{1+\beta^2(2|g_K|^2 P_c\nu_1^2 + 2P_p)}\right), \\ \frac{1}{1+K}\log\left(1 + \frac{\beta^2 P_c/2(1-\nu_1^2)(4-3\nu_2^2)}{1+\beta^2(2|g_K|^2 P_c\nu_1^2 + 2P_p)}\right) \end{array}\right\} \tag{13}$$

and $P_c + P_p = P$.

Sketch of proof: Rate (12)-(13) is achieved via block-Markov encoding using rate and power splitting, in a way that resembles the achievability scheme for the two-user Gaussian

MAC channel described in Sec. II-A. Specifically, as for that basic scenario, each MS divides its resources between transmission of *private* and *common* information, where the latter is transmitted cooperatively by the MSs to the BSs. Moreover, in order to enable cooperation, the MSs exchange *signalling* information about the common messages. The main issue is how to perform this task in an effective manner. As explained below, this can be done by using decode-and-forward techniques and exploiting the side information available at each MS regarding the signals generated at the MS itself or already decoded by it.

To be more specific, fix an integer $K > 0$. Encoding is performed in $B + K$ blocks. To generate such codebooks, a number of auxiliary variables per MS are defined:

- U_m accounts for the common information of the mth MS that is cooperatively sent by a number of other MSs to the BSs. Specifically, cooperation takes place with $2K$ other MSs, K on each side of the mth MS (see also [20]);
- $V_{r,m}$ and $V_{l,m}$ represent the signalling information that the mth MS sends to the neighboring MSs to the right (the $(m+1)$th MS) and left (the $(m-1)$th MS), respectively, to enable cooperation. More specifically, as explained below, $V_{r,m}$ is used by the mth MS to propagate signalling information from left to right; similarly, $V_{l,m}$ propagates signalling information from right to left. Note that the decoding of such signalling information at each MS uses the fact that the codewords generated from $V_{r,m+1}$ and $V_{l,m-1}$ are known at the mth MS as they carry information that has been previous routed through the mth MS (analog network coding); and
- $V_{d,m}$ accounts for signalling information generated locally by the mth MS.

The joint distribution of all random variables is chosen to be complex Gaussian: $U_m,\ V_{r,m},\ V_{l,m},\ V_{d,m},\ U_{d,m}$ are independent distributed as $\mathcal{CN}(0,1)$ and

$$V_m = \nu_1\sqrt{P_c}\sum_{k=-K}^{K} g_k U_{m-k} + \nu_2\sqrt{\frac{P_c}{2}(1-\nu_1^2)}(V_{r,m} + V_{l,m}) + \sqrt{P_c(1-\nu_1^2)(1-\nu_2^2)}V_{d,m} \quad (14)$$

$$X_m = V_m + \sqrt{P_p}U_{d,m}, \quad (15)$$

where the parameters are constrained as in Proposition 1. Note that P_p and P_c correspond, respectively, to the powers used for transmitting *private* and *common* information: the former is sent to the BSs without any cooperation from other MSs, unlike the latter, where cooperation occurs with $2K$ other MSs (see the discussion above). Furthermore, the common-part power P_c is divided between the power used for cooperative transmission to the BSs, given by $\nu_1^2 P_c$ (the first term in (14)), and the power used for signalling to other MSs so as to enable cooperation, given by $(1-\nu_1^2)P_c$. The latter power is in turn split between the power employed to forward signalling information received from neighbors ($\nu_2^2 P_c(1-\nu_1^2)$, the second term in (14)) and locally generated common information ($P_c(1-\nu_1^2)(1-\nu_2^2)$, the last term in (14)). We emphasize that the parameter ν_1 is especially critical as it accounts for the trade-off between power used for cooperative transmission to the BSs and that used for signalling among MSs.

As further shown in Appendix-B, using an appropriately designed block-Markov coding strategy and backward decoding at the destination, rates (12)-(13) can be achieved. It is noted that, similarly to (4)-(5) for the two-user Gaussian MAC, a proper choice of the common rate R_c as in (13) guarantees correct decoding of the signalling messages at the MSs, while condition (12) enables correct decoding at the central processor.

Remark 3: As discussed in Remark 1, the scheme achieving rate (12) fails to be optimal even in the case of the two-user Gaussian MAC. However, as will be discussed in the next section, it obtains relevant gains with respect to a non-cooperative scenario.

Remark 4: In Remark 2, it was noted that for a two-user Gaussian MAC, if the MS measurements are of good enough quality, the scheme at hand is able to attain the upper bound corresponding to full cooperation. The same conclusion applies to the rate (4)-(5) for the cellular MAC of Fig. 1. In fact, if β is large enough in (7), by selecting $P_c = P$ (and $P_p = 0$) and ν_1^2 sufficiently close to 1, one can get arbitrarily close to the upper bound (10). To see this, observe that: (*i*) For K large enough, we can have $|G(f)|^2 \simeq S(f)$ in (12) since a finite-impulse-response filter $\mathbf{g}$ can approximate any frequency response (e.g., the waterfilling solution $S(f)$) if the number of taps is large enough (see also [20]); and (*ii*) One can set $g_K = 0$ so as to avoid interference in the decoding of the inter-MS signalling messages (see (13)): by setting $g_K = 0$, every MS effectively cooperates with only $2(K-1)$ MS, rather than $2K$, thus sacrificing some cooperative gain to enable the two neighboring MSs to cancel all the interference caused by such cooperative signals (see the proof in Appendix-B and the discussion above for details).

V. NUMERICAL RESULTS

In this section, we provide some numerical results to obtain insight into the performance and limitations of the achievable rate derived above. For comparison, we consider the rate (13) achievable with no cooperation, which sets a lower bound, and the upper bound (10) corresponding to full cooperation. To reduce the number of optimization variables, we fix $\nu_2^2 = (K-1)/K$, which essentially corresponds to dividing the power used for signalling in an equal fashion among the common messages communicated at each block. Moreover, the filter $\mathbf{g}$ is chosen (suboptimally) using a frequency-sampling method with target function given by the waterfilling solution (11) as in [20]. The remaining parameters specifying the per-cell achievable rate (12), namely the power allocation (P_c, P_p) and the fraction v_1 of the common power used for cooperative transmission to the BSs or signalling (recall (14)), are optimized numerically. Note that the noise correlation coefficient ρ in (8) plays no role in either the achievable rate or the bounds, and can thus be set to an arbitrary value in the following.

Fig. 3 shows the achievable rate (12) versus the signal-to-noise ratio P as compared to lower and upper bounds for $\alpha = 0.8$, $\beta^2 = 20dB$ [1] and for different values of the number of cooperating terminals $K = 1, 2$, and 3. It is noted that the optimal fraction of common power v_1 used for cooperative transmission decreases with K (not shown), as expected, since more power is required for signalling as the number of common messages to be delivered increases. We also remark that increasing the number of cooperating MSs beyond $K = 3$ is deleterious in terms of achievable rates in this example, due to the limitations in terms of resources for signalling. Also shown is the case $\beta^2 = 30dB$, $K = 2$. It is seen that, if β^2 is sufficiently large, the proposed scheme enables relevant rate gains with respect to no cooperation, and allows the system to partially bridge the gap to the upper bound corresponding to full cooperation.

This fact is further investigated in Fig. 4 where the rates discussed above are shown versus β^2 for $\alpha = 0.6$ and $P = -2dB$. Following Remark 4, we set $g_K = 0$ and obtain the remaining $2(K-1)+1$ taps in $\mathbf{g}$ according to the frequency-sampling as above. This is motivated by the fact that we are interested in achieving the upper bound of full cooperation for sufficiently large β^2. As discussed above, a full optimization over all the parameters has the potential of further increasing the achievable rate. It should also be emphasized that the choice to operate in the low-SNR regime is motivated by the fact that only in this regime are gains from cooperation attainable: In fact, for sufficiently large SNR P, the upper (10) and lower (9) bounds coincide, implying that optimal performance is achieved without any cooperation among MSs. Fig. 4 confirms that, for sufficiently large β and K, the performance of the proposed scheme attains the upper bound of full cooperation.

VI. Conclusions

The presence of feedback signals can be exploited by the MSs of a cellular system to set up cooperation. The achievable rate derived in this paper extends previously proposed techniques for two-user Gaussian MACs by exploiting the regular cellular structure to effectively communicate among nearby MSs via "analog network coding" techniques. Numerical results show that, even when accounting for the resources needed to set up cooperation via inter-MS signalling, feedback-based techniques can achieve rate gains in the low-SNR regime, and can achieve the upper bound of full cooperation if the MS measurement channels are "good" enough. The results here provide a more complete picture regarding the gains from MS cooperation predicted by [20], where the inter-MS channel used for signalling were provided as extra resources orthogonal to the main uplink channel.

Interesting extensions of this work include devising feedback strategies similar to [3] [4] based on local feedback from BSs to corresponding MSs and extending the results to two-dimensional cellular models [10].

[1] β^2, when measured in dB, is defined as $10\log_{10}\beta^2$, where the latter is in linear scale.

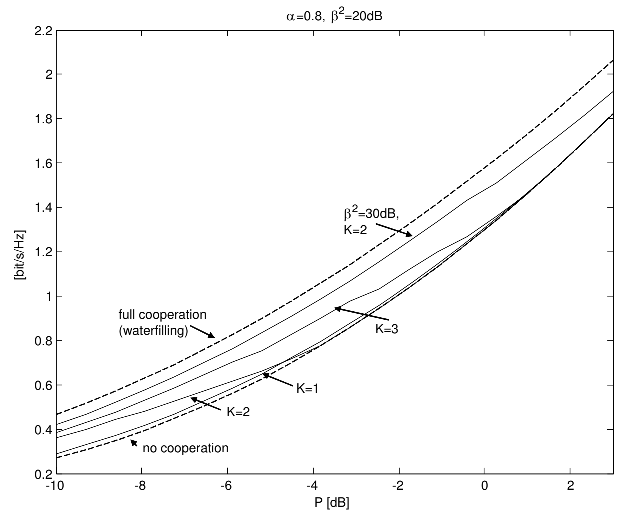

Fig. 3. Achievable rate (12) versus P as compared to lower (9) (no cooperation) and upper bound (10) (full cooperation) for $\alpha = 0.8$ and $\beta^2 = 20dB$ and for different values of the number of cooperating MSs $K = 1, 2, 3$.

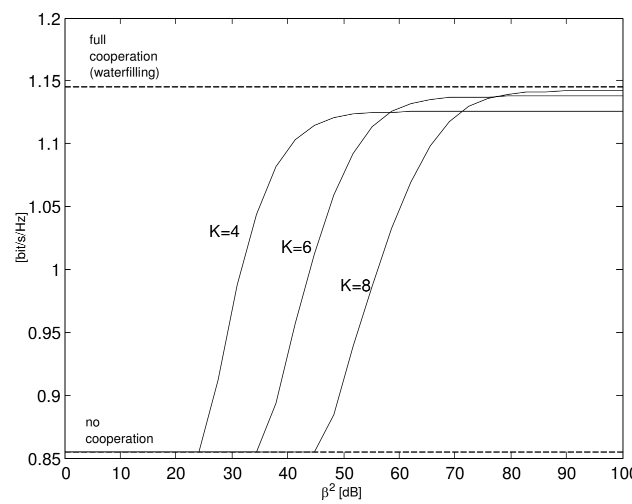

Fig. 4. Achievable rate (12) versus β^2 as compared to lower (9) (no cooperation) and upper bound (10) (full cooperation) for $\alpha = 0.6$ and $P = -2dB$.

Appendix

A. Brief Derivation of (4)-(5)

Using the achievable rates of [5]–[7] (see also [8]), we obtain the following conditions on the equal rate R (recall definitions in Sec. II-A):

$$R \le I(X_1;Y|UV_1V_2X_2) + I(V_1;Y_2|UX_2) \quad (16a)$$
$$R \le I(X_2;Y|UV_1V_2X_1) + I(V_2;Y_1|UX_1) \quad (16b)$$
$$R \le \frac{1}{2}I(X_1X_2;Y|UV_1V_2) \quad (16c)$$
$$+ \frac{1}{2}I(V_1;Y_2|UX_2) + \frac{1}{2}I(V_2;Y_1|UX_1) \quad (16d)$$
$$R \le \frac{1}{2}I(X_1X_2;Y) \quad (16e)$$

to be maximized over auxiliary random variables U, V_1, V_2 satisfying the Markov chain relationship $(V_1X_1)-U-(V_2X_2)$ and the power constraint $E[|X_i|^2] \le P_i$. As discussed in Remark 1, the Markov condition limits the correlation structure between the codewords of the two sources. Now for the following choice of auxiliary random variables ($i = 1, 2$), we have

$$V_i = \nu\sqrt{P_c}U + \sqrt{(1-\nu^2)P_c}V_i' \quad (17a)$$
$$X_i = V_i + \sqrt{P_p}U_i' \quad (17b)$$

with $0 \le \nu \le 1$, $P_c + P_p = P$ and U, V_i' and U_i' ($i = 1, 2$) independent $\mathcal{CN}(0,1)$; defining $R_c = I(V_1;Y_2|UX_2) = I(V_2;Y_1|UX_1)$, one gets (4)-(5).

B. Proof of Proposition 1

Referring to Sec. IV for basic definitions, here we complete the proof.

Codebook generation:

Split the per-cell rate R as $R = R_p + R_c$ (private and common parts, $R_p, R_c \ge 0$).

- Generate 2^{nR_c} codewords $u_m^n(\tilde{w}_{c,m})$, $\tilde{w}_{c,m} = 1, 2, ..., 2^{nR_c}$, by choosing each symbol independently according to $\mathcal{CN}(0,1)$;
- Generate $2^{n(K-1)R_c}$ codewords $v_{r,m}^n(\{w_{c,m-k}\}_{k=1}^{K-1})$ for $w_{c,m-k} = 1, 2, ..., 2^{nR_c}$ ($k = 1, ..., K-1$) by choosing independent $\mathcal{CN}(0,1)$ symbols. Proceed similarly for codewords $v_{l,m}^n(\{w_{c,m+k}\}_{k=1}^{K-1})$, $w_{c,m+i} = 1, 2, ..., 2^{nR_c}$. Finally, generate 2^{nR_c} codewords $v_{d,m}^n(w_{c,m})$, $w_{c,m} = 1, 2, ..., 2^{nR_c}$, again by choosing independent $\mathcal{CN}(0,1)$ symbols;
- For each value of $\tilde{\mathbf{w}}_{c,m} = (\{\tilde{w}_{c,m+k}\}_{k=-K}^{K})$ and $\mathbf{w}_{c,m} = (\{w_{c,m+k}\}_{k=-K+1}^{K-1})$ create a codeword $v_m^n(\tilde{\mathbf{w}}_{c,m}, \mathbf{w}_{c,m})$ by summing the corresponding $u_m^n(\tilde{\mathbf{w}}_{c,m})$, $v_{r,m}^n(\{w_{c,m-k}\}_{k=1}^{K-1})$, $v_{l,m}^n(\{w_{c,m+k}\}_{k=1}^{K-1})$, $v_{d,m}^n(w_{c,m})$ according to (14);
- For each value of $\tilde{\mathbf{w}}_{c,m}$ and $\mathbf{w}_{c,m}$ generate 2^{nR_p} codewords $x_m^n(\tilde{\mathbf{w}}_{c,m}, \mathbf{w}_{c,m}, w_{p,m})$ with $w_{p,m} = 1, 2, ..., 2^{nR_p}$ by choosing each symbol independently according to the complex Gaussian distribution $f_{X|V}(\cdot|v_{m,i}(\tilde{\mathbf{w}}_{c,m}, \mathbf{w}_{c,m}))$ defined as in (15).

Encoders: We use Block-Markov encoding over $B + K$ blocks. Each MS message w_m carries $n((B+K)R_p+BR_c) = n(BR + KR_p)$ bits and it is split into two parts, one private $w_{p,m}$ with $n(B + K)R_p$ and one common with nBR_c bits. The private message is then further split into $B + K$ blocks $w_{p,m}^{(b)}$, $b = 1, 2, ..., B + K$ (each with nR_p bits), while the common message is split into B blocks $w_{c,m}^{(b)}$, $b = 1, 2, ..., B$, each with nR_c bits. Note that the overall rate is

$$\bar{R}(K) = \frac{BR + KR}{B},$$

so that $\bar{R}(K) \to R$ for $B \to \infty$ and *fixed* K.

Define $\tilde{\mathbf{w}}_{c,m}^{(b)} = (\{w_{c,m+k}^{(b-K)}\}_{k=-K}^{K})$ and $\mathbf{w}_{c,m}^{(b)} = (\mathbf{w}_{c,r,m}^{(b)}, w_{c,m}^{(b)}, \mathbf{w}_{c,l,m}^{(b)})$ with $\mathbf{w}_{c,r,m}^{(b)} = \{w_{c,m-k}^{(b-k)}\}_{k=1}^{K-1}$, $\mathbf{w}_{c,l,m}^{(b)} = \{w_{c,m+k}^{(b-k)}\}_{k=1}^{K-1}$. Assume that at block b each mth MSs has correctly estimated $\tilde{\mathbf{w}}_{c,m}^{(b)}$ (it clearly already knows $w_{c,m}^{(b-K)}$) and $\mathbf{w}_{c,m}^{(b)}$ (it knows $w_{c,m}^{(b)}$) (see the description of decoding below). The mth encoder transmits $x_m^n(\tilde{\mathbf{w}}_{c,m}^{(b)}, \mathbf{w}_{c,m}^{(b)}, w_{p,m})$ at block b, where $w_{c,m}^{(b')} = 1$ for $b' \le 0$ and $b' \ge B$. See Fig. 5 for an illustration for $B = 2$ and $K = 2$.

Decoders:

Decoders at the mth MS: The mth MS at block b, based on the observation of $\mathcal{Y}_m^{n(b)}$ is interested in decoding $\mathbf{w}_{c,l,m+1}^{(b)}$, $\mathbf{w}_{c,r,m-1}^{(b)}$, $w_{c,m-1}^{(b)}$ and $w_{c,m+1}^{(b)}$. This is done via a joint typicality decoder that attempts to find the variables mentioned above such that the sequences

$$\{\{u_{m+k}^n(w_{c,m+k}^{(b-K)})\}_{k=-K}^{K},\ v_{r,m-1}^n(\mathbf{w}_{c,r,m-1}^{(b)}),$$
$$v_{l,m-1}^n(\mathbf{w}_{c,l,m-1}^{(b)}),\ v_{r,m+1}^n(\mathbf{w}_{c,r,m+1}^{(b)}),\ v_{l,m+1}^n(\mathbf{w}_{c,l,m+1}^{(b)}),$$
$$v_{d,m-1}^n(w_{c,m-1}^{(b)}),\ v_{d,m+1}^n(w_{c,m+1}^{(b)}), \mathcal{Y}_m^{n(b)}\}$$

are jointly typical. Notice that messages $\{w_{c,m+k}^{(b-K)}\}_{k=-K}^{K}$, $\mathbf{w}_{c,l,m-1}^{(b)}$ and $\mathbf{w}_{c,r,m+1}^{(b)}$ are known by the mth MS at the bth block as they have either been decoded or generated by the mth MS at previous blocks.

Decoder at the CP: The CP uses backward decoding and joint typicality detection [7].

Analysis of error probability:

At the mth MS: There are 15 disjoint error events and keeping only the dominant ones, we get

$$R_c \le \frac{1}{2}\log\left(1 + \frac{2\beta^2 P_c(1-\nu_1^2)(1-\nu_2^2)}{1+\beta^2(2|g_K|^2P_c\nu_1^2 + 2P_p)}\right)$$
$$R_c \le \frac{1}{2(K-1)}\log\left(1 + \frac{\beta^2 P_c(1-\nu_1^2)\nu_2^2}{1+\beta^2(2|g_K|^2P_c\nu_1^2 + 2P_p)}\right)$$
$$R_c \le \frac{1}{2K}\log\left(1 + \frac{\beta^2 P_c(1-\nu_1^2)(2-\nu_2^2)}{1+\beta^2(2|g_K|^2P_c\nu_1^2 + 2P_p)}\right)$$
$$R_c \le \frac{1}{2K-1}\log\left(1 + \frac{\beta^2 P_c(1-\nu_1^2)}{1+\beta^2(2|g_K|^2P_c\nu_1^2 + 2P_p)}\right)$$
$$R_c \le \frac{1}{1+K}\log\left(1 + \frac{\beta^2 P_c/2(1-\nu_1^2)(4-3\nu_2^2)}{1+\beta^2(2|g_K|^2P_c\nu_1^2 + 2P_p)}\right).$$

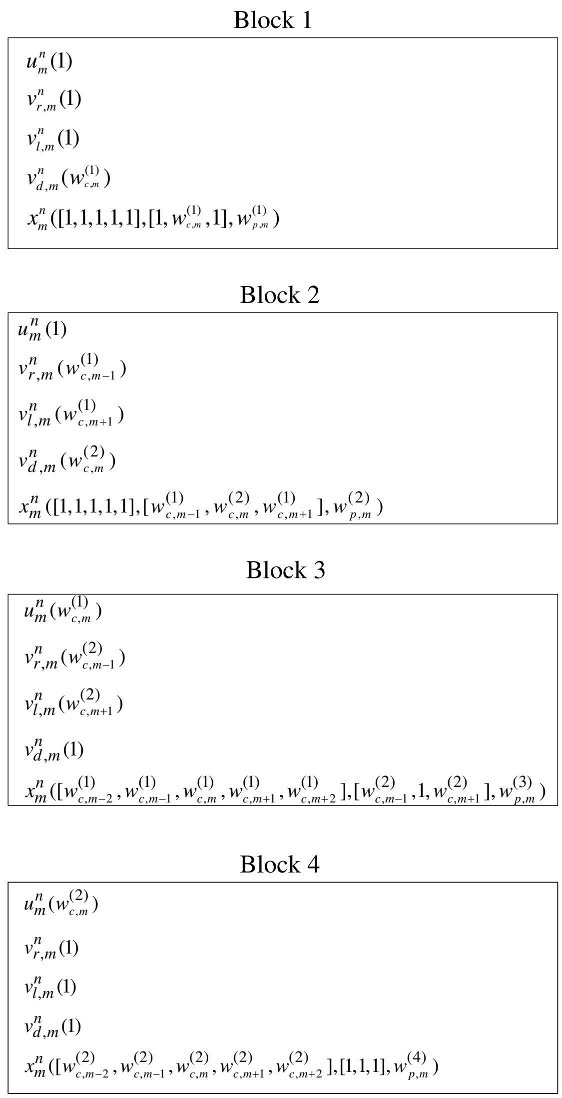

Fig. 5. Block-Markov encoding strategy for $B = 2$ and $K = 2$.

At the BSs: Using backward decoding, the analysis of the error probability reduces to the one carried out in [20] so that

$$R_p + R_c \leq \int_0^1 \log(1 + P_p H(f)^2 + \nu_1^2 P_c |G(f)|^2 H(f)^2) df$$
$$R_p \leq \int_0^1 \log(1 + P_p H(f)^2) df,$$

from which Proposition 1 is proved.

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