Joint Source-Channel Coding with One-Bit ADC Front End

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***Abstract*—This paper considers the zero-delay transmission of a Gaussian source over an additive white Gaussian noise (AWGN) channel with a one-bit analog-to-digital converter (ADC) front end. The optimization of the encoder and decoder is tackled under both the mean squared error (MSE) distortion and the outage distortion criteria with an average power constraint. For MSE distortion, the optimal transceiver is identified over the space of symmetric encoders. This result demonstrates that the linear encoder, which is optimal with a full-precision front end, approaches optimality only in the low signal-to-noise ratio (SNR) regime; while, digital transmission is optimal in the high SNR regime. For the outage distortion criterion, the structure of the optimal encoder and decoder are obtained. In particular, it is shown that the encoder mapping is piecewise constant and can take only two opposite values when it is non-zero.**

Index Terms-Joint source channel coding, zero-delay transmission, average distortion, outage distortion, one-bit ADC.

I. Introduction

An important practical constraint on the implementation of digital receivers in communication systems is the power consumed by analog-to-digital converters (ADCs) [1]. Therefore, a practical solution to reduce the power consumption is to keep the ADC resolution low. Motivated by this observation, in [2]–[7], communication systems with a one-bit ADC front end have been studied for different scenarios such as low-power systems, ultra-wideband links, millimetre-wave communication, and massive multiple-input multiple-output (MIMO) systems. For example, for an AWGN channel with a one-bit ADC, it is shown in [2] that bipolar (BPSK) transmission achieves the capacity.

While the previous literature focuses on the reliable transmission of digital information over long blocks, in applications such as the Internet of Things, cyber-physical systems or wireless sensor networks, low-delay transfer of analog measurements is a more relevant communication task [8]. In light of this, here we consider the zero-delay transmission of an analog Gaussian source over an AWGN channel in the presence of a one-bit ADC (see Fig. 1). As a reference, we note that, for this problem, in the presence of a full resolution front end, and under the mean squared error (MSE) distortion measure, linear transmission and minimum MSE (MMSE) estimation are the optimal encoder and decoder, respectively [9].

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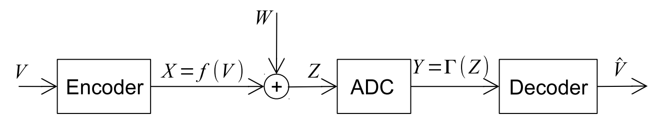

Figure 1. System model for the transmission of a single Gaussian source over an AWGN channel with a one-bit ADC receiver front end.

We study the optimization of encoding and decoding functions under two different criteria, namely MSE and distortion outage probability. Among other results, we show that, under MSE distortion, linear transmission only approaches optimality in the low signal-to-noise ratio (SNR) regime. Also, for the outage distortion criterion, structure of the optimal encoder and decoder are obtained, and it is shown that the encoder mapping is piecewise constant, taking only two opposite values when it is non-zero.

The paper is organized as follows. In Section II, we introduce the system model. In Section III, we consider the transmission under the MSE distortion criterion. As reference, we also compare the resulting performance with linear and digital encoders. In Section IV, we consider the transmission under the outage distortion criterion. In Section V, numerical results are provided, followed by conclusions in Section VI. Details of the proofs are omitted due to space limitations, and will be provided in a longer version of this paper.

II. System Model

We consider the system in Fig. 1, in which a single sample of Gaussian source $V \sim \mathcal{N}(0, \sigma_v^2)$, is transmitted to a receiver that makes a single quantized observation over an AWGN channel. The encoded signal is given as $X = f(V)$, where $f : \mathbb{R} \to \mathbb{R}$ is a mapping from the source sample to the channel input, whose average transmission power is constrained by P, i.e., $\mathbb{E}[f(V)^2] \leq P$. The noisy signal

$$Z = f(V) + W, \tag{1}$$

with noise $W \sim \mathcal{N}(0, \sigma_w^2)$, is quantized with a one-bit ADC, $\Gamma(\cdot)$, producing the single observed bit

$$Y = \Gamma(Z), \tag{2}$$

where $\Gamma(x) = 1$ if $x < 0$, and $\Gamma(x) = 0$ otherwise. We define $\text{SNR} = P/\sigma_w^2$. From the quantized signal Y, the decoder produces an estimate $\hat{V}$ of V using a decoding function $g : \{0, 1\} \to \mathbb{R}$, i.e., $\hat{V} = g(Y)$.

Two performance criteria are considered, namely, the MSE

$$\bar{D} = \mathbb{E}[(V - \hat{V})^2], \tag{3}$$

and the distortion outage probability, defined as

$$\epsilon(D) = \Pr((V - \hat{V})^2 \geq D), \tag{4}$$

for some specified target distortion D. In either case, we aim at identifying the optimal function $f(\cdot)$ along with the corresponding estimator at the receiver.

III. Average Distortion

In this section, we study the design of the encoder and decoder under the MSE criterion (3). We first derive the optimal encoder and decoder mappings under the constraint that the encoder mapping $f(\cdot)$ be symmetric. We then consider the conventional linear transmission and digital modulation schemes as reference.

A. Optimal Symmetric Encoder and Decoder

We consider the problem of minimizing the MSE under an average power constraint

$$\begin{aligned} &\underset{f,g}{\text{minimize}} \quad \bar{D} \\ &\text{subject to} \quad \mathbb{E}[f(V)^2] \leq P. \end{aligned} \tag{5}$$

Without loss of generality, we write the receiver mapping as

$$\hat{V} = g(Y) = \begin{cases} \hat{v}_0 & \text{if} \quad Y = 0 \\ \hat{v}_1 & \text{if} \quad Y = 1 \end{cases}, \tag{6}$$

which is defined by the pair of parameters $(\hat{v}_0, \hat{v}_1)$. We observe that, for any encoder mapping $f(\cdot)$, the MMSE estimator at the receiver is optimal for problem (5). Therefore, we can set $\hat{v}_0 = \mathbb{E}[V|Y = 0]$ and $\hat{v}_1 = \mathbb{E}[V|Y = 1]$ without loss of optimality.

Due to the symmetry of the problem, we limit the transmitter to odd mapping functions $f(\cdot)$, i.e., $-f(v) = f(-v)$. We leave it as an open issue to assess whether odd mappings are optimal (see [10] for a counterexample in a related problem). Under this restriction, we can calculate the optimal decoder (6) as

$$\hat{v}_0 = \mathbb{E}[V|Y = 0] \tag{7a}$$

$$= \frac{\frac{1}{\sigma_v}\int_{-\infty}^{\infty} v\Phi\left(\frac{v}{\sigma_v}\right)\Pr(Y = 0\big|V = v)dv}{\Pr(Y = 0)} \tag{7b}$$

$$= \frac{2}{\sigma_v} - \frac{4}{\sigma_v}\int_{0}^{\infty} v\Phi\left(\frac{v}{\sigma_v}\right)Q\left(\frac{f(v)}{\sigma_w}\right)dv, \tag{7c}$$

where $\Phi(\cdot)$ and $Q(\cdot)$ represent the probability density function (PDF) and complementary cumulative distribution function (CCDF) of a standard Gaussian random variable $\mathcal{N}(0, 1)$, respectively. We also have $\hat{v}_0 = -\hat{v}_1$. Moreover, we can restrict without loss of generality the optimization space to mapping functions $f(v)$ that satisfy $\hat{v}_0 \geq 0$. The average distortion can be written as

$$\bar{D} \overset{(a)}{=} \sigma_v^2 - \mathbb{E}[V\hat{V}] \tag{8a}$$

$$\overset{(b)}{=} \sigma_v^2 - \frac{1}{2}\left(\hat{v}_0\mathbb{E}[V|\hat{V} = \hat{v}_0] + \hat{v}_1\mathbb{E}[V|\hat{V} = \hat{v}_1]\right) \tag{8b}$$

$$\overset{(c)}{=} \sigma_v^2 - \hat{v}_0^2, \tag{8c}$$

where (a) is due to the orthogonality property of MMSE estimation; (b) follows from the discussed symmetry of the encoder; and (c) is due to the chain of equalities $\mathbb{E}[V|\hat{V} = \hat{v}_0] = \mathbb{E}[V|Y = 0] = \hat{v}_0 = -\hat{v}_1 = -\mathbb{E}[V|\hat{V} = \hat{v}_1]$. Given (8c) and the non-negativity of $\hat{v}_0$, minimizing $\bar{D}$ is equivalent to maximizing $\hat{v}_0$.

In light of the discussion above, we propose to tackle problem (5) by minimizing the following Lagrangian function:

$$\underset{f}{\text{minimize}} \quad -\hat{v}_0 + \lambda E[f(V)^2]. \tag{9}$$

where $\lambda > 0$ is a Lagrange multiplier. The next proposition provides the optimal encoder mapping that solves problem (9).

Proposition III.1. *The optimal odd mapping $f(\cdot)$ for problem (9) is unique*[1] *and is defined by the implicit equation*

$$f(v)e^{\frac{f(v)^2}{2\sigma_w^2}} = \frac{v}{\sqrt{2\pi}\sigma_w\lambda}. \tag{10}$$

Proof: The problem (9) satisfies the conditions under which the Weierstrass' theorem applies, since the objective function is continuous and coercive (see [11, Section A.2] for definitions). Therefore, there exists a function $f(\cdot)$ that solves problem (9). By applying the Euler-Lagrange equations [12, Section 7.5], we obtain the necessary condition in (10) for the optimal $f(\cdot)$. Finally, we observe that, due to the monotonicity of the left-hand side of (10) as a function of $f(\cdot)$, there is only one function $f(v)$ that satisfies the necessary Euler-Lagrange equation (10). This, along with the existence of an optimal solution, completes the proof. □

From Proposition III.1, it can be verified that, in the low SNR regime, namely for $\sigma_w^2 \to \infty$ (or SNR $\to 0$), the optimal mapping for problem (9) is linear, i.e., $f(v) \propto v$. This conclusion is in line with the results obtained in [13, Theorem 8]. Examples of the optimal encoding functions obtained numerically from (10) are provided in Section V.

B. Linear transmission

The encoder mapping for linear transmission is given by

$$f(v) = \sqrt{P/\sigma_v^2}v. \tag{11}$$

As seen in Sec. III-A, linear transmission is optimal in the low-SNR regime. Here we obtain its performance for any given channel SNR. The MSE distortion $\bar{D}_l$ achieved by linear transmission can be found by substituting (11) in (8c). Since the resulting expression is not in closed form, we now derive analytical upper and lower bounds that can be useful to obtain additional insights. Using the lower bound in [14] for the CCDF $Q(\cdot)$, namely $Q(x) \geq \beta e^{-\frac{kx^2}{2}}$, where

[1]We have uniqueness if we constrain the mapping to be monotone increasing.

$\beta = \frac{e^{(\pi(k-1)+2)^{-1}}}{2k}\sqrt{\frac{1}{\pi}(k-1)(\pi(k-1)+2)}$ for any $k \geq 1$, the distortion of linear transmission can be lower bounded as

$$\bar{D}_l \geq \sigma_v^2 \left(1 - \frac{2}{\pi}\left(1 - \frac{2\beta}{1 + k \cdot \mathrm{SNR}}\right)^2\right). \tag{12}$$

Instead, using the inequality $Q(x) \leq \frac{1}{4}(e^{-x^2} + e^{-\frac{x^2}{2}})$ [15] we have the upper bound

$$\bar{D}_l \leq \sigma_v^2 \left(1 - \frac{1}{2\pi}\left(\frac{\mathrm{SNR}(3 + 8\mathrm{SNR})}{(\mathrm{SNR}+1)(2\mathrm{SNR}+1)}\right)\right). \tag{13}$$

Using these bounds we observe that, in the asymptotic limit of low SNR, when SNR $\to 0$, we have the average distortion $\bar{D}_l = \sigma_v^2$, while, when SNR $\to \infty$, we obtain $\bar{D}_l = \sigma_v^2(1 - 2/\pi)$. Both distortions can be argued to be the minimal in the low SNR and high SNR regimes, respectively. In fact, for zero SNR, the MMSE estimate, even with an infinite-resolution front end, is given by $\hat{V} = 0$ which yields $\bar{D} = \sigma_v^2$. Instead, for infinite SNR, the best mapping is given by the optimal binary quantizer, which yields $\bar{D} = \sigma_v^2(1 - 2/\pi)$ (see, e.g., [16, Section 10.1]).

C. Digital Transmission

Here we consider a scheme based on source quantization and mapping to a discrete constellation for transmission. Accordingly, the source is quantized to one of M levels, each characterized by the intervals $(d_{i-1}, d_i]$ for $i = 1, ..., M$, where $d_0 = -\infty$, $d_M = \infty$, and $d_{i-1} \leq d_i$ for all $i = 1, ..., M$. Each interval $(d_{i-1}, d_i]$ is mapped to the corresponding channel input $X = x_i$. We take the constellation of possible transmission points to be $\{x_i = (2i - 1 - M)A, i = 1, ..., M\}$, for some parameter $A \geq 0$. Note that, when M is even, this corresponds to the M-ary pulse amplitude modulation (M-PAM), while if M is odd, the constellation includes the zero-power signal $x_{\frac{M+1}{2}} = 0$. The transmission power can be written as $E[X^2] = \sum_{i=1}^{M} x_i^2 \cdot \Pr(X = x_i)$, where $\Pr(X = x_i) = \frac{1}{\sigma_v}\int_{d_{i-1}}^{d_i} \Phi\left(\frac{v}{\sigma_v}\right) dv$. The MSE distortion $\bar{D}_{d,M}$ achieved by this scheme is given by (8c), where

$$\hat{v}_0 = \frac{2\sigma_v}{\sqrt{2\pi}} \sum_{i=1}^{M} \left(e^{-\frac{d_{i-1}^2}{2\sigma_v^2}} - e^{-\frac{d_i^2}{2\sigma_v^2}}\right) Q\left(\frac{-x_i}{\sigma_w}\right). \tag{14}$$

As a special case, when $M = 2$, setting the quantization threshold $d_1 = 0$, the scheme simplifies to BPSK transmission, and the achievable distortion can be computed as

$$\bar{D}_{d,2} = \sigma_v^2 \left(1 - \frac{2}{\pi}\left(1 - 2Q\left(\sqrt{\mathrm{SNR}}\right)\right)^2\right). \tag{15}$$

We observe that, as for linear transmission, for SNR $\to \infty$, we have $\bar{D}_{d,2} = \sigma_v^2(1 - 2/\pi)$, and, for SNR $= 0$, we have $\bar{D}_{d,2} = \sigma_v^2$.

IV. Outage Distortion

In this section we study the optimal encoder and decoder under the outage distortion criterion (4). We specifically focus on the minimization of the Lagrangian functional below

$$\underset{f,g}{\text{minimize}} \quad \epsilon(D) + \lambda \mathbb{E}[f(V)^2], \tag{16}$$

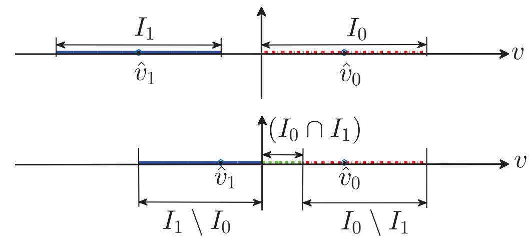

Figure 2. Illustration of the intervals I_0, I_1 and $(I_0 \cap I_1)$ that characterize the optimal encoder for the outage distortion criterion, for two different cases depending on the $(\hat{v}_0,\ \hat{v}_1)$ values.

where $\lambda > 0$ is a Lagrange multiplier, and the decoder is given, with no loss of generality, as in (6). We first assume that the reconstruction points $(\hat{v}_0, \hat{v}_1)$ are fixed arbitrarily, and focus on the optimization of the encoder mapping $f(\cdot)$ for a given decoder in (6). We then tackle the problem of minimizing the outage probability over the reconstruction points $(\hat{v}_0, \hat{v}_1)$.

To elaborate, we define the intervals

$$I_i = \{v : (v - \hat{v}_i)^2 \leq D\} \tag{17}$$

for $i = 0, 1$, which are depicted in Fig. 2. Each interval I_i, corresponds to the set of source values that are within the allowed distortion D of the reconstruction point $\hat{v}_i$. The following claims hold: (*i*) For all source outputs v in the set $(I_0 \cup I_1)^C = \{v : \min_{i=0,1}(v - \hat{v}_i)^2 > D\}$, outage occurs (superscript C denotes the complement set). We refer to this event as *source outage*. (*ii*) For all source values in the interval $I_0 \cap I_1$, either of the reconstruction points yield a distortion no more than the target value D. Therefore, regardless of which value $(\hat{v}_0, \hat{v}_1)$ is selected by the receiver, no outage occurs. From observations (*i*) and (*ii*), it easily follows that, for all source values v inside the intervals $(I_0 \cup I_1)^C$ and $(I_0 \cap I_1)$, the optimal mapping is $f(v) = 0$, since, for both intervals, the occurrence of an outage event is independent of the transmitted signal.

From the discussion above, we only need to specify the optimal mapping for the intervals $I_0 \backslash I_1$ and $I_1 \backslash I_0$. This should be done by accounting not only for the source outage event mentioned above, but also for the channel outage events. In particular, the distortion outage probability $\epsilon(D)$ can be written as

$$\begin{aligned} \epsilon(D) = &\Pr\left(V \in (I_0 \cup I_1)^C\right) \\ &+ \Pr\left(V \in (I_0 \setminus I_1), \hat{V} = \hat{v}_1\right) \\ &+ \Pr\left(V \in (I_1 \setminus I_0), \hat{V} = \hat{v}_0\right), \end{aligned} \tag{18}$$

where the first term accounts for the source outage event, while the second and third terms are the probabilities of outage due to channel transmission errors. For instance, the second term is the probability that the decoder selects $\hat{V} = \hat{v}_1$ while V is in the interval $I_0 \setminus I_1$, see Fig. 2. The next proposition characterizes the optimal encoder mapping.

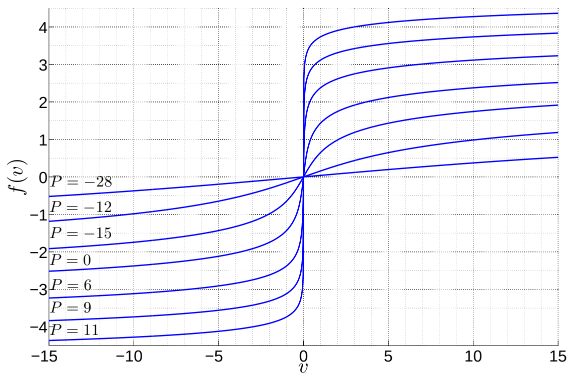

Figure 3. Optimal encoder mapping for the MSE distortion criterion under different power constraints P (dB) ($\sigma_w^2 = \sigma_v^2 = 1$).

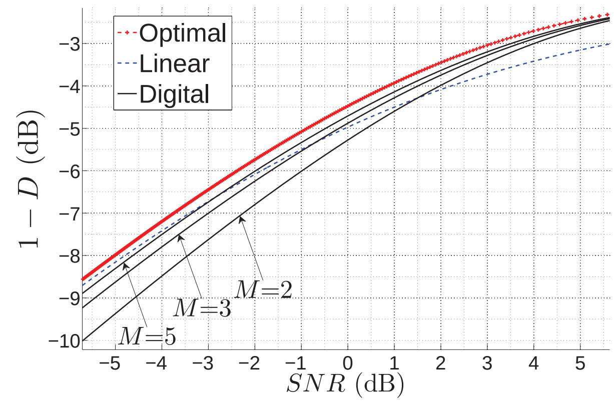

Figure 4. Complement of the MSE $1 - \bar{D}$ (dB), versus the SNR (dB) for the optimal symmetric encoder obtained in Proposition III.1 as well as for the linear and digital schemes ($\sigma_w^2 = \sigma_v^2 = 1$).

Proposition IV.1. *Given a target distortion D, and arbitrary reconstruction points $\hat{v}_0$ and $\hat{v}_1$, the optimal mapping $f(\cdot)$ for problem (16) is given by*

$$f(v) = \begin{cases} 0 & \text{if} \quad v \in (I_0 \cup I_1)^C \cup (I_0 \cap I_1) \\ -u & \text{if} \quad v \in (I_1 \backslash I_0) \\ u & \text{if} \quad v \in (I_0 \backslash I_1) \end{cases} \tag{19}$$

where u is the unique solution of

$$ue^{\frac{u^2}{2\sigma_w^2}} = \frac{1}{2\sqrt{2\pi}\sigma_w \lambda}. \tag{20}$$

We note here that, for given λ, the optimal u is independent of the values of $\hat{v}_0$ and $\hat{v}_1$. Examples of optimal encoders will be provided in Section V. In the next proposition, we turn to the optimization of the reconstruction levels $(\hat{v}_0, \hat{v}_1)$.

Proposition IV.2. *The optimal reconstruction points $(\hat{v}_0, \hat{v}_1)$, are given by*

$$\hat{v}_0 = \sqrt{D} - a^* \tag{21a}$$
$$\hat{v}_1 = -\hat{v}_0 \tag{21b}$$

where a^* is obtained from

$$a^* = \underset{a \in [0, \sqrt{D}]}{\text{arg min}} \quad 2Q\left(\frac{2\sqrt{D} - a}{\sigma_v}\right) + 2\left(Q\left(\frac{u}{\sigma_w}\right) + \lambda u^2\right) \cdot \left(Q\left(\frac{a}{\sigma_v}\right) - Q\left(\frac{2\sqrt{D} - a}{\sigma_v}\right)\right), \tag{22}$$

where u is obtained by solving (20).

To summarize, the optimal encoder and decoder are obtained as follows. First, given the Lagrange multiplier λ, the value of u is obtained by solving (20). Then the decoder's reconstruction points $(\hat{v}_0, \hat{v}_1)$ are computed from (21)-(22). Finally, the optimal encoder mapping is given by (19).

The next remark elaborates on the optimal encoder and decoder in two asymptotic SNR regimes.

Remark IV.1. *If λ is large, i.e., in the low-SNR regime, we have $\mathrm{u} \approx 0$ from (20), and hence, from (22) we obtain*

$$a^* \approx \underset{a \in [0, \sqrt{D}]}{\text{arg min}} \quad Q\left(\frac{a}{\sigma_v}\right) + Q\left(\frac{2\sqrt{D} - a}{\sigma_v}\right), \tag{23}$$

yielding $\hat{v}_0 = \hat{v}_1 = 0$, that is, $I_0 = I_1$ and $\epsilon(D) = 2Q\left(\frac{\sqrt{D}}{\sigma_v}\right)$. On the other hand, for small values of λ, corresponding to the high-SNR regime, we have that u is large and

$$a^* \approx \underset{a \in [0, \sqrt{D}]}{\text{arg min}} \quad 2Q\left(\frac{2\sqrt{D} - a}{\sigma_v}\right), \tag{24}$$

which yields distinct intervals I_0 and I_1 with $\hat{v}_0 = -\hat{v}_1 = \sqrt{D}$, and $\epsilon(D) = 2Q\left(\frac{2\sqrt{D}}{\sigma_v}\right)$.

V. Numerical Results

Here we provide some illustrations of the results derived above by means of numerical examples. We start by plotting in Fig. 3 the optimal mapping functions obtained from Proposition III.1 under the MSE criterion for different values of P, with $\sigma_w^2 = 1$. The value of the Lagrange multiplier λ in (10) is obtained by means of bisection so as to satisfy the power constraint $\mathbb{E}[f(V)^2] = P$. As discussed in Section III-A, the optimal mapping function for low SNR tends to linear transmission. This is reflected in Fig. 3 by the fact that for smaller values of P the function $f(v)$ approaches a straight line. Instead, for larger values of P, which corresponds to larger SNR, the mapping function tends to resemble a step function, which corresponds to digital transmission with $M = 2$, as discussed in Section III-C.

In Fig. 4, the MSE of optimal, linear and digital transmission schemes is plotted versus the SNR for $\sigma_w^2 = 1$ and $\sigma_v^2 = 1$. For clarity of illustration, we plot the accuracy measure $1 - \bar{D}$, where we note that $\bar{D} = \sigma_v^2 = 1$ is achievable by setting $\hat{V} = 0$ irrespective of the received signal. As discussed, linear transmission approaches optimality at low SNR, whereas, for higher SNR, digital schemes outperform linear transmission.

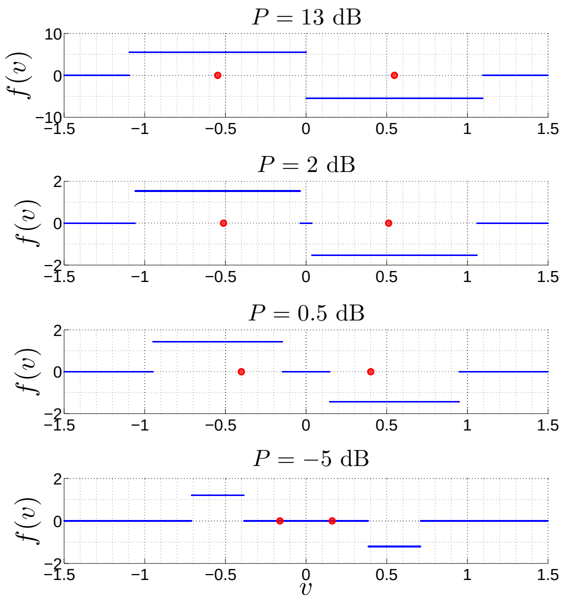

Figure 5. Optimal encoder mapping for different values of P ($\sigma_w^2 = \sigma_v^2 = 1$ and $D = 0.3$). The dots are the reconstruction points $\hat{v}_0$ and $\hat{v}_1$.

Also, it is seen that for digital transmission, increasing the number of constellation points improves the performance, while in the high SNR regime binary transmission is sufficient to achieve the optimal performance.

In Fig. 5, the optimal encoding function $f(\cdot)$ and the corresponding reconstruction points $(\hat{v}_0, \hat{v}_1)$ are shown under the distortion outage criteria for four different values of the power constraint P. It is seen that, as P decreases, the optimal reconstruction points $\hat{v}_0$ and $\hat{v}_1$, tend to zero as discussed in Remark IV.1. This can be explained, since, for low P, the optimal solution aims at minimizing the probability of source outage events, rather than the probability of error due to channel noise (recall (18)). In contrast, for significantly large P, as for Remark IV.1, we obtain $\hat{v}_0 = -\hat{v}_1 = \sqrt{D}$.

Finally, in Fig. 6, the complement of the outage probability $1 - \epsilon(D)$ is plotted with respect to SNR for different values of D. As the SNR decreases, the distortion outage probability tends to the source outage term in (23), which decreases with D.

VI. Conclusions

In this paper, we considered the zero-delay transmission of a single sample of a Gaussian source over an AWGN channel followed by a one-bit ADC at the receiver. We studied this scenario under an average power constraint for two performance criteria; namely, the MSE, and distortion outage probability. For the MSE, over the space of symmetric encoder mappings, we obtained the optimal encoder structure. We observed that in the low SNR regime linear transmission approaches the optimal performance whereas digital transmission becomes optimal in the high SNR regime. For the outage distortion criterion, we first obtained the optimal structure of the encoder given arbitrary reconstruction points. Then, we optimized the

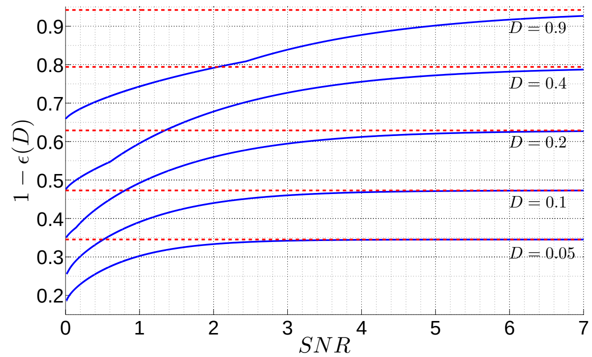

Figure 6. Complement of the outage probability $1-\epsilon(D)$ versus SNR ($\sigma_w^2 = \sigma_v^2 = 1$). Dashed lines represent the outage distortion probability in the high SNR regime for the corresponding distortion target.

reconstruction points for a given power constraint. We showed that the optimal encoder function is symmetric, and partially constant with respect to the value of the reconstruction points.

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