Optimization of Multistatic Cloud Radar with Multiple-Access Wireless Backhaul

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***Abstract*—A multistatic cloud radar system is investigated, where receive antennas (RAs), or sensors, communicate with a fusion center (FC) over a multiple-access wireless backhaul. Each RA receives a measurement of the signal sent by a transmit antenna (TA) and reflected from target, possibly in the presence of clutter and interference, amplifies it, and forwards it to the FC on a wireless fading channel. The FC receives the signals transmitted by the RAs and determines the presence of a target. The problem of maximizing the Bhattacharyya distance as the detection performance metric under power constraints for the TA and RAs is formulated with respect to the transmitted code vector and the gains applied at the RAs. A short-term adaptive design is first considered that leverages the instant gain of the RAs-to-FC channels, and then a long-term adaptive design is considered that uses only stochastic channel state information (CSI). Algorithmic solutions for both scenarios are proposed based on successive convex approximation and the performance is evaluated via numerical results.**

***Index Terms*—Multistatic radar, cloud radar, detection, code design, power allocation.**

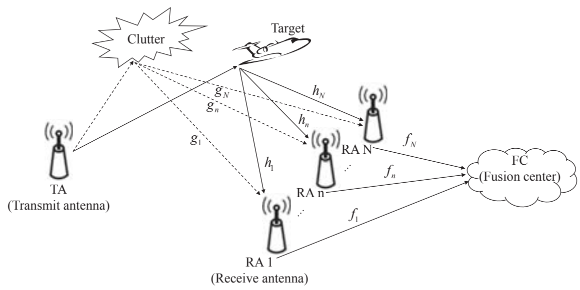

Fig. 1. Illustration of the considered multistatic cloud radar system, which consists of a TA, N RAs, and a FC. All the nodes are configured with a single antenna. The RAs are connected to the FC via a multiple-access wireless channel.

I. Introduction

In radar systems, the design of the transmitted waveform has received significant interest due to its role in determining detection performance by controlling the response to the target and to clutter [1], [2]. For monostatic radar systems, the waveform design in terms of the Neyman Pearson (NP) criterion is studied in [3]. In a multistatic radar scenario, however, the performance of the NP optimal detector is in general too complex to be suitable as a design metric. As a result, various information-theoretic criteria such as the Bhattacharyya distance, the Kullback Leibler divergence, the J-divergence and the mutual information, which can be shown to provide various bounds to the probability of error (missed detections and false alarms), have been considered as alternative design metrics [4]–[7].

In a multistatic radar system, multiple sensors, or receive antennas (RAs), receive the signal sent by a transmit antenna (TA), and reflected from a target and clutter. Signals received at the RAs are communicated to a fusion center (FC), where target detection is performed. The RAs and FC are connected via wired or wireless backhaul links, which prior work such as [5], [6] assume to be ideal. Inspired by the cloud radio access architecture in cellular communication systems [8], the concept of multistatic *cloud radar* is introduced in [7]. In a multistatic cloud radar, the RAs are connected to the FC via non-ideal connections. In particular, reference [7] assumes orthogonal, finite-capacity RAs-to-FC backhaul links. In order to comply with such capacity limitation, the RAs quantize the received baseband signals prior to transmission to the FC. In [7], significant performance advantages are demonstrated by jointly optimizing the Bhattacharyya distance criterion over waveforms implemented as code vectors and the backhaul quantization strategy.

In this paper, we consider a different implementation of cloud radar, in which, as shown in Fig. 1, a (non-orthogonal) multiple-access wireless backhaul channel connects the RAs and the FC. As in [7], each RA takes noisy measurements of the signal reflected by target and clutter. However, unlike [7], in order to communicate over the non-orthogonal wireless backhaul, each RA amplifies and forwards the received signal to the FC. A similar set up has been studied in [9], [10], but only the power allocation of the RAs was optimized for detection. In contrast, here we jointly optimize the code vector used at the TA and amplifying gains used at the RAs. Furthermore, we consider an adaptive design in which the code vector and gains may depend on the instantaneous gains of the channel state information (CSI) of the RAs-to-FC channels. We refer to this approach as short-term adaptive design. A long-term adaptive design is also proposed, which depends only on the stochastic CSI of the RAs-to-FC channels. In another departure from [9], [10], the detection performance, with the Bhattacharyya distance as its proxy, is the objective of the optimization, rather than the minimum mean square error (MMSE) of the estimated signal. The rest of the paper is organized as follows. In Section II, we present the detection problem as well as review the optimal detector. Then, we formulate the problems of the interest in Section III, and propose solutions based on successive convex approximations [11], [12] for both short-term and long-term adaptive designs in Section IV and Section V, respectively. Numerical results are provided in Section VI, and finally conclusions are drawn in Section VII.

II. System Model

We consider a cloud radar system consisting of a TA, N sensors, or RAs, and a FC, as illustrated in Fig. 1. The RAs communicate to a FC over a multiple-access wireless backhaul. All the nodes are equipped with a single antenna, and the set of RAs is denoted as $\mathcal{N} = \{1, \ldots, N\}$.

The system aims to detect the presence of a single stationary target in a homogeneous clutter field. Each RA receives a noisy version of the signal transmitted by the TA and reflected from the surveillance area, amplifies it and forwards it to the FC via a multiple-access wireless channel. It is assumed that ideal timing information is available at the FC, such that samples of the received signal may be associated with locations in some coordinate system. For such a location, and based on all the amplified signals from the different RAs, the FC makes a decision about the presence of the target.

A. Signal Model

Let the transmitted waveform be a train of K standard pulses with complex amplitudes $\boldsymbol{x} = [x_1, \ldots, x_K]^T$. The set of amplitudes $\boldsymbol{x}$ is referred to as a code vector [6], [7]. The pulses may form a continuous waveform or serve as individual pulses in a coherent pulse interval. The design of the code vector $\boldsymbol{x}$ determines the range resolution and clutter response, and thus has a key role in performance of the radar system.

The Swerling I model is assumed for the amplitude of the target echo, and hence the return has a Rayleigh envelope, which is fixed during the observation interval. The clutter is homogeneous and fixed over the observation interval with a complex-valued Gaussian distribution across the sensors. The returns are assumed to be independent between RAs and between target and clutter. Finally, each RA observes time-correlated complex Gaussian noise that captures the possible presence of various types of interference and jamming.

Following the discussion above, the $K \times 1$ discrete received signal by RA n, for $n \in \mathcal{N}$, after matched filtering and symbol-rate sampling, is given by [6], [7]

$$\mathcal{H}_0 : \boldsymbol{r}_n = \boldsymbol{c}_n + \boldsymbol{w}_n, \tag{1a}$$

$$\mathcal{H}_1 : \boldsymbol{r}_n = \boldsymbol{s}_n + \boldsymbol{c}_n + \boldsymbol{w}_n, \quad n \in \mathcal{N}, \tag{1b}$$

where $\mathcal{H}_0$ and $\mathcal{H}_1$ represent the hypotheses under which the target is absent or present, respectively, and $\boldsymbol{s}_n$ denotes the signal received from the target at RA n given as

$$\boldsymbol{s}_n = h_n \boldsymbol{x}, \tag{2}$$

with $h_n \sim \mathcal{CN}(0, \sigma_{t,n}^2)$ the random complex amplitude of the target return. The vector $\boldsymbol{c}_n$ represents the clutter contribution, modeled as

$$\boldsymbol{c}_n = g_n \boldsymbol{x}, \tag{3}$$

with $g_n \sim \mathcal{CN}(0, \sigma_{c,n}^2)$ the random complex amplitude of the clutter. The term $\boldsymbol{w}_n \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{C}_{w,n})$ represents signal-independent interference, which includes the contributions of thermal noise, interference and jamming, and whose temporal correlation is described by the covariance matrix $\boldsymbol{C}_{w,n}$.

The RA n communicates the received signal $\boldsymbol{r}_n$ in (1) to the FC after amplification. The amplified signal received at the FC is given as

$$\begin{aligned}\mathcal{H}_0 : \tilde{\boldsymbol{r}} &= \sum_{n=1}^{N} f_n \alpha_n \boldsymbol{r}_n + \boldsymbol{z} \\ &= \sum_{n=1}^{N} (f_n \alpha_n \boldsymbol{c}_n + f_n \alpha_n \boldsymbol{w}_n) + \boldsymbol{z}\end{aligned} \tag{4a}$$

$$\begin{aligned}\mathcal{H}_1 : \tilde{\boldsymbol{r}} &= \sum_{n=1}^{N} f_n \alpha_n \boldsymbol{r}_n + \boldsymbol{z} \\ &= \sum_{n=1}^{N} (f_n \alpha_n \boldsymbol{s}_n + f_n \alpha_n \boldsymbol{c}_n + f_n \alpha_n \boldsymbol{w}_n) + \boldsymbol{z},\end{aligned} \tag{4b}$$

where $f_n \sim \mathcal{CN}(0, \sigma_{f_n}^2)$ is the complex-valued channel between RA n and the FC. The channels f_n are assumed to be independent. The gains α_n are the subject of the design. The noise vector $\boldsymbol{z} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{C}_z)$ is temporally correlated with correlation matrix $\boldsymbol{C}_z$.

The variables h_n, g_n, $\boldsymbol{w}_n$, f_n and $\boldsymbol{z}$ for all $n \in \mathcal{N}$ are assumed to be mutually independent. Based on prior information or measurements, the second-order statistics of the channel gains between the target and the RAs, and of the noise terms, namely $\sigma_{t,n}^2$, $\sigma_{c,n}^2$, $\boldsymbol{C}_{w,n}$ and $\boldsymbol{C}_z$, are assumed to be known to the FC for all $n \in \mathcal{N}$ (see [6] and references therein). The RAs-to-FC channels $\boldsymbol{f} = [f_1 \cdots f_N]^T$ are assumed known at the FC, for example, via training and channel estimation. Since only the second-order statistics of the channel gains h_n, $n \in \mathcal{N}$, are known to the RAs and the FC, no coherent gains can be achieved by optimizing the amplifying gains, and hence one can focus, without loss of optimality, only on the RA's power gains $\boldsymbol{p} = [p_1 \cdots p_N]^T$, with $p_n = |\alpha_n|^2$, $n \in \mathcal{N}$.

We can write the hypotheses (4) in a standard form by whitening the signal received at the FC, and consequently the detection problem can be expressed as

$$\mathcal{H}_0 : \boldsymbol{y} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}), \tag{5a}$$

$$\mathcal{H}_1 : \boldsymbol{y} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{DSD} + \boldsymbol{I}), \tag{5b}$$

where $\boldsymbol{y} = \boldsymbol{D}\tilde{\boldsymbol{r}}$; $\boldsymbol{D} = (\sum_{n=1}^{N}(|f_n|^2 p_n \sigma_{c,n}^2 \boldsymbol{x}\boldsymbol{x}^H + |f_n|^2 p_n \boldsymbol{C}_{w,n}) + \boldsymbol{C}_z)^{-1/2}$ is the whitening filter with respect to the overall additive noise $\sum_{n=1}^{N} (f_n \alpha_n \boldsymbol{c}_n + f_n \alpha_n \boldsymbol{w}_n) + \boldsymbol{z}$, and $\boldsymbol{S} = \sum_{n=1}^{N} |f_n|^2 p_n \sigma_{t,n}^2 \boldsymbol{x}\boldsymbol{x}^H$ is the correlation matrix of the desired signal part. The detection problem described by (5) has the standard estimator-correlator solution given by the test

$$\boldsymbol{y}^H \boldsymbol{T} \boldsymbol{y} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma, \tag{6}$$

where we have defined $\boldsymbol{T} = \boldsymbol{DSD}(\boldsymbol{DSD} + \boldsymbol{I})^{-1}$, and the threshold γ is set based on the tolerated false alarm probability [13].

III. Problem formulation

We seek to optimize the detection performance with respect to the code vector $\boldsymbol{x}$ and the power gains $\boldsymbol{p}$, under power constraints on the TA and RAs. As in [4], [6], [7], we adopt the Bhattacharyya distance as the performance metric, which is a suitable and tractable information-theoretic measure of the performance of the optimal test (6).

The Bhattacharyya distance $\mathcal{B}$ between two multivariate Gaussian distributions, $\mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma}_1)$ and $\mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma}_2)$ is given by $\mathcal{B} = |0.5(\boldsymbol{\Sigma}_1 + \boldsymbol{\Sigma}_2)|/\sqrt{|\boldsymbol{\Sigma}_1||\boldsymbol{\Sigma}_2|}$ [4]. From (5a) and (5b), the Bhattacharyya distance between the distributions (5) under the two hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ can be calculated as

$$\mathcal{B}(\boldsymbol{x}, \boldsymbol{p}; \boldsymbol{f}) = \log\left(\frac{|\boldsymbol{I} + 0.5\boldsymbol{DSD}|}{\sqrt{|\boldsymbol{I} + \boldsymbol{DSD}|}}\right) = \log\frac{1 + 0.5\lambda}{\sqrt{1 + \lambda}}, \quad (7)$$

where $\lambda = \boldsymbol{f}^H\boldsymbol{P}\boldsymbol{\Sigma}_t\boldsymbol{f}\boldsymbol{x}^H(\boldsymbol{f}^H\boldsymbol{P}\boldsymbol{\Sigma}_c\boldsymbol{f}\boldsymbol{x}\boldsymbol{x}^H + (\boldsymbol{f} \otimes \boldsymbol{I}_K)^H(\boldsymbol{P} \otimes \boldsymbol{I}_K)\boldsymbol{C}_w(\boldsymbol{f} \otimes \boldsymbol{I}_K) + \boldsymbol{C}_z)^{-1}\boldsymbol{x}$, $\boldsymbol{\Sigma}_t = \mathrm{diag}\{\sigma_{t,1}^2 \ldots \sigma_{t,N}^2\}$ and $\boldsymbol{\Sigma}_c = \mathrm{diag}\{\sigma_{c,1}^2 \ldots \sigma_{c,N}^2\}$ are the diagonal matrices whose components are the second-order statistics of channel amplitudes of target return and clutter, respectively, $\boldsymbol{C}_w = \mathrm{diag}\{\boldsymbol{C}_{w,1} \ldots \boldsymbol{C}_{w,N}\} \in R^{NK \times NK}$ is a block diagonal matrix containing all the noise covariance matrices at the RAs, and $\boldsymbol{P} = \mathrm{diag}\{\boldsymbol{p}\} \in R^{N \times N}$ is the diagonal matrix that contains the RA's power gains. Note that we have made explicit the dependence of the Bhattacharyya distance $\mathcal{B}(\boldsymbol{x}, \boldsymbol{p}; \boldsymbol{f})$ on the channel gains $\boldsymbol{f}$ at the FC, as well as on the code vector $\boldsymbol{x}$ and the RAs' power gains $\boldsymbol{p}$. In the following, we formulate the problems for short-term (Section III-A) and long-term (Section III-B) adaptive designs pursuing the system objective.

A. Short-term adaptive design

In this section, we consider the case in which design of the code vector and of the RAs' gains depends on the instantaneous gain of the CSI of the RAs-to-FC channels $\boldsymbol{f}$. Note that this design requires to modify the solution vector $(\boldsymbol{x}, \boldsymbol{p})$ at the time scale at which the channel vector $\boldsymbol{f}$ varies, hence entailing a potentially large feedback overhead from the FC to the RAs and the TA. Defining the maximum powers available at the TA and across all the RAs as P_T and P_R, respectively, the problem of maximizing the Bhattacharyya distance (7) over the code vector $\boldsymbol{x}$ and the power gains $\boldsymbol{p}$ under the power constraints for TA and RAs, is stated as

$$\begin{aligned} \underset{\boldsymbol{x}, \boldsymbol{p}}{\text{minimize}} \quad & \bar{\mathcal{B}}(\boldsymbol{x}, \boldsymbol{p}; \boldsymbol{f}) && (8a)\\ \text{s.t.} \quad & \boldsymbol{x}^H\boldsymbol{x} \leq P_T, && (8b)\\ & \mathbf{1}^T\boldsymbol{p} \leq P_R, && (8c)\\ & p_n \geq 0, \quad n \in \mathcal{N}, && (8d) \end{aligned}$$

where we have defined $\bar{\mathcal{B}}(\boldsymbol{x}, \boldsymbol{p}; \boldsymbol{f}) = -\mathcal{B}(\boldsymbol{x}, \boldsymbol{p}; \boldsymbol{f})$ to formulate the problem as the minimization of the negative Bhattacharyya distance $\bar{\mathcal{B}}(\boldsymbol{x}, \boldsymbol{p}; \boldsymbol{f})$. We observe that the problem (8) can be easily modified to include individual power constraints at the RAs, but this is not further explored here. Note also that the problem (8) is not a convex program, since the objective function (8a) is not convex.

B. Long-term adaptive design

Here, in order to avoid the possibly excessive feedback overhead between FC and the TA and RAs of the short-term adaptive solution, we adopt the average Bhattacharyya distance, as the performance criterion, where the average is taken with respect to the distribution of the RAs-to-FC channels $\boldsymbol{f}$. In this way, the code vector $\boldsymbol{x}$ and RAs' gains $\boldsymbol{p}$ have to be updated only at the time scale at which the statistics of channels and noise terms vary. Then, the problem for the long-term adaptive design is formulated from problem (8) by substituting the objective function $\bar{\mathcal{B}}(\boldsymbol{x}, \boldsymbol{p}; \boldsymbol{f})$ with $E_{\boldsymbol{f}}[\bar{\mathcal{B}}(\boldsymbol{x}, \boldsymbol{p}; \boldsymbol{f})]$, yielding

$$\begin{aligned} \underset{\boldsymbol{x}, \boldsymbol{p}}{\text{minimize}} \quad & E_{\boldsymbol{f}}\left[\bar{\mathcal{B}}(\boldsymbol{x}, \boldsymbol{p}; \boldsymbol{f})\right] && (9a)\\ \text{s.t.} \quad & (8b) - (8d). && (9b) \end{aligned}$$

Note that the problem (9) is a stochastic program with a non-convex objective function (9a).

IV. Short-term adaptive design of code vector and amplifying gains

In the following, we propose an algorithm to solve the optimization problem (8). Since the problem is not convex, and hence it is difficult to obtain a global optimal solution, we aim to develop algorithms that target local optimal solutions. To this end, we adopt an alternating optimization scheme coupled with the MM method. The method solves a sequence of convex problems alternating over the code vector $\boldsymbol{x}$ and over the power gains $\boldsymbol{p}$. We first present the optimization over the code vector $\boldsymbol{x}$ given the gains $\boldsymbol{p}$ via MM algorithm in Section IV-A, and then describe the optimization over $\boldsymbol{p}$ with fixed $\boldsymbol{x}$ via MM algorithm in Section IV-B. The proposed algorithm is summarized in Table Algorithm 1. Note that we use the superscript i to identify the iterations of the outer loop of Algorithm 1, and the superscript j as the index of the inner iteration of the MM algorithm (e.g., $\boldsymbol{x}^{(i,j)}$ indicates the code vector optimized at the jth iteration of the inner loop of the MM algorithm and the ith iteration of the outer loop).

A. Optimization over $\boldsymbol{x}$

Here, the goal is to optimize the objective function (8a) over the code vector $\boldsymbol{x}^{(i)}$ given the gains $\boldsymbol{p} = \boldsymbol{p}^{(i-1)}$. For this purpose, we apply the MM algorithm. Specifically, at the jth iteration of the MM algorithm and the ith iteration of the outer loop, the MM algorithm solves a convex quadratically constrained quadratic program (QCQP) and obtains a solution $\boldsymbol{x}^{(i,j)}$ by substituting the non-convex objective function $\bar{\mathcal{B}}(\boldsymbol{x}, \boldsymbol{p}; \boldsymbol{f})$ with a tight upper bound $\mathcal{U}(\boldsymbol{x}, \boldsymbol{p}; \boldsymbol{f}|\boldsymbol{x}^{(i,j-1)})$ around the current iterate $\boldsymbol{x}^{(i,j-1)}$. This bound is obtained by following the same steps as in [6, Section III, IV] and is given by

$$\begin{aligned} &\mathcal{U}(\boldsymbol{x}, \boldsymbol{p}; \boldsymbol{f}|\boldsymbol{x}^{(i,j-1)})\\ &= \phi^{(i,j-1)}\boldsymbol{x}^H\left((\boldsymbol{f} \otimes \boldsymbol{I}_K)^H(\boldsymbol{P} \otimes \boldsymbol{I}_K)\boldsymbol{C}_w(\boldsymbol{f} \otimes \boldsymbol{I}_K)\right.\\ &\qquad \left. + \boldsymbol{C}_z\right)^{-1}\boldsymbol{x} - \mathrm{Re}\left\{\left(\boldsymbol{d}^{(i,j-1)}\right)^H\boldsymbol{x}\right\}, \quad (10) \end{aligned}$$

where

$$\phi^{(i,j-1)} = \frac{\beta}{1+\beta y^{(i,j-1)}} + \beta(1+0.5\gamma) + \frac{0.5\gamma}{1+\lambda^{(i,j-1)}} \frac{\beta}{\left(1+\beta y^{(i,j-1)}\right)^2};$$

$$\boldsymbol{d}^{(i,j-1)} = \left(\frac{2\beta\,(1+0.5\gamma)}{1+\beta y^{(i,j-1)}\,(1+0.5\gamma)} + 2\beta\,(1+0.5\gamma)\right) \left((\boldsymbol{f}\otimes\boldsymbol{I}_K)^H\,(\boldsymbol{P}\otimes\boldsymbol{I}_K)\,\boldsymbol{C}_w\,(\boldsymbol{f}\otimes\boldsymbol{I}_K)+\boldsymbol{C}_z\right)^{-1}\boldsymbol{x}^{(i,j-1)};$$

$$\beta = \boldsymbol{f}^H\boldsymbol{P}\boldsymbol{\Sigma}_c\boldsymbol{f};$$

$$\gamma = \frac{\boldsymbol{f}^H\boldsymbol{P}\boldsymbol{\Sigma}_t\boldsymbol{f}}{\beta};$$

$$y^{(i,j-1)} = \left(\boldsymbol{x}^{(i,j-1)}\right)^H \left((\boldsymbol{f}\otimes\boldsymbol{I}_K)^H\,(\boldsymbol{P}\otimes\boldsymbol{I}_K)\,\boldsymbol{C}_w\,(\boldsymbol{f}\otimes\boldsymbol{I}_K) + \boldsymbol{C}_z\right)^{-1}\boldsymbol{x}^{(i,j-1)};\ \text{and}$$

$$\lambda^{(i,j-1)} = \gamma - \frac{\gamma}{1+\beta y^{(i,j-1)}}.$$

At the jth iteration of the MM algorithm and the ith outer loop, we evaluate the new iterate $\boldsymbol{x}^{(i,j)}$ by solving the following QCQP problem

$$\boldsymbol{x}^{(i,j)} \leftarrow \underset{\boldsymbol{x}}{\operatorname{argmin}} \quad \mathcal{U}(\boldsymbol{x},\boldsymbol{p};\boldsymbol{f}|\boldsymbol{x}^{(i,j-1)}) \tag{11a}$$

$$\text{s.t.} \quad \boldsymbol{x}^H\boldsymbol{x} \le P_T \tag{11b}$$

The MM algorithm obtains the solution $\boldsymbol{x}^{(i)}$ for the ith iteration of the outer loop by solving the problem (11) iteratively over j until a convergence criterion is satisfied.

B. Optimization over $\boldsymbol{p}$

Here, we consider the optimization of the gains $\boldsymbol{p}^{(i)}$, when the code vector $\boldsymbol{x} = \boldsymbol{x}^{(i)}$ is given. Similar to the optimization over $\boldsymbol{x}^{(i)}$ in the previous section, we also use the MM algorithm for the optimization over $\boldsymbol{p}$. Towards this goal, we obtain the upper bound $\mathcal{U}(\boldsymbol{x},\boldsymbol{p};\boldsymbol{f}|\boldsymbol{p}^{(i,j-1)})$ of the objective function $\bar{\mathcal{B}}(\boldsymbol{x},\boldsymbol{p};\boldsymbol{f})$ around the current iterate $\boldsymbol{p}^{(i,j-1)}$. This bound is derived by writing $\bar{\mathcal{B}}(\boldsymbol{x},\boldsymbol{p};\boldsymbol{f})$ as the difference of convex functions of $\boldsymbol{P}$ given as $\bar{\mathcal{B}}(\boldsymbol{x},\boldsymbol{p};\boldsymbol{f}) = -\ln|\boldsymbol{f}^H\boldsymbol{P}(0.5\boldsymbol{\Sigma}_t+\Sigma_c)\boldsymbol{f}\boldsymbol{x}\boldsymbol{x}^H+(\boldsymbol{f}\otimes\boldsymbol{I}_K)^H(\boldsymbol{P}\otimes\boldsymbol{I}_K)\boldsymbol{C}_w(\boldsymbol{f}\otimes\boldsymbol{I}_K)+\boldsymbol{C}_z| + 0.5\ln|\boldsymbol{f}^H\boldsymbol{P}(\Sigma_t+\Sigma_c)\boldsymbol{f}\boldsymbol{x}\boldsymbol{x}^H+(\boldsymbol{f}\otimes\boldsymbol{I}_K)^H(\boldsymbol{P}\otimes\boldsymbol{I}_K)\boldsymbol{C}_w(\boldsymbol{f}\otimes\boldsymbol{I}_K)+\boldsymbol{C}_z| + 0.5\ln|\boldsymbol{f}^H\boldsymbol{P}\boldsymbol{\Sigma}_c\boldsymbol{f}\boldsymbol{x}\boldsymbol{x}^H+(\boldsymbol{f}\otimes\boldsymbol{I}_K)^H(\boldsymbol{P}\otimes\boldsymbol{I}_K)\boldsymbol{C}_w(\boldsymbol{f}\otimes\boldsymbol{I}_K)+\boldsymbol{C}_z|$, and by linearizing the difference of convex functions via the first-order Taylor approximation [11]. The following bound can then be obtained

$$\begin{aligned}
&\mathcal{U}(\boldsymbol{x},\boldsymbol{p};\boldsymbol{f}|\boldsymbol{p}^{(i,j-1)}) = \\
&-\ln\left|\boldsymbol{f}^H\boldsymbol{P}\,(0.5\Sigma_t+\Sigma_c)\,\boldsymbol{f}\boldsymbol{x}\boldsymbol{x}^H + (\boldsymbol{f}\otimes\boldsymbol{I}_K)^H\,(\boldsymbol{P}\otimes\boldsymbol{I}_K)\,\boldsymbol{C}_w \right.\\
&\qquad \left.(\boldsymbol{f}\otimes\boldsymbol{I}_K)+\boldsymbol{C}_z\right| \\
&+0.5\mathrm{tr}\left\{\left(\boldsymbol{f}^H\boldsymbol{P}^{(i,j-1)}\,(\Sigma_t+\Sigma_c)\,\boldsymbol{f}\boldsymbol{x}\boldsymbol{x}^H + (\boldsymbol{f}\otimes\boldsymbol{I}_K)^H\right.\right.\\
&\qquad \left.\left(\boldsymbol{P}^{(i,j-1)}\otimes\boldsymbol{I}_K\right)\boldsymbol{C}_w\,(\boldsymbol{f}\otimes\boldsymbol{I}_K)+\boldsymbol{C}_z\right)^{-1}\\
&\qquad \left(\boldsymbol{f}^H\boldsymbol{P}\,(\Sigma_t+\Sigma_c)\,\boldsymbol{f}\boldsymbol{x}\boldsymbol{x}^H + (\boldsymbol{f}\otimes\boldsymbol{I}_K)^H\,(\boldsymbol{P}\otimes\boldsymbol{I}_K)\,\boldsymbol{C}_w\right.\\
&\qquad \left.\left.(\boldsymbol{f}\otimes\boldsymbol{I}_K))\right\}\\
&+0.5\mathrm{tr}\left\{\left(\boldsymbol{f}^H\boldsymbol{P}^{(i,j-1)}\Sigma_c\boldsymbol{f}\boldsymbol{x}\boldsymbol{x}^H + (\boldsymbol{f}\otimes\boldsymbol{I}_K)^H\right.\right.\\
&\qquad \left.\left(\boldsymbol{P}^{(i,j-1)}\otimes\boldsymbol{I}_K\right)\boldsymbol{C}_w\,(\boldsymbol{f}\otimes\boldsymbol{I}_K)+\boldsymbol{C}_z\right)^{-1}\\
&\qquad \left.\left(\boldsymbol{f}^H\boldsymbol{P}\Sigma_c\boldsymbol{f}\boldsymbol{x}\boldsymbol{x}^H + (\boldsymbol{f}\otimes\boldsymbol{I}_K)^H\,(\boldsymbol{P}\otimes\boldsymbol{I}_K)\,\boldsymbol{C}_w\,(\boldsymbol{f}\otimes\boldsymbol{I}_K)\right)\right\}.
\end{aligned} \tag{12}$$

Then, the new iterate $\boldsymbol{p}^{(i,j)}$ at the jth iteration of the MM algorithm and the i iteration of the outer loop can be obtained by solving the following optimization problem:

$$\boldsymbol{p}^{(i,j)} \leftarrow \underset{\boldsymbol{p}}{\operatorname{argmin}} \quad \mathcal{U}(\boldsymbol{x},\boldsymbol{p};\boldsymbol{f}|\boldsymbol{p}^{(i,j-1)}) \tag{13a}$$

$$\text{s.t.} \quad \mathbf{1}^T\boldsymbol{p} \le P_R, \tag{13b}$$

$$p_n \ge 0, \quad n \in \mathcal{N}. \tag{13c}$$

By repeating the procedure (13) over j until a convergence criterion is satisfied, the solution $\boldsymbol{p}^{(i)}$ is determined for the ith outer loop.

C. Proposed Algorithm

In summary, in order to solve problem (8), we propose a algorithm (described in Table Algorithm 1) that alternates between the optimization over $\boldsymbol{x}$, described in Section IV-A and the optimization over $\boldsymbol{p}$, discussed in Section IV-B. In particular, at the ith iteration of the outer loop, the iterate $\boldsymbol{x}^{(i)}$ is obtained by solving a sequence of convex problems (11) via the MM algorithm for a fixed $\boldsymbol{p} = \boldsymbol{p}^{(i-1)}$. Then, the iterate $\boldsymbol{p}^{(i)}$ is found by solving a sequence of convex problems (13) via the MM algorithm with $\boldsymbol{x} = \boldsymbol{x}^{(i)}$ attained in the previous step. According to the the properties of the MM algorithm [11], the proposed scheme yields a non-increasing objective function along the outer and inter iterations, hence ensuring convergence.

Algorithm 1 Short-term adaptive design of code vector and amplifier gain (8))

Initialization (outer loop): Initialize $\boldsymbol{x}^{(0)} \in C^{K\times 1}$, $\boldsymbol{p}^{(0)} \succeq 0$ and set $i = 0$.
Repeat
  $i \leftarrow i+1$
  Initialization (inner loop): Initialize $\boldsymbol{x}^{(i,0)} = \boldsymbol{x}^{(i-1)}$ and set $j = 0$.
  Repeat (MM algorithm for $\boldsymbol{x}^{(i)}$)
    $j \leftarrow j+1$
    Find $\boldsymbol{x}^{(i,j)}$ by solving the problem (11) with $\boldsymbol{p} = \boldsymbol{p}^{(i-1)}$ (see (11)).
  Until a convergence criterion is satisfied.
  Update $\boldsymbol{x}^{(i)} \leftarrow \boldsymbol{x}^{(i,j)}$
  Initialization (inner loop): Initialize $\boldsymbol{p}^{(i,0)} = \boldsymbol{p}^{(i-1)}$ and set $j = 0$.
  Repeat (MM algorithm for $\boldsymbol{p}^{(i)}$)
    $j \leftarrow j+1$
    Find $\boldsymbol{p}^{(i,j)}$ by solving the problem (13) with $\boldsymbol{x} = \boldsymbol{x}^{(i)}$ (see (13)).
  Until a convergence criterion is satisfied.
  Update $\boldsymbol{p}^{(i)} \leftarrow \boldsymbol{p}^{(i,j)}$
Until a convergence criterion is satisfied.
Solution: $\boldsymbol{x} \leftarrow \boldsymbol{x}^{(i)}$ and $\boldsymbol{p} \leftarrow \boldsymbol{p}^{(i)}$

V. Long-term adaptive design of code vector and amplifying gains

In order to prevent the possibly excessive feedback overhead from the FC to the RAs and the TA, we consider a long-term adaptive design and solve the problem (9). Since the stochastic program (9) has a non-convex objective function, we apply the stochastic successive upper-bound minimization method (SSUM) [12], which minimizes an approximate ensemble average at each step of a locally tight upper bound of the objective function. Specifically, based on SSUM, we develop an alternating optimization scheme similar to the one detailed in Table Algorithm 1. The proposed method for the problem (9) solves a sequence of convex problems alternating over the code vector $\boldsymbol{x}$ and over the power gains $\boldsymbol{p}$. We first present the optimization over the code vector $\boldsymbol{x}$ given the gains $\boldsymbol{p}$ via SSUM, and then describe the optimization over $\boldsymbol{p}$ with fixed $\boldsymbol{x}$ via SSUM. Here, similar Algorithm 1, we use the superscript i to identify the iterations of the outer loop, and the superscript j as the index of the inner iterations of SSUM.

At the jth iteration of SSUM and the ith outer loop, we optimize the code vector $\boldsymbol{x}^{(i,j)}$ given $\boldsymbol{p} = \boldsymbol{p}^{(i-1)}$ by solving the following convex problem

$$\boldsymbol{x}^{(i,j)} \leftarrow \underset{\boldsymbol{x}}{\text{argmin}} \quad \frac{1}{j}\sum_{l=1}^{j}\mathcal{U}^{(l)}(\boldsymbol{x},\boldsymbol{p};\boldsymbol{f}^{(l)}|\boldsymbol{x}^{(i,l-1)}) \tag{14a}$$

$$\text{s.t.} \quad \boldsymbol{x}^H\boldsymbol{x} \leq P_T \tag{14b}$$

where $\boldsymbol{f}^{(l)}$ denotes a channel vector $\boldsymbol{f}$ for the FC that is randomly and independently generated at the lth iteration according to the known distribution, and $\mathcal{U}^{(l)}(\boldsymbol{x},\boldsymbol{p};\boldsymbol{f}^{(l)}|\boldsymbol{x}^{(i,l-1)})$ is the locally tight convex upper bound defined in (10) on the negative Bhattacharyya distance around the point $\boldsymbol{x}^{(i,l-1)}$ obtained at the $(l-1)$th iteration. Note that the objective function (14a) depends on all the realizations of the channel vectors $\boldsymbol{f}^{(l)}$ for $l = 1,\ldots,j$. The jth iteration of SSUM achieves the solution $\boldsymbol{x}^{(i)}$ for the ith iteration of the outer loop by solving the problem (14) iteratively over j, until a convergence criterion is satisfied.

With the optimized code vector $\boldsymbol{x} = \boldsymbol{x}^{(i)}$, the SSUM calculates the iterates $\boldsymbol{p}^{(i,j)}$ by solving the following problem

$$\boldsymbol{p}^{(i,j)} \leftarrow \underset{\boldsymbol{p}}{\text{argmin}} \quad \frac{1}{j}\sum_{l=1}^{j}\mathcal{U}^{(l)}(\boldsymbol{x},\boldsymbol{p};\boldsymbol{f}^{(l)}|\boldsymbol{p}^{(i,l-1)}) \tag{15a}$$

$$\text{s.t.} \quad \mathbf{1}^T\boldsymbol{p} \leq P_R, \tag{15b}$$

$$p_n \geq 0, \quad n \in \mathcal{N}, \tag{15c}$$

where $\mathcal{U}^{(l)}(\boldsymbol{x},\boldsymbol{p};\boldsymbol{f}^{(l)}|\boldsymbol{p}^{(i,l-1)})$ represents the convex upper bound (12) on the negative Bhattacharyya distance around the point $\boldsymbol{p}^{(i,l-1)}$. The iterate $\boldsymbol{p}^{(i)}$ is obtained by solving the problem iteratively over j (15) until the convergence. The final algorithm for long-term adaptive design can be summarized as in Table Algorithm 1 by substituting (11) and (13) with (14) and (15), respectively.

VI. Numerical Results

In this section, we evaluate the performance of the proposed algorithms that perform the joint optimization of the code vector $\boldsymbol{x}$ and of the amplifying power gains $\boldsymbol{p}$ for the short-term (Section IV) and long-term (Section V) adaptive designs. For reference, we consider the following schemes; *(i) No optimization* (No opt.): Set $\boldsymbol{x} = \sqrt{P_T/K}\mathbf{1}_K$ and $\boldsymbol{p} = P_R/N\mathbf{1}_N$; *(ii) Code vector optimization* (Code opt.): Optimize the code vector $\boldsymbol{x}$ as per Algorithm 1 (with (14) in lieu of (11) for the long-term adaptive design) with $\boldsymbol{p} = P_R/N\mathbf{1}_N$; and *(iii) Gain optimization* (Gain opt.): optimize the gains $\boldsymbol{p}$ as per Algorithm 1 (with (15) in lieu of (13) for the long-term adaptive design) with $\boldsymbol{x} = \sqrt{P_T/K}\mathbf{1}_K$. We set the length of the code vector to $K = 6$ and the variances of the target amplitudes as $\sigma^2_{t,n} = 1$ for $n \in \mathcal{N}$. Moreover, we model the noise with covariance matrices $[\boldsymbol{C}_{w,n}]_{i,j} = (1-0.12n)^{|i-j|}$ and $[\boldsymbol{C}_z]_{i,j} = (1-0.45)^{|i-j|}$ [6], [7]. The channel coefficients f_n have unit variance, i.e., $\sigma^2_{f_n} = 1$.

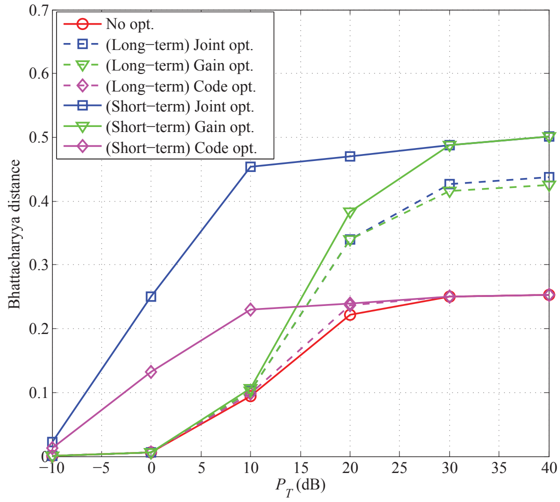

Fig. 2. Bhattacharyya distance vs. the value of P_T with $P_R = 10$ dB, $N = 3$, $\sigma^2_{c,1} = 0.125$, $\sigma^2_{c,2} = 0.25$ and $\sigma^2_{c,3} = 0.5$.

Fig. 2 shows the Bhattacharyya distance as a function of the TA's power P_T, with $P_R = 10$ dB, $N = 3$, $\sigma^2_{c,1} = 0.125$, $\sigma^2_{c,2} = 0.25$ and $\sigma^2_{c,3} = 0.5$. For smaller values of P_T, optimizing the code vector is more advantageous than optimizing the amplifying gains, due to the fact that performance is limited by the TA-to-RAs connection. In contrast, for sufficiently large values of P_T, the optimization of the RAs' gains is to be preferred, since the performance becomes limited by the channels between the RAs and the FC. Joint optimization significantly outperforms all other schemes, except in the very low- and large-power regimes, in which, as discussed, the performance is limited by either the TA-to-RAs or the RAs-to-FC channels. In addition, we observe that the long-term adaptive scheme loses about 10% in terms of the Bhattacharyya distance with respect to the short-term adaptive design in the high SNR regime.

Fig. 2 also points to an interesting property of cloud radar: properties of the communications channel affect the design of the radar waveform. From the figure, it is observed that different Bhattacharyya distances are obtained by the code only optimizations for the short-term versus the long-term

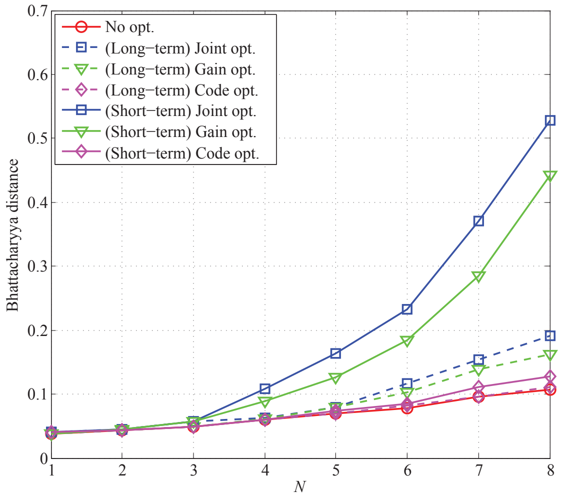

Fig. 3. Bhattacharyya distance vs. the value of N with $P_T = 12$ dB, $P_R = 10$ dB, $\sigma_{c,1}^2 = 1$, $\sigma_{c,2}^2 = 0.9$, $\sigma_{c,3}^2 = 0.75$, $\sigma_{c,4}^2 = 0.5$, $\sigma_{c,5}^2 = 0.35$, $\sigma_{c,6}^2 = 0.25$ and $\sigma_{c,7}^2 = 0.125$, $\sigma_{c,8}^2 = 0.05$.

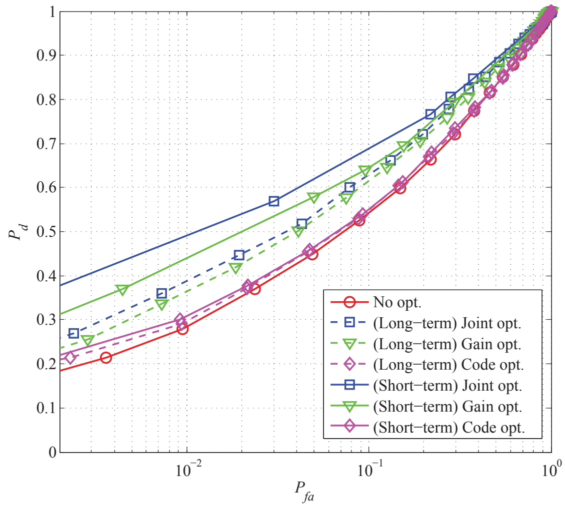

Fig. 4. ROC curves with $P_T = 20$ dB, $P_R = 10$ dB, $N = 3$, $\sigma_{c,1}^2 = 0.125$, $\sigma_{c,2}^2 = 0.25$ and $\sigma_{c,3}^2 = 0.5$.

adaptive designs (particularly at lower values of transmitted power). This may be explained by noting that the code vector is designed such that the transmitted power is reduced at frequencies with large interference. Thus, sensors that experience high interference at some frequencies, have more effect on the signal design. Now, sensors that experience high interference and have a high channel gain to the FC, may deliver a high amount of interference to the decision processing. The optimization seeks to design the transmitted signal $\boldsymbol{x}$ such that it reduces the power transmitted at these frequencies.

In Fig. 3, the Bhattacharyya distance is plotted versus the number RAs N with $P_T = 12$ dB, $P_R = 10$ dB, $\sigma_{c,1}^2 = 1$, $\sigma_{c,2}^2 = 0.9$, $\sigma_{c,3}^2 = 0.75$, $\sigma_{c,4}^2 = 0.5$, $\sigma_{c,5}^2 = 0.35$, $\sigma_{c,6}^2 = 0.25$, $\sigma_{c,7}^2 = 0.125$ and $\sigma_{c,8}^2 = 0.05$. Optimizing the RAs' power gains is seen to be especially beneficial at large N, due to the ability to allocate more power to the RAs with lower measurement noise. For instance, even with the long-term adaptive design, optimizing the RAs' power gains outperforms code optimization with short-term adaptive design, for sufficiently large N.

Fig. 4 plots the Receiving Operating Characteristic (ROC), i.e., the detection probability P_d versus false alarm probability P_{fa} with $P_T = 20$ dB, $P_R = 10$ dB, $N = 3$, $\sigma_{c,1}^2 = 0.125$, $\sigma_{c,2}^2 = 0.25$ and $\sigma_{c,3}^2 = 0.5$. The curve was evaluated via Monte Carlo simulations by implementing the optimum test detector (6). It can be observed that the gains observed in the previous figures directly translate into a better ROC performance of joint optimization. Note also that power gain optimization is seen to be advantageous due to large value of P_T predicted based on Fig. 2.

VII. Conclusions

We have studied a multistatic cloud radar in which, unlike [7], where the RAs and FC are connected via a non-orthogonal multiple-access wireless backhaul channel. Each RA amplifies and forwards the signal sent by the TA to a FC, which collects the amplified signals from all the RAs and determines the target's presence. The problems of maximizing the Bhattacharyya distance as performance metric over the TA's code vector and RAs' power gains under the power constraints for the TA and RAs are formulated for the short-term and long-term adaptive designs, respectively. Then, algorithmic solutions are proposed for both cases based on successive convex approximation, whose performance is verified via numerical results.

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