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Stable throughput of cognitive radios with relaying capability

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***Abstract*— A cognitive interference channel consists of two single-user links, one licensed to use the spectral resource (primary) and one unlicensed (secondary or cognitive). According to the cognitive radio principle, the activity of the secondary link should not interfere with the performance of the primary. The cognitive transmitter is allowed to access the channel only when sensed idle. In this paper, the advantages of having the cognitive transmitter acting as a "transparent relay" for the packets of the primary are investigated in terms of stable throughput (packets/slot). The analysis accounts for random packet arrivals, sensing errors due to fading at the secondary link, and power allocation at the secondary transmitter based on long-term measurements.**

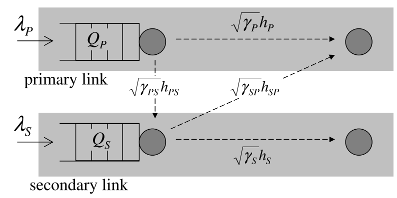

Fig. 1. MAC and physical layer view of a cognitive interference channel.

I. INTRODUCTION

According to the principle informing cognitive radio, nodes of a radio network are divided into licensed (or primary) and unlicensed (secondary or cognitive). While the first group is allowed to access the spectrum any time, the second seeks opportunities for transmission by exploiting the idle periods of primary nodes [1]. The main requirement is that the activity of the secondary nodes should be "transparent" to the primary, so as not to interfere with the licensed use of the spectrum. Toward this end, cognitive radios follow an *observation/ decision* cycle that prescribes to first measure (observe) relevant radio parameters (e.g., detect the activity of primary users), and then decide the appropriate action at all layers of the protocol stack according to the quality-of-service requirements of the targeted application.

A first attempt to study a cognitive scenario from an information theoretic standpoint has been presented in the landmark work [2] and in [3], where a basic channel consisting of two source-destination links, one primary and one secondary, is considered (cognitive interference channel). The basic assumption in [2] [3] is that the cognitive transmitter has perfect prior information about the message transmitted by the primary (see also [4]).

In [5], a stability analysis (in the sense of [6]) of the cognitive interference channel has been reported, by considering random packet arrivals, slotted time, and measurement errors at the cognitive transmitter (see fig. 1). It was shown that measurement errors limit the throughput attainable by the cognitive link. The analysis was therein carried out by allowing the cognitive transmitter to control its transmitted power based on long-term measurements in order to guarantee "transparency" of the activity of the secondary link.

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In this paper, we reconsider the cognitive interference channel of fig. 1 under the same assumptions of [5] by adding the capability at the cognitive transmitter to act as a "transparent" relay for the packets of the primary (see fig. 2). More precisely, we let the secondary transmitter forward packets of the primary user that it has been able to decode, but have not been successfully received by the intended destination. The rationale for this choice is that, if the propagation channel from the primary transmitter to the secondary transmitter is advantageous with respect to the direct (primary) channel, having packets relayed by the secondary can help emptying the queue of the primary, thus creating transmitting opportunities for the secondary. Clearly, this increased number of available slots for the cognitive node has to be shared between transmission of own packets and relayed packets. Assessing the benefits of this modified structure is then not trivial, and is the subject of this paper.

II. SYSTEM MODEL

In order to present the system model, it is expedient to first consider the basic scenario in fig. 1 (treated in [5]) and then extend it in order to add relaying capability to the secondary link as in fig. 2.

A. The basic cognitive interference channel

Let us first consider the scenario in fig. 1. Both primary and secondary transmitting nodes have a buffer of infinite capacity to store incoming packets. Time is slotted and transmission of each packet takes one slot (all packets have the same number of bits). The packet arrival processes at each node are independent and stationary with mean λ_P [packets/slot] for the primary user and λ_S [packets/slot] for the cognitive (see fig. 1). According to the cognitive principle, the primary link employs the channel whenever it has some packets to transmit in its queue. On the other hand, the secondary (cognitive) transmitter senses the channel in each slot and, if it detects an idle slot, transmits a packet (if there is any) from its queue.

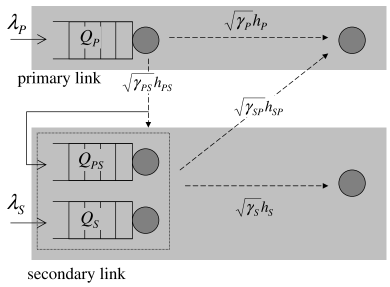

Fig. 2. A modified interference cognitive channel where the secondary transmitter may act as a relay for the primary link.

Radio propagation between any pair of nodes is assumed to be affected by *independent stationary Rayleigh fading* channels $h_i(t)$ with $E[|h_i(t)|^2] = 1$ (t denotes time and runs over time-slots). The average channel power gain (due to shadowing and path loss) is denoted as γ_i, where subscript i identifies the link as shown in fig. 1. The primary node transmits with normalized power $P_P = 1$ and, without loss of generality, the noise power spectral density at all receivers is also normalized to 1. The power transmitted by the secondary node (when active) is $P_S \leq 1$.

Transmission of a given packet is considered successful if the instantaneous received signal-to-noise ratio $\gamma_i|h_i(t)|^2 P_i$ is above a given threshold β_i, that is fixed given the choice of the transmission mode. Therefore, the *probability of outage* (unsuccessful packet reception) on the primary or secondary link reads (i equals "P" or "S")

$$\mathrm{P}_{out,i} = \mathrm{P}[\gamma_i|h_i(t)|^2 P_i < \beta_i] = 1 - \exp\left(-\frac{\beta_i}{\gamma_i P_i}\right). \quad (1)$$

Notice that the primary and secondary links can in general employ transmission modes with different signal-to-noise ratio requirements, $\beta_P \neq \beta_S$. Unsuccessful reception is signalled by a NACK message and requires retransmission.

The cognitive transmitter is able to correctly detect the transmission of the primary user if the instantaneous channel gain $\gamma_{PS}|h_{PS}|^2$ is larger than a threshold α (recall that $P_P = 1$). It follows that the probability of error in the detection process is

$$\mathrm{P}_e = \mathrm{P}[\gamma_{PS}|h_{PS}(t)|^2 < \alpha] = 1 - \exp\left(-\frac{\alpha}{\gamma_{PS}}\right). \quad (2)$$

Moreover, it is assumed that whenever the primary user is not transmitting, the secondary user is able to detect an idle slot with zero probability of error. This assumption is reasonable in the scenario at hand where interference from other systems is assumed to be negligible.

B. Adding relaying capabilities to the secondary transmitter

Here, the model presented above is modified in order to account for the added relaying capability at the secondary transmitter. The only differences concern the transmission strategy of the cognitive node and details on the exchange of ACK/ NACK messages. With reference to fig. 2, the cognitive node has two queues, one collecting own packets ($Q_S(t)$) and one containing packets received by the primary transmitters to be relayed to the primary destination ($Q_{PS}(t)$). A packet transmitted by the primary node can in fact be erroneously received by the intended destination (that signals the event with a NACK message) but correctly received by the secondary transmitter (that sends an ACK message). In this case, the primary source drops the packet from its queue, as if correctly received by the destination[1], and the secondary puts it in its queue $Q_{PS}(t)$. Notice that if both primary destination and secondary transmitter correctly decode the signal, the secondary does not include the latter in its queue (upon reception of the ACK message from the destination).

Whenever the secondary node senses an idle slot (and it does so with error probability P_e in (2)), *it transmits a packet from queue $Q_{PS}(t)$ (primary's packets) with probability ε and from the second queue $Q_S(t)$ (own packets) with probability $1 - \varepsilon$.*

C. Problem formulation

As elaborated below, the primary users selects a throughput λ_P based on its quality-of-service requirements, being oblivious to the presence of the secondary link. The secondary node then select its transmission power $P_S \leq 1$ *and* the probability ε based on the knowledge of the statistics of the channels ($\gamma_P, \gamma_S, \gamma_{SP}, \gamma_{PS}$) and the system parameters ($\alpha, \beta_P, \beta_S, \lambda_P$) towards the following conflicting goals: (*i*) ensuring "transparency" of its activity; (*ii*) maximizing its own stable throughput.

In this paper, "transparency" of the cognitive node to the primary user is defined in terms of *stability* of the queue of the primary user. That is, as a result of the activity of the secondary, the primary node is guaranteed that its queue will remain stable. However, no constraints are imposed on the average delay.

Knowledge of channel parameters is assumed at the cognitive link under the premise that in the assumed stationary fading scenario, the cognitive node will have enough time to infer these parameters during the observation phase of the cognitive cycle. Possible solutions to achieve this goal can build on collaboration with the secondary receiver in idle slots (see [11] for further information on cooperative detection in cognitive networks). Moreover, system parameter β_P is assumed to be part of the prior knowledge available at the secondary link about the primary communication. Finally, the throughput λ_P selected by the primary link can be estimated by observing the fraction of idle slots and measuring the ACK/ NACK messages sent by the secondary receiver.

III. Stability throughput of the cognitive link

In this Section, we investigate the solution to the problem described in Sec. II-C.

[1]This a small deviation from the cognitive radio principle of transparency of the secondary user to the primary: in fact, because of the secondary activity, the primary might receive two acknowledgments for the same packet. In this case, it will simply consider the packet as correctly received if at least one acknowlegment is positive.

A. Some definitions and analytical tools

Stability is defined as the state where all the queues in the system are stable (see [6] for rigorous definitions). If *arrival and departure rates of a queuing system are stationary*, then stability can be checked by using *Loynes' theorem* [7]. This states that, under the said assumption, if the average arrival rate λ_i is less than the average departure rate μ_i, $\lambda_i < \mu_i$, then the ith queue is stable; on the other hand, if the average arrival rate λ_i is greater than the average departure rate μ_i, the queue is unstable; finally, if $\lambda_i = \mu_i$, the queue can be either stable or unstable. Whenever the Loynes' theorem is applicable, we define the average departure rate μ_i as the *maximum stable throughput* of the ith queue.

Application of the Loynes' theorem largely simplifies the task of studying the stability of a queuing process. However, while stationarity of arrival processes is guaranteed by the model assumptions, the same cannot be ensured for departure rates, notwithstanding the assumed stationarity of the channel fading processes. This is due to the interaction between the queue at the primary and secondary transmitting nodes (see [6] [8]). In such a case, a powerful tool that enables to study stability by still exploiting the Loynes' theorem has been developed in [6]. The idea is to analyze a transformed system, referred to as *dominant*, that has the same stability properties as the original system and, at the same time, presents non-interacting queues.

B. The point of view of the primary user

According to the cognitive principle, the primary link is unaware of the presence of a secondary node willing to use the bandwidth whenever available. Therefore, as far as the primary link is concerned, the system consists of a single queue (its own), characterized by a stationary departure rate (due to the stationarity of the channel fading process $h_P(t)$) with average (recall (1)) $\mu_P^{\max} = 1 - \mathrm{P}_{out,P} = \exp(-\beta_P/\gamma_P)$. Moreover, by the Loynes' theorem, rate $\mu_P^{\max}$ is the maximum stable throughput as "perceived" by the primary user. In other words, the primary user is allowed to select any rate λ_P that satisfies:

$$\lambda_P < \mu_P^{\max} = \exp\left(-\frac{\beta_P}{\gamma_P}\right). \quad (3)$$

C. System analysis

In order to investigate the stability of the system at hand, we employ the concept of *dominant system*. We recall that a dominant system is characterized by the properties that: (*i*) it is stable if and only if the original system is; (*ii*) its queuing processes are non-interacting, thus enabling the possibility to use Loynes' theorem in order to study stability. Similarly to [6] [8] [10], in the scenario of fig. 2, the dominant system can be defined by this simple modification to the original system: if $Q_{PS}(t) = 0$ (or $Q_S(t) = 0$), the secondary user continues to transmit "dummy" packets whenever it senses an idle channel and the first (or second queue) is selected, thus continuing to possibly interfere with the primary user whether its queues are empty or not. Property (*i*) can be demonstrated as in [6], whereas property (*ii*) will be shown while proving the two main Propositions discussed below.

The primary selects an average rate in the range (3). The following proposition determines the maximum power P_S that the secondary node is allowed to transmit in order to guarantee stability of the primary queue (i.e., "transparency" of its activity).

Proposition 1: Given the channel parameters $(\gamma_P, \gamma_{PS}, \gamma_{SP})$ and system parameters $(\alpha, \beta_P, \lambda_P)$:

- if $\lambda_P < \mu_P^{\max}\exp(-\alpha/\gamma_{PS}) + \Delta\mu_P$ with
$$\Delta\mu_P = \exp\left(-\frac{\alpha+\beta_P}{\gamma_{PS}}\right)\left(1-\exp\left(-\frac{\beta_P}{\gamma_P}\right)\right), \quad (4)$$
the secondary user can employ any power P_S without harming the stability of the queue of the primary node, and in particular we can set P_S equal to its maximum, $P_S = 1$;
- if $\mu_P^{\max}\exp(-\alpha/\gamma_{PS}) + \Delta\mu_P \leq \lambda_P < \mu_P^{\max}$, the maximum power that the cognitive node can employ is
$$P_S < \left(\frac{\mu_P^{\max} + \Delta\mu_P - \lambda_P}{\lambda_P - \mu_P^{\max}\exp\left(-\frac{\alpha}{\gamma_{PS}}\right) - \Delta\mu_P}\right)\frac{\gamma_P/\beta_P}{\gamma_{SP}}. \quad (5)$$

Proof: see Appendix-A.

As a direct consequence of Proposition 1, the secondary can employ its maximum power $P_S = 1$ without affecting the stability of the primary's queue for $\lambda_P < \bar{\lambda}_P^{rel}$ where

$$\bar{\lambda}_P^{rel} = \bar{\lambda}_P + \Delta\mu_P \quad (6)$$

and

$$\bar{\lambda}_P = \mu_P^{\max}\left[\frac{\gamma_P/\beta_P + \exp\left(-\frac{\alpha}{\gamma_{PS}}\right)\gamma_{SP}}{\gamma_{SP} + \gamma_P/\beta_P}\right]. \quad (7)$$

From [5], the rate (7) represents the maximum throughput of the primary at which the secondary is allowed to use $P_S = 1$ in case of no relaying (i.e., in the scenario depicted in fig. 1). By comparing this result with (6), we can conclude that relaying enhances the average departure rate of the primary by $\Delta\mu_P$, thus increasing by the same amount the range of primary user throughputs at which the cognitive node is allowed to transmit at full power.

Let us now turn to the analysis of two queuing processes at the cognitive node. The following Proposition addresses the problem raised in Sec. II-C.

Proposition 2: Given the channel parameters $(\gamma_P, \gamma_{PS}, \gamma_{SP}, \gamma_S)$ and system parameters $(\alpha, \beta_P, \beta_S, \lambda_P)$, under the assumption that the stability of the queue of the primary user is preserved ("transparency" of the cognitive node), the maximum stable throughput of the cognitive user is the result of the following optimization problem

$$\max_{P_S} \mu_S(P_S) \text{ (see (23))} \quad (8)$$

$$\text{s.t.} \begin{cases} P_S < \left(\frac{\mu_P^{\max}+\Delta\mu_P-\lambda_P}{\lambda_P-\mu_P^{\max}\exp\left(-\frac{\alpha}{\gamma_{PS}}\right)-\Delta\mu_P}\right)\frac{\gamma_P/\beta_P}{\gamma_{SP}} \\ \quad \text{if } \lambda_P \leq \bar{\lambda}_P^{rel} \\ P_S \leq 1 \\ \quad \text{if } \lambda_P > \bar{\lambda}_P^{rel} \end{cases}$$

$$\varepsilon = \frac{\lambda_P\left(1-\exp\left(-\frac{\beta_P}{\gamma_P}\right)\right)\exp\left(-\frac{\beta_P}{\gamma_{PS}}\right)}{(\mu_P(P_S)+\Delta\mu_P-\lambda_P)\exp\left(-\frac{\beta_P}{\gamma_{SP}P_S}\right)} < 1$$

This problem is convex and can be solved by using standard methods [13].

Proof: see Appendix-B.

In (8) the first constraint limits the transmitted power, according to the results in Proposition 1, so as to ensure the stability of the queue of the primary $Q_P(t)$. On the other hand, the second constraint imposes that the probability ε (of serving queue $Q_{PS}(t)$) that guarantees stability of the queue $Q_{PS}(t)$ is in fact a probability (equality is excluded since it would lead to $\mu_S = 0$).

The optimization problem (8) might not have feasible solutions for some λ_P due to the latter constraint on the stability of $Q_{PS}(t)$, i.e., on ε. For instance, assume that the probability of outage between primary transmitter and secondary transmitter $\mathrm{P}_{out,PS}$ is much smaller than $\mathrm{P}_{out,P}$ and, at the same time, the probability of outage between secondary transmitter and primary receiver $P_{out,SP}$ is large; in this case, it is apparent that most of the traffic passes through queue $Q_{PS}(t)$ that overflows due to the small departure rate towards the secondary receiver.

IV. Numerical results

In this Section, the conclusions stated in the previous section are corroborated by numerical results. Fig. 3 shows the optimal power P_S, the probability ε and the maximum stable throughput μ_S obtained from Proposition 2 versus the throughput selected by the primary node λ_P. Parameters are selected as $\gamma_P = 4dB$, $\gamma_S = \gamma_{SP} = \gamma_{PS} = 10dB$ and $\alpha = 0dB$, $\beta_P = \beta_S = 4dB$. Notice that in this example the average channel gain to and from the "relay" are $6dB$ better than the direct primary link γ_P. The upper figure reveals that, while in the non-relaying mode the cognitive node can transmit maximum power only up to around $\bar{\lambda}_P = 0.34$ (recall (7)), in the relaying case the cognitive node can transmit at the maximum power in the whole range (3). Moreover, from the middle part of fig. 3, queue $Q_{PS}(t)$ in this case is always stabilizable, i.e., the optimal probability ε resulting from (8) is less than one in the range of interest. Finally, the lower part of fig. 3 compares the maximum throughput for the no-relaying case (Proposition 2) and for the relaying case (Proposition 1), showing the relevant advantages of relaying for sufficiently large λ_P.

The performance advantage of using relaying is further illustrated by fig. 4, where the maximum throughput of the secondary user μ_S is plotted for a fixed $\lambda_P = \mu_P^{\max}$ (3)[2] in case of relaying. Notice that for $\lambda_P = \mu_P^{\max}$, the throughput of the cognitive node with no relaying is zero [5], and, therefore, the figure at hand measures the gain obtained by relaying (see also the lower part of fig. 3). Where not stated otherwise, parameters are selected as in the example above. The figure shows that increasing (at the same rate) the quality of the channel to and from the cognitive node (γ_{SP} and γ_{PS}) with respect to γ_P increases the gain of relaying, and that the advantage is more relevant if the direct channel gain is smaller (compare the two curves with $\gamma_P = 4dB$ and $7dB$). Moreover, the issue of feasibility and stability mentioned above is illustrated by fig. 4: if the channels γ_{SP} and γ_{PS} are not good enough to sustain the extra traffic coming from the primary, then the optimization problem (8) does not have any feasible solution and the throughput of the secondary node is zero.

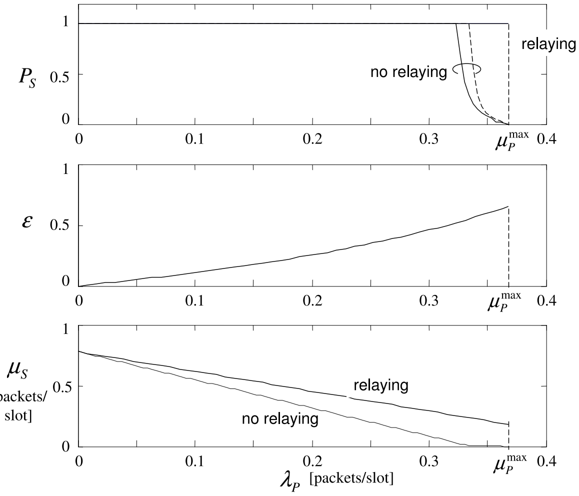

Fig. 3. Optimal power P_S (upper figure), probability ε and maximum stable throughput μ_S obtained from Proposition 2 versus the throughput selected by the primary node λ_P ($\gamma_P = 4dB$, $\gamma_S = \gamma_{SP} = \gamma_{PS} = 10dB$ and $\alpha = 0dB$, $\beta_P = \beta_S = 4dB$).

V. Concluding remarks

The advantages of allowing the secondary transmitter to act as a "transparent" relay for the traffic of the primary link have been investigated. This analysis is meant to shed some light on performance and possible technologies that will enable the application of the cognitive principle. Toward this goal, this point of view should be complemented by feasibility studies from different standpoints such as security and pricing.

VI. Appendix

A. Proof of Proposition 1

The queue of the primary transmitter evolves as

$$Q_P(t) = (Q_P(t-1) - X_P(t))^+ + Y_P(t), \quad (9)$$

[2] To be precise, we should write $\lambda_P = \mu_P^{\max} - \delta$ for an arbitrarily small $\delta > 0$ since the average arrival rate is limited by the model to (3).

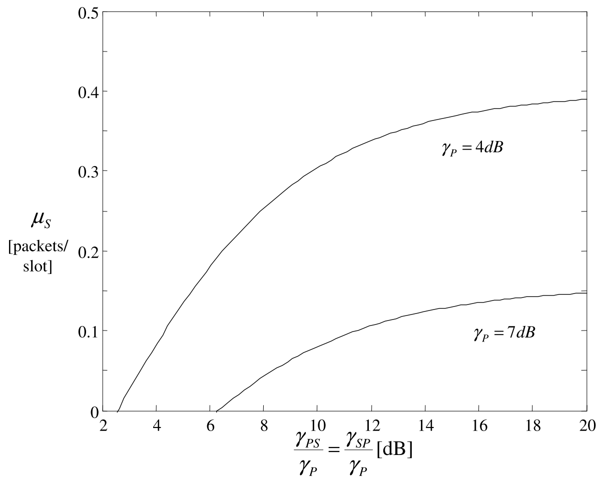

Fig. 4. Maximum throughput of the secondary user μ_S for a fixed $\lambda_P = \mu_P^{\max}$ versus the average channel gain ratios $\gamma_{PS}/\gamma_P = \gamma_{SP}/\gamma_P$ ($\gamma_S = 10dB$ and $\alpha = 0dB$, $\beta_P = \beta_S = 4dB$).

where the departure rate $X_P(t)$ satisfies

$$X_P(t) = \mathbf{1}\{\mathcal{O}_D(t) \cap \mathcal{O}''_P(t)\} + \mathbf{1}\{\mathcal{O}^c_D(t) \cap \mathcal{O}'_P(t)\}, \quad (10)$$

with the following definitions. $\mathbf{1}\{\cdot\}$ is the indicator function of the event enclosed in the brackets; $\mathcal{O}_D(t)$ denotes the event that the cognitive node correctly identifies the ongoing activity of the primary user (and so it does not interfere with it), which happens with probability $1 - \mathrm{P}_e$ in (2); $\mathcal{O}''_P(t)$ represents the event of a successful transmission by the primary user (assuming that the secondary does not interfere), which happens with probability

$$\mathrm{P}''_{out,P} = 1 - \exp\left(-\frac{\beta_P}{\gamma_P}\right) - \exp\left(-\frac{\beta_P}{\gamma_{PS}}\right) \quad (11)$$

$$+\exp\left(-\frac{\beta_P}{\gamma_P} - \frac{\beta_P}{\gamma_{PS}}\right). \quad (12)$$

Notice that the outage probability (11) takes into account the fact that transmission by the primary node is considered successful when the packet is correctly received either by the intended destination (with probability $\exp(-\beta_P/\gamma_P)$) or by cognitive node (with probability $\exp(-\beta_P/\gamma_{PS})$). $\mathcal{O}^c_D(t)$ is the complement of $\mathcal{O}_D(t)$. $\mathcal{O}'_P(t)$ represents the event of a successful transmission by the primary user, in case the secondary interferes. The probability of the latter event is evaluated in the Appendix-C by assuming that the interference from the cognitive node to the primary receiver is Gaussian distributed, and shown to be $1 - \mathrm{P}'_{out,P}$, where the probability of outage in presence of the interference reads:

$$\mathrm{P}'_{out,P} = 1 - \frac{\exp\left(-\frac{\beta_P}{\gamma_P}\right)}{1 + \beta_P \frac{\gamma_{PS}}{\gamma_P} P_S}. \quad (13)$$

According to the discussion above, $X_P(t)$ is a stationary process with mean

$$\mu_P^{rel}(P_S) = E[X_P(t)] = (1-\mathrm{P}_e)(1-\mathrm{P}''_{out,P}) + \mathrm{P}_e(1-\mathrm{P}'_{out,P}). \quad (14)$$

By using (11) in (14), it is easily found that the average departure rate at the primary in the considered relaying scenario reads

$$\mu_P^{rel}(P_S) = \mu_P(P_S) + \Delta\mu_P, \quad (15)$$

having defined $\Delta\mu_P$ as in (4) and $\mu_P(P_S)$ as

$$\mu_P(P_S) = \mu_P^{\max} \cdot \frac{\gamma_P/\beta_P + \exp\left(-\frac{\alpha}{\gamma_{PS}}\right)\gamma_{PS}P_S}{\gamma_P/\beta_P + \gamma_{PS}P_S}, \quad (16)$$

which is shown in [5] to be the maximum stable throughput for the primary in the scenario with no relaying depicted in fig. 1. In other words, cooperation leads to an additive increase of the throughput of the primary user which is independent on P_S. Taking this conclusion into account, Proposition 1 easily follows.

B. Proof of Proposition 2

The first queue size $Q_{PS}(t)$ evolves as $Q_{PS}(t) = (Q_{PS}(t-1) - X_{PS}(t))^+ + Y_{PS}(t)$ where the arrival rate $Y_{PS}(t)$ can be written as

$$Y_{PS}(t) = \mathbf{1}\{\{Q_P(t) \neq 0\} \cap \mathcal{O}^c_P(t)(t) \cap \mathcal{O}_{PS}(t)\}. \quad (17)$$

From the stationarity of the fading processes and stability of the queue of the primary user, the departure process $Y_{PS}(t)$ is stationary with mean (using the Little's theorem, $P[Q_P(t) = 0] = 1 - \lambda_P/\mu_P^{rel}$ [9])

$$\lambda_{PS} = \left(1 - \frac{\lambda_P}{\mu_P^{rel}}\right) \cdot \mathrm{P}_{out,P} \cdot (1 - \mathrm{P}_{out,PS}), \quad (18)$$

with

$$\mathrm{P}_{out,PS} = 1 - \exp\left(-\frac{\beta_P}{\gamma_{PS}}\right).$$

Notice that in writing (17), it is assumed that whenever the secondary node is able to decode the signal of the primary, it is also able to detect its presence, i.e., $\alpha < \beta_P$. On the other hand, the departure process is

$$X_{PS}(t) = \mathbf{1}\{\mathcal{A}_{PS}(t) \cap \mathcal{O}_{SP}(t)\}, \quad (19)$$

with $\mathcal{A}_{PS}(t)$ denoting the event that the tth time slot is available for transmission by the first queue of the secondary, which happens with probability $\mathrm{P}[Q_P(t) = 0] \cdot \varepsilon$. Therefore, the departure process $X_{PS}(t)$ is stationary and its mean reads

$$\mu_{PS}(P_S, \varepsilon) = E[X_{PS}(t)] = \mathrm{P}[Q_P(t) = 0] \cdot (1 - \mathrm{P}_{out,SP}) \cdot \varepsilon, \quad (20)$$

with

$$\mathrm{P}_{out,SP} = 1 - \exp\left(-\frac{\beta_P P_S}{\gamma_{SP}}\right).$$

As far as the first queue $Q_{PS}(t)$ is concerned, stability is guaranteed if the condition $\lambda_{PS} < \mu_{PS}(P_S, \varepsilon)$ holds (Loynes' theorem), which in turn from (18) and (20) entails the following condition on ε and P_S:

$$\varepsilon > \frac{\lambda_P\left(1 - \exp\left(-\frac{\beta_P}{\gamma_P}\right)\right)\exp\left(-\frac{\beta_P}{\gamma_{PS}}\right)}{\left(\mu_P^{rel} - \lambda_P\right)\exp\left(-\frac{\beta_P}{\gamma_{SP}P_S}\right)}. \quad (21)$$

The departure process $X_S(t)$ is stationary with mean

$$\mu_S(P_S,\varepsilon)=\left(1-\frac{\lambda_P}{\mu_P^{rel}}\right)\exp\left(-\frac{\beta_S}{\gamma_S P_S}\right)(1-\varepsilon). \quad (22)$$

Optimizing the stable throughput of the cognitive node amounts to maximizing $\mu_S(P_S,\varepsilon)$ with respect to ε and P_S since from the Loynes' theorem $\lambda_S < \mu_S(P_S,\varepsilon)$. The maximum achievable throughput $\mu_S(P_S,\varepsilon)$ (22) is a decreasing function of ε. Therefore, in order to maximize $\mu_S(P_S,\varepsilon)$, we can set ε equal to its minimum value (21), thus obtaining

$$\begin{aligned}\lambda_S \quad < \quad & \mu_S(P_S)=\left[\frac{\left(\mu_P^{rel}-\lambda_P\right)}{\mu_P^{rel}}\exp\left(-\frac{\beta_P}{\gamma_{SP}P_S}\right)\right. \quad (23)\\ & \left.-\frac{\lambda_P}{\mu_P^{rel}}\left(1-\exp\left(-\frac{\beta_P}{\gamma_P}\right)\right)\exp\left(-\frac{\beta_P}{\gamma_{PS}}\right)\right]\\ & \exp\left(-\frac{\beta_S}{\gamma_S P_S}+\frac{\beta_P}{\gamma_{SP}P_S}\right).\end{aligned}$$

From the discussion above, Proposition 4 easily follows.

C. *Proof of (13)*

In case the transmission of the primary user is intefered by the cognitive transmitter, the signal-to-interference-plus-noise ratio (SINR) at the cognitive receiver reads:

$$\begin{aligned}SINR_P \quad &= \quad \frac{\gamma_P 1|h_P|^2}{1+\gamma_{SP}|h_S|^2 P_S}=\\ &= \quad \frac{\gamma_P}{\gamma_{SP}P_S}\frac{|h_P|^2}{\frac{1}{\gamma_{SP}P_S}+|h_S|^2}. \quad (24)\end{aligned}$$

Using the results in [12] (namely, eq. (15)) the cumulative distribution function is easily evaluated as:

$$\mathrm{P}\left[SINR_P<x\right]=1-\frac{\exp\left(-x\frac{1}{\gamma_P}\right)}{1+x\frac{\gamma_{SP}P_S}{\gamma_P}}. \quad (25)$$

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