On the Spectrum of Large Random Hermitian Finite-Band Matrices

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***Abstract*— The open problem of calculating the limiting spectrum (or its Shannon transform) of increasingly large random Hermitian finite-band matrices is described. In general, these matrices include a finite number of non-zero diagonals around their main diagonal regardless of their size. Two different communication setups which may be modeled using such matrices are presented: a simple cellular uplink channel, and a time varying inter-symbol interference channel. Selected recent information-theoretic works dealing directly with such channels are reviewed. Finally, several characteristics of the still unknown limiting spectrum of such matrices are listed, and some reflections are touched upon.**

I. Problem Description

Consider a linear channel of the form

$$\boldsymbol{y} = \boldsymbol{H}_N \boldsymbol{x} + \boldsymbol{z} \ , \tag{1}$$

where $\boldsymbol{x}$ is the $NK \times 1$ zero-mean complex Gaussian input vector $\boldsymbol{x} \sim \mathcal{CN}(0, \frac{P}{K}\boldsymbol{I}_{NK})$ [1], $\boldsymbol{y}$ is the $N \times 1$ output vector, and $\boldsymbol{z}$ denotes the $N \times 1$ zero-mean complex Gaussian additive noise vector $\boldsymbol{z} \sim \mathcal{CN}(0, \boldsymbol{I}_{NK})$, which is independent of $\boldsymbol{x}$ and $\boldsymbol{H}_N$. Accordingly $\rho = \frac{P}{K}$ is the transmitted signal-to-noise ratio (SNR). In addition, the $N \times NK$ channels transfer matrix $\boldsymbol{H}_N$ is defined by

$$\boldsymbol{H}_N = \begin{pmatrix} \boldsymbol{a}_1 & \beta \boldsymbol{c}_1 & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ \alpha \boldsymbol{b}_2 & \boldsymbol{a}_2 & \beta \boldsymbol{c}_2 & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \alpha \boldsymbol{b}_3 & \boldsymbol{a}_3 & \beta \boldsymbol{c}_3 & \ddots & \vdots \\ \vdots & \boldsymbol{0} & \alpha \boldsymbol{b}_4 & \ddots & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \vdots & \ddots & \ddots & \boldsymbol{a}_{N-1} & \beta \boldsymbol{c}_{N-1} \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \alpha \boldsymbol{b}_N & \boldsymbol{a}_N \end{pmatrix} , \tag{2}$$

where $\{\boldsymbol{a}_i,\ \boldsymbol{b}_i,\ \boldsymbol{c}_i\}$ are statistically independent $1 \times K$ random row vectors with independent identically distributed (i.i.d.) entries $a_{i,j} \sim \pi_a$, $b_{i,j} \sim \pi_b$, and $c_{i,j} \sim \pi_c$. For simplicity, we assume that the power moments of the entries for any finite order are bounded. Finally, $\alpha,\ \beta \in [0,1]$ are constants.

The normalized input-output mutual information of (1) conditioned on $\boldsymbol{H}_N$ (also known as the Shannon transform) is[2]

$$\begin{aligned} \frac{1}{N} I(\boldsymbol{x};\boldsymbol{y}|\boldsymbol{H}_N) &= \frac{1}{N} \log\det\left(\boldsymbol{I}_N + \rho \boldsymbol{H}_N \boldsymbol{H}_N^\dagger\right) \\ &= \frac{1}{N}\sum_{i=1}^{N} \log\left(1 + \rho\lambda_i(\boldsymbol{H}_N\boldsymbol{H}_N^\dagger)\right) \\ &= \int_0^\infty \log(1+\rho x) d\mathrm{F}_{\boldsymbol{H}_N\boldsymbol{H}_N^\dagger}(x) \ , \end{aligned} \tag{3}$$

where $\lambda_i(\boldsymbol{H}_N\boldsymbol{H}_N^\dagger)$ denotes the ith eigenvalue of the Hermitian *five-diagonal* matrix $\boldsymbol{H}_N\boldsymbol{H}_N^\dagger$. Furthermore, denoting the indicator function by $1\{\cdot\}$,

$$\mathrm{F}_{\boldsymbol{H}_N\boldsymbol{H}_N^\dagger}(x) = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}\{\lambda_i(\boldsymbol{H}_N\boldsymbol{H}_N^\dagger) \leq x\} \tag{4}$$

is the empirical cumulative distribution function of the eigenvalues (also referred to as the spectrum or empirical distribution) of $\boldsymbol{H}_N\boldsymbol{H}_N^\dagger$. Fixing K and assuming that $\mathrm{F}_{\boldsymbol{H}_N\boldsymbol{H}_N^\dagger}(x)$ converges almost surely (a.s.) to a unique limiting spectrum $\mathrm{F}_{\boldsymbol{H}_N\boldsymbol{H}_N^\dagger}(x) \xrightarrow[N\to\infty]{\text{a.s.}} F(x)$, it can be shown that the expectation of (3) with respect to (w.r.t.) the distribution of $\boldsymbol{H}_N$ converges as well. This is since (3) is uniformly integrable due to the Hadamard inequality and the bounded power moment assumption, and hence the a.s. convergence implies convergence in expectation [1].

In Section II it will be realized that if the channel $\boldsymbol{H}_N$ is known at the receiver and its variation over time is stationary and ergodic, then the expectation of (3) w.r.t. the distribution of $\boldsymbol{H}_N$ is the per-cell sum-rate capacity of a certain cellular uplink channel model. In another setting (see Section II), the same expectation may be interpreted as the capacity of a certain time variant inter-symbol interference (ISI) channel, assuming again that the channel is known at the receiver.

A. Analytical Difficulty

Many recent studies have analyzed the asymptotic rates of various vector channels using results from the theory of (large) random matrix (see [2] for a recent review). In those cases, the number of random variables involved is of the order of the number of elements in the matrix $\boldsymbol{H}_N$, and self-averaging is strong enough to ensure convergence of the

[1]An $N \times N$ identity matrix is denoted by $\boldsymbol{I}_N$.

[2]Unless explicitly denoted otherwise a natural base logarithm is used throughout this presentation.

empirical measure of eigenvalues, and to derive equations for the limiting spectrum (or its Stieltjes transform). In particular, this is the case if the normalized continuous power profile of $\boldsymbol{H}_N$, which is defined with $r,\ t \in [0,1]$ as

$$\mathcal{P}_N(r,t) \triangleq \mathbb{E}(|[\boldsymbol{H}_N]_{i,j}|^2) \quad \frac{i-1}{N} \le r < \frac{i}{N}\ ,\ \frac{j-1}{NK} \le t < \frac{j}{NK}\ , \tag{5}$$

converges uniformly to a bounded, piecewise continuous function as $N \to \infty$, see e.g. [2, Theorem 2.50]. In the case under consideration here, it is easy to verify that for K fixed, $\mathcal{P}_M(r,t)$ does *not* converge uniformly, and other techniques are required.

Remark: It is noted that the setting of (1) can be extended in many ways such as increasing the number of non-zero block diagonals, or replacing each K-dimensional random row vector with an $n \times m$ random matrix. Such settings result in $\boldsymbol{H}_N\boldsymbol{H}_N^\dagger$ which includes more than five non-zero diagonals and are referred to as Hermitian *finite-band* random matrices; the resulting matrices contain only zero entries outside a finite band (finite number of non-zero diagonals) around their main diagonal regardless of N.

To the best of the authors' knowledge, neither the limiting spectrum of Hermitian finite-band random matrices, nor the expectation of the normalized input-output conditional mutual information (3), is known in general except for a few special cases (see Section III). Moreover, even the high-SNR regime characterization (defined in [3][4]) of the latter is known only for a few special cases (see Section III) and remains an open problem in general.

II. Motivation

In this section we present two different multi-access communication channels whose channel transfer matrices are finite-band.

a) Cellular uplink: Motivated by the fact that a mobile user in a cellular system effectively "sees" only a finite number of base-stations, a simplified cellular model family has been introduced by Wyner in [5] (see also [6] for an independent earlier work which deals with similar setups). According to the original linear variant setup presented in [5], the K homogenous users of each cell are collocated at the cell's center and "see" their local base-station antenna and the antennas of the two adjacent base-stations only. While the signals travel to the local antenna with no path-loss, the path-loss to the two adjacent cells' antennas is characterized by a single parameter $\alpha \in [0,\ 1]$. Wyner assumed that the users cannot cooperate in any way and that all the base-stations are connected to a central receiver via an ideal error-free infinite capacity backhaul network. With optimal joint processing of all the received signals, the channel can be considered as a multiple-access channel whose vector representation is given by (1). The non-fading setup of [5] was extended to include flat fading channels in [7][8]. Considering an infinite number of cells and assuming that the channel state information is known by the central receiver, the per-cell sum-rate capacity of the Wyner model is given by setting $\beta = \alpha$ and $\pi_a = \pi_b = \pi_c = \pi$, and averaging the mutual information of (3) over the entries of $\boldsymbol{H}_N$. It is noted that the basic model can be extended to cases where each mobile "sees" any finite number of cell-site antennas and the resulting $\boldsymbol{H}_N\boldsymbol{H}_N^\dagger$ is a finite-band matrix.

Remark: Using the uplink-downlink duality (e.g. [9]), the per-cell sum-rate capacity of the Wyner uplink channel is also an achievable per-cell sum-rate (a lower bound of the per-cell capacity) of the reciprocal Wyner downlink channel, assuming the joint multicell transmitter has full channel state information (CSI) while each mobile is aware of its own CSI only.

Since its introduction in [5], the Wyner model family has provided a powerful framework for research assessing the performance of various joint multicell processing schemes (see [11] and [12] for recent surveys). Overcoming the analytical difficulties relating to these models and calculating the spectra (or their transforms) of the resulting finite-band matrices, would greatly enhance our understanding and insight into the theoretical performance of future cellular (and wireless) systems.

b) Time varying ISI channels: Here we consider K homogenous users communicating with a receiver over an L-tap time varying ISI channel. Assuming that the channel taps are i.i.d. between different users and also i.i.d. in the time index it is easily verified that the received signal is given by (1). Assuming that $L = 3$, the sum-rate of this multiple access channel is given by averaging the mutual information of (3) over the entries of $\boldsymbol{H}_N$. This setup may describe a "fast" multipath fading channel where the channel taps are independent over the time index. As with the previous setup for any finite L the resulting $\boldsymbol{H}_N\boldsymbol{H}_N^\dagger$ is a finite-band matrix. In contrast to the previous model where the entries of the received signal are in the spatial domain, the entries of the received signal here are in the time domain.

III. Selected Prior Work

In this section we briefly review selected previous works dealing with the spectrum of finite-band matrices, its Shannon transform, and related issues. The reader is referred to [11] and [12] for detailed surveys of relevant information-theoretic works.

The non-fading (or deterministic) case was analyzed by Wyner in [5] for the special case of $\beta = \alpha$. Setting $a_{i,j} = b_{i,j} = c_{i,j} = 1$ we get that $\frac{1}{K}\boldsymbol{H}_N\boldsymbol{H}_N^\dagger$ becomes a five-diagonal Toeplitz matrix with non-zero entries $(\alpha^2,\ 2\alpha,\ 1 + 2\alpha^2,\ 2\alpha,\ \alpha^2)$. Using well known results regarding the limiting spectrum of large Toeplitz matrices (Szegö's Theorem [13]), Wyner showed that the per-cell sum-rate capacity approaches as $N \to \infty$ to

$$C = \int_0^1 \log\left(1 + P(1 + 2\alpha\cos(2\pi f))^2\right) df\ . \tag{6}$$

It is noted that the result is independent of K as long as the total transmit power per-cell P is fixed. The reader is referred to [14] for a derivation of the Stieltjes transform of the spectrum for similar five-diagonal Toeplitz matrices.

The infinite linear Wyner model in the presence of flat fading channels is considered in [8]. For the special case of $\beta = \alpha$, $\pi_a = \pi_b = \pi_c = \pi$ and $K = 1$ it is shown that the unordered eigenvalue distribution $\mathbb{E}(F_{\boldsymbol{H}_N \boldsymbol{H}_N^\dagger})$ converges weakly to a unique distribution. It is conjectured that using similar methods the spectrum can be proved to converge a.s. to a unique limit as well. In addition, using a standard weighted paths summation over a restricted grid, the limiting values of the first several moments of this distribution were calculated for the special case in which the amplitude of an individual fading coefficient is statistically independent of its uniformly distributed phase (e.g. *Rayleigh* fading $\pi = \mathcal{CN}(0,1)$). For example, listed below are the first three limiting moments:

$$\begin{aligned}\mathcal{M}_1 =& m_2 + 2m_2\alpha^2 \\ \mathcal{M}_2 =& m_4 + 8m_2^2\alpha^2 + (4m_2^2 + 2m_4)\alpha^4 \\ \mathcal{M}_3 =& m_6 + (6m_2^3 + 12m_2m_4)\alpha^2 + (36m_2^3 + 12m_2m_4)\alpha^4 \\ &+ (6m_2^3 + 12m_2m_4 + 2m_6)\alpha^6 \ ,\end{aligned} \tag{7}$$

where m_i is the i-th power moments of the amplitude of an individual fading coefficient. It is noted that this procedure can be extended in principle, although in a tedious manner, for any finite K or also for $\boldsymbol{H}_N$ to include more than three non-zero block diagonals. Since the limiting moments of increasing order are functions of increasing orders of the moments of the fading coefficients, it is conjectured that the limiting distributions (and also the spectra) of finite-band matrices depend on the *actual* fading distribution and not just on its few first moments. Focusing on the case in which K is large while P is kept constant, and applying the strong law of large numbers (SLLN), the entries of $\frac{1}{K}\boldsymbol{H}_N\boldsymbol{H}_N^\dagger$ consolidate a.s. to their mean values and the latter becomes a Toeplitz matrix. By applying Szegö's Theorem for $N \to \infty$ it is shown in [8] that the per-cell sum-rate capacity is given by

$$\begin{aligned}C = \int_0^1 &\log\Big(1 + P\Big[\ \sigma^2(1+2\alpha^2) \\ &+ |m_1|^2\left(1 + 2\alpha\cos(2\pi\theta)\right)^2\Big]\Big)\, d\theta \ ,\end{aligned} \tag{8}$$

where $\sigma^2 = m_2 - |m_1|^2$ is the variance of an individual fading coefficient.

An alternative approach which replaces the role of the eigenvalues of $\boldsymbol{H}_N\boldsymbol{H}_N^\dagger$ with the diagonal elements of its *Cholesky* decomposition, is presented by Narula [15]. With $\alpha = 1$, $\beta = 0$, $\pi_a = \pi_b = \pi$, and $K = 1$, the resulting $\boldsymbol{H}_N\boldsymbol{H}_N^\dagger$ is a three-diagonal matrix (also known as *Jacobi* matrix). Originally, Narula has studied the capacity of a "fast" time varying two-tap ISI channel, where the channel coefficients are i.i.d. zero-mean complex Gaussian (i.e. $\pi = \mathcal{CN}(0,1)$). Following [15], the diagonal entries of the *Cholesky* decomposition applied to the covariance matrix $\left(\boldsymbol{I}_N + P\boldsymbol{H}_N\boldsymbol{H}_N^\dagger\right) = \boldsymbol{L}_N\boldsymbol{D}_N\boldsymbol{U}_N$, are given by

$$d_n = 1 + P|a_n|^2 + P|b_n|^2\left(1 - P\frac{|a_{n-1}|^2}{d_{n-1}}\right) \ , \ n = 2, \ldots, N \ , \tag{9}$$

with an initial condition $d_1 = 1 + P|a_1|^2 + P|b_1|^2$. Thus, the diagonal entries $\{d_m\}$ form a discrete-time continuous space Markov chain. Remarkably, Narula managed to prove that this Markov chain possesses a unique ergodic stationary distribution, given by

$$f_d(x) = \frac{\log(x)e^{-\frac{x}{P}}}{\mathrm{Ei}\left(\frac{1}{P}\right)\bar{P}} \quad ; \quad x \geq 1 \ , \tag{10}$$

where $\mathrm{Ei}(x) = \int_x^\infty \frac{\exp(-t)}{t}dt$ is the exponential integral function. Further, it is proven in [15] that the SLLN holds for the sequence $\{\log d_n\}$ as $N \to \infty$, and the channel capacity is

$$C = \int_1^\infty \frac{(\log(x))^2 e^{-\frac{x}{P}}}{\mathrm{Ei}\left(\frac{1}{P}\right)\bar{P}}dx \ . \tag{11}$$

It is noted that Narula's approach is closely matched to the above setting and any attempt so far to change a key parameter in this setting (such as the entries' distribution, the number of users per-cell, and the number of non-zero diagonals) leads to an analytically intractable derivation. This is probably related to the unique properties of Jacobi matrices which does not apply to finite-band matrices in general. For example, the determinant of a Jacobi matrix is equal to a weighted sum of the determinants of its *two* largest principal sub-matrices. In addition, Narula's analysis provides additional evidence to support the conjecture that the limiting spectrum of finite-band random matrices is dependent on the distribution of their entries. On this note, in [16] an equivalent cellular uplink setup but with uniform phase fading ($|a_{i,j}|^2 = 1$ and $\theta_{i,j} = \measuredangle a_{i,j} \sim U[0, 2\pi]$) known at the joint receiver is considered, and the per-cell sum-rate capacity is shown to coincide with the non-fading setup for $N \to \infty$. It is worth mentioning that the latter result holds only for the tridiagonal case.

As an alternative to deriving exact analytical results, some works focus on extracting parameters that characterize the channel capacity under extreme SNR scenarios (see [3] - [4] for more details on the extreme SNR characterization). The low-SNR regime is characterized through the minimum transmit E_b/N_0 that enables reliable communications, i.e., $E_b/N_{0_{\min}}$, and the low-SNR spectral efficiency slope $\mathcal{S}_0$. Assuming full receiver CSI and no user cooperation, it is shown in [17] that the derivation of the low-SNR parameters reduces to the calculation of $\mathrm{tr}\left(\mathbb{E}(\boldsymbol{H}_N^\dagger\boldsymbol{H}_N)\right)$ and $\mathrm{tr}\left(\mathbb{E}\left(\boldsymbol{H}_N^\dagger\boldsymbol{H}_N\right)^2\right)$. For example, the low-SNR parameters for the capacity of the Wyner setup are given for $N \to \infty$ by [18]

$$\begin{aligned}\frac{E_b}{N_0}_{\min} &= \frac{\log 2}{m_2(1+2\alpha^2)} \\ \mathcal{S}_0 &= \frac{2K(1+2\alpha^2)^2}{\mathcal{K} + K - 1 + 4(1+K)\alpha^2 + 2(\mathcal{K} + 2K)\alpha^4} \ ,\end{aligned} \tag{12}$$

where the *kurtosis* of an individual fading coefficient is defined as $\mathcal{K} = m_4/(m_2)^2$. This result can be extended in a straightforward yet tedious manner to general finite-band matrices.

The high-SNR regime is characterized through the high-SNR slope $\mathcal{S}_\infty$ (also referred to as the "multiplexing gain") and the high-SNR power offset $\mathcal{L}_\infty$. Recently [1], the per-cell capacity high-SNR parameters for a two diagonal $\boldsymbol{H}_N$

($K=1$, $\alpha=1$, and $\beta=0$) were calculated for $N\to\infty$ and rather general fading distributions:

$$\mathcal{S}_\infty = 1 \quad ; \quad \mathcal{L}_\infty = -2\max\left(\mathbb{E}_{\pi_a}\log_2|x|, \mathbb{E}_{\pi_b}\log_2|x|\right) \ . \qquad (13)$$

The main idea is to link the spectral properties of $\boldsymbol{H}_N\boldsymbol{H}_N^\dagger$ with the exponential growth of the elements of its eigenvectors. Since $\boldsymbol{H}_N\boldsymbol{H}_N^\dagger$ in this case is an *Hermitian Jacobi* matrix, and hence is tridiagonal, its eigenvectors can be considered to be sequences with second order linear recurrence. Therefore, the problem reduces to the study of the exponential growth of products of two by two matrices. This is closely related to the evaluation of the top *Lyapunov* exponent of the product; The explicit link between the Shannon transform (3) and the top Lyapunov exponent is the *Thouless* formula [19]. Moreover, for arbitrary finite K, it is shown in [1] that $\mathcal{S}_\infty=1$ while the power offset is bounded by a sequence of explicit upper- and lower-bounds; the gap between the lower and the upper bounds decreases with the bounds' order and complexity. It is noted that calculating exact expressions for the high-SNR parameters of channels with *general* fading distribution and *arbitrary finite* K remains an open problem even for the tridiagonal case. In addition, (13) also further supports the conjecture made regarding the dependency of the limiting spectrum of finite-band matrices on their entries' distribution.

Recently [14], the limiting spectrum of $\frac{1}{1+2\alpha^2}\boldsymbol{H}_N\boldsymbol{H}_N^\dagger$ for the Wyner setup and complex Gaussian vectors, has been loosely shown by *free probability* tools to be approximated by the Marŏenko-Pastur distribution with parameter K. The approximation, is shown to fairly well match the spectrum by Monte-Carlo simulations only for relatively large values of α. It should be emphasized that such a match is not guaranteed for other fading distributions excluding the complex Gaussian distribution (i.e. Rayleigh fading). A possible reasoning for the approximation inaccuracy in the low α regime is that in the extreme case of $\alpha=0$, the eigenvalues are evidently exponentially distributed, with no finite support (in contrast to the Marŏenko-Pastur distribution).

IV. Concluding Remarks

The limiting spectrum (or its Shannon transform) of certain large finite-band Hermitian random matrices is known for a few limited cases and remains an open problem in general. Moreover, even the high-SNR characterization of their Shannon transforms is still unsolved. Due to their special power profile, standard tools from the theory of random matrices cannot be used for this problem. It is conjectured that unlike "full" random matrices, the limiting spectra of finite-band random matrices depend on the actual distribution of their entries. It seems that unconventional methods such as the method used by Narula, replacing the role of eigenvalues with the diagonal elements of the Cholesky decomposition, are required to shed light on this problem. Nevertheless, it is noted that the tri-diagonal (Jacobi matrices) case is unique and these techniques may not apply to general finite-band matrices. Finally, we note that solving the problem would facilitate analytical treatment, which in turn gains much insight into the effect of key system parameters on the performance of certain cellular uplink channels and time varying ISI channels.

Acknowledgment

The research was supported in part by a Marie Curie Outgoing International Fellowship and the NEWCOM++ network of excellence both within the 6th European Community Framework Programme, by the U.S. National Science Foundation under Grants CNS-06-25637 and CNS-06-26611, and also by the REMON Consortium.

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