# Boundaries and supercurrent multiplets in 3D Landau-Ginzburg models

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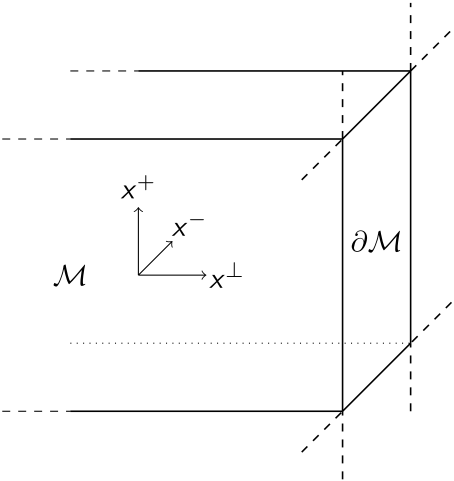
# Introduction

- Recent work with Ilka Brunner and Alexander Tabler
- Landau-Ginzburg models in 3D with 2D boundary
- Study the operator cohomology of the supercharges
- Study supercurrent multiplets in this setting to access cohomology

# Outline

# General Setup

Bulk: flat Minkowski space, $x^\mu = (x^+, x^-, x^\perp)$, boundary at $x^\perp = 0$



# General Setup

- Pure bulk theory has 3D $\mathcal{N} = 2$ supersymmetry

$$\{Q_\pm, \overline{Q}_\pm\} = -4P_\pm, \quad \{Q_+, \overline{Q}_-\} = 2P_\perp$$

- Restrict to $\{x^\perp \leq 0\}$ and possibly introduce localized degrees of freedom at $x^\perp = 0$
  $\Rightarrow P_\perp$ no longer generates a symmetry
- SUSY breaks to a 2D $(0, 2)$ or 2D $(1, 1)$ subalgebra:

$$(0, 2): \quad \{Q_+, \overline{Q}_+\} = -4P_+$$

# Landau-Ginzburg model

- Half-space $M = \{\, x \in \mathbb{R}^{1,2} \mid x^\perp \leq 0 \,\}$, bulk and boundary Lagrangian

$$S = \int_M \mathcal{L}_{\text{bulk}} + \int_{\partial M} \mathcal{L}_{\text{bdy}}$$

- $\mathcal{L}_{\text{bdy}}$ contains couplings of *bulk* fields at the boundary, and localized degrees of freedom
- Bulk theory: One chiral field $\Phi = \phi + \sqrt{2}\theta\psi + \dots$, superpotential $W(\Phi)$:

$$\mathcal{L}_{\text{bulk}} = \underbrace{\int d^4\theta \overline{\Phi}\Phi}_{\mathcal{L}_{\text{kin.}}} + \underbrace{\int d^2\theta W(\Phi) + \text{cc.}}_{\mathcal{L}_W}$$

- Assume global $R$-symmetry in bulk and boundary

# Landau-Ginzburg model

- Full SUSY cannot be preserved, look at the SUSY variation under the $(0, 2)$ subalgebra $\delta_{(0,2)}$
- Bulk theory is SUSY:

$$\delta_{(0,2)}\mathcal{L}_{\text{bulk}} = \partial_\mu V^\mu$$

- In the presence of a boundary, the action is *not* necessarily symmetric:

$$\delta_{(0,2)}S = \int_M \partial_\mu V^\mu + \int_{\partial M} \delta_{(0,2)}\mathcal{L}_{\text{bdy}} = \int_{\partial M} (V^\perp + \delta_{(0,2)}\mathcal{L}_{\text{bdy}})$$

$\Rightarrow$ the boundary Lagrangian must *compensate* the bulk variation

# Landau-Ginzburg model

- $\mathcal{L}_{\text{kin.}}$ can always be compensated by a coupling term of bulk fields localized at the boundary
- The compensation of $\mathcal{L}_W$ depends on the boundary conditions of the bulk fields $\phi$ and $\psi$
- Fixing $\phi|_\partial$ (generalized Dirichlet condition): $\delta_{(0,2)}\mathcal{L}_W$ vanishes at the boundary
- Fixing $\partial_\perp\phi|_\partial$ (generalized Neumann condition): We need additional degrees of freedom on the boundary to compensate $\delta_{(0,2)}\mathcal{L}_W$ $\Rightarrow (0,2)$ Fermi multiplet: $E$- and $J$-potential, need *matrix factorization* $E \cdot J = W$ at the boundary
- straightforward generalization to arbitrary LG models and defects

# Accessing cohomology

- Boundary operator cohomology and indices computed in Dirichlet and Neumann $W = 0$ case (Dimofte, Gaiotto, Paquette, '17)
- hard in general Neumann / factorization case
- In 2D $\mathcal{N} = (0, 2)$, the algebra of supercurrents gives access to cohomology; can this argument be lifted?
  $\Rightarrow$ Study supercurrent multiplets in this setting

# Supercurrent multiplets in 3D bulk theories

In any supersymmetric theory on 3D Minkowski space, there exists a superfield with the following properties (Dumitrescu, Seiberg, '11):

1. Energy momentum tensor $T^{\mu\nu} \in$ multiplet.
2. The supersymmetry currents $S_\alpha^\mu \in$ multiplet.
3. all other components of the multiplet have spin $\leq 1$.
4. only unique up to improvements (related to the improvements of the currents).
5. The multiplet is *indecomposable* (but usually not irreducible).

# Supercurrent multiplets in 3D bulk theories

If an $R$-symmetry is present:
Superfields $\mathcal{R}_{\alpha\beta}$, $\chi_\alpha$ with constraints

$$\begin{aligned} \overline{D}^\beta \mathcal{R}_{\alpha\beta} &= \chi_\alpha, \quad \overline{D}_\alpha \chi_\beta = 0, \\ D^\alpha \chi_\alpha + \overline{D}^\alpha \overline{\chi}_\alpha &= 0. \end{aligned} \tag{1}$$

- Contains currents $j^\mu$, $S^\mu_\alpha$, $T^{\mu\nu}$; conserved by eq. (1)
- Brane currents $H_\mu$, $F_{\mu\nu}$ and other terms

$\implies$ What happens to conserved currents when a boundary is introduced?

# Currents in presence of a boundary

- Generic bulk currents:

$$\partial_\mu J_B^\mu = 0 \ , \quad Q_B = \int_\Sigma J_B^0 \ ,$$

  no longer conserved after the introduction of a boundary $\partial\Sigma$.

- If the boundary preserves the symmetry: Introduce 2D boundary current $J_\partial^{\hat{\mu}} = (J_\partial^+, J_\partial^-)$ localized at boundary,

$$\partial_\mu J_B^\mu = 0, \quad \partial_{\hat{\mu}} J_\partial^{\hat{\mu}} = J_B^\perp|_\partial .$$

- Full current and charge:

$$J^\mu = J_B^\mu + \delta(x^\perp)\mathcal{P}^\mu{}_{\hat{\mu}} J_\partial^{\hat{\mu}}, \quad Q = \int_\Sigma J_B^0 + \int_{\partial\Sigma} J_\partial^0 \quad \text{conserved}$$

- Boundary Noether: Explicit formula for $J_\partial^{\hat{\mu}}$ in Lagrangian theories

# Boundary Supercurrent multiplets

Search for a multiplet that contains the preserved bulk-boundary supercurrents and energy-momentum tensor:

- Expect bulk and boundary parts for supercurrent multiplet:

$$\text{“}\mathcal{R}_\mu^{\text{full}} = \mathcal{R}_\mu^B + \delta(x^\perp)\mathcal{P}^{\hat{\mu}}{}_\mu \mathcal{R}_{\hat{\mu}}^\partial\text{”},$$

$$\text{“}\chi_\alpha^{\text{full}} = \chi_\alpha^B + \delta(x^\perp)\chi_\alpha^\partial\text{”}$$

- Problem: Bulk has 3D $\mathcal{N} = 2$ superspace structure, boundary has 2D $\mathcal{N} = (0,2)$
- Strategy: Decompose bulk fields according to their $(0,2)$ substructure, then construct bulk-boundary structures

# Bulk decomposition

Multiplet decomposition:

$$\mathcal{R}_\mu^B(x,\theta,\overline{\theta}) = \mathcal{R}_\mu^{B(0)} + \theta^- \mathcal{R}_\mu^{B(1)} - \overline{\theta}^- \overline{\mathcal{R}_\mu^{B(1)}} + \theta^- \overline{\theta}^- \mathcal{R}_\mu^{B(2)},$$

$$\chi_\alpha^B(x,\theta,\overline{\theta}) = \chi_\alpha^{B(0)} + \theta^- \chi_\alpha^{B(1a)} + \overline{\theta}^- \chi_\alpha^{B(1b)} + \theta^- \overline{\theta}^- \chi_\alpha^{B(2)},$$

where $\mathcal{R}_\mu^{B(*)}(x,\theta^+,\overline{\theta}^+)$, $\chi_\alpha^{B(*)}(x,\theta^+,\overline{\theta}^+)$ are $(0,2)$-multiplets.

# Bulk decomposition

3D constraint equations:

$$\overline{D}^{\beta}\mathcal{R}_{\alpha\beta} = \chi_{\alpha}, \quad \overline{D}_{\alpha}\chi_{\beta} = 0,$$

$$D^{\alpha}\chi_{\alpha} + \overline{D}^{\alpha}\overline{\chi}_{\alpha} = 0.$$

In (0,2) superspace: 12 equations (fully equivalent)

$$0 = \overline{D}_{+}\chi_{\alpha}^{B(0)},$$

$$0 = \overline{D}_{+}\chi_{\alpha}^{B(1a)} + 2i\partial_{\perp}\chi_{\alpha}^{B(0)},$$

$$(\dots)$$

$$\chi_{\alpha}^{B(2)} = \overline{D}_{+}\mathcal{R}_{\alpha-}^{B(2)} + 2i\partial_{\perp}\overline{\mathcal{R}_{\alpha-}^{B(1)}} + 2i\partial_{-}\overline{\mathcal{R}_{\alpha+}^{B(1)}}.$$

# Boundary multiplet

Ansatz for the boundary part of the multiplet:

$$\mathcal{R}^{\partial}_{\hat{\mu}}(x,\theta,\overline{\theta}) = \mathcal{R}^{\partial(0)}_{\hat{\mu}} + \cancel{\theta^{-}\mathcal{R}^{\partial(1)}_{\hat{\mu}} - \overline{\theta}^{-}\overline{\mathcal{R}^{\partial(1)}_{\hat{\mu}}}} + \theta^{-}\overline{\theta}^{-}\mathcal{R}^{\partial(2)}_{\hat{\mu}},$$

$$\chi^{\partial}_{\alpha}(x,\theta,\overline{\theta}) = \chi^{\partial(0)}_{\alpha} + \theta^{-}\chi^{\partial(1a)}_{\alpha} + \overline{\theta}^{-}\chi^{\partial(1b)}_{\alpha} + \theta^{-}\overline{\theta}^{-}\chi^{\partial(2)}_{\alpha},$$

containing boundary parts of currents.

$\mathcal{R}^{B(1)}_{\mu} \sim (S^{B}_{\mu})_{-} + \theta^{+} T_{\mu\perp} + \dots$ contains only “broken” currents
$\implies$ No boundary part $\mathcal{R}^{\partial(1)}_{\hat{\mu}}$

# Boundary multiplet constraints

Adapt the bulk constraint equations to boundary constraint equations:

- Adapt such that bulk-boundary conservation $\partial_{\hat{\mu}} J_\partial^{\hat{\mu}} = J_B^\perp|_\partial$ is implied for all symmetries preserved by the boundary
- Demand internal consistency

$\Rightarrow$ We find a matching set of equations and bulk-boundary multiplets

$$\begin{aligned} \mathcal{R}_\mu^{(*)\mathrm{full}} &= \mathcal{R}_\mu^{(*)B} + \delta(x^\perp)\mathcal{P}^{\hat{\mu}}{}_\mu \mathcal{R}_{\hat{\mu}}^{(*)\partial} \\ \chi_\alpha^{(*)\mathrm{full}} &= \chi_\alpha^{(*)B} + \delta(x^\perp)\chi_\alpha^{(*)\partial} \end{aligned} \quad + \text{ equations}$$

# Cohomology in 2D

In *pure* 2D $\mathcal{N} = (0,2)$ theories, the $\mathcal{R}$-multiplet has the form

$$\partial_{--}\mathcal{R}^{(0,2)}_{++} + \partial_{++}\mathcal{R}^{(0,2)}_{--} = 0,$$

$$\boxed{\overline{D}_+(\mathcal{T}^{(0,2)}_{----} - \tfrac{i}{2}\partial_{--}\mathcal{R}^{(0,2)}_{--}) = 0.} \qquad (2)$$

Implies *twisted* EM tensor in the cohomology (Dedushenko '15):

$$\widetilde{T}_{++} = T_{++} + \tfrac{i}{2}\partial_+ j_+ \quad \overline{Q}_+\text{-exact},$$

$$\widetilde{T}_{+-} = T_{+-} - \tfrac{i}{2}\partial_- j_+ \quad \overline{Q}_+\text{-exact},$$

$$\widetilde{T}_{--} = T_{--} - \tfrac{i}{2}\partial_- j_- \quad \overline{Q}_+\text{-closed}.$$

# Cohomology in 3D

In our 3D full multiplet, we find in analogy

$$\overline{D}_+(\mathcal{R}_{--}^{\text{full}(2)} + 2i\partial_- \mathcal{R}_{--}^{\text{full}(0)}) = -2i\partial_\perp \overline{\mathcal{R}_{--}^{B(1)}} + 2i\delta(x^\perp)\overline{\mathcal{R}_{--}^{B(1)}}|_\partial$$

Not closed, but closed after integration along $x^\perp$.

# Integrated current multiplet

Integrated multiplet:

$$\mathcal{R}_{\hat{\mu}}^{\text{int.}} := \mathcal{R}_{\hat{\mu}}^{\partial} + \int dx^{\perp} \mathcal{R}_{\hat{\mu}}^{B}.$$

- Contains *integrated currents*

$$J_{\text{int.}}^{\hat{\mu}} = J_{\partial}^{\hat{\mu}} + \int dx^{\perp} J_{B}^{\mu}, \qquad \partial_{\hat{\mu}} J_{\text{int.}}^{\hat{\mu}} = 0.$$

- Currents lead to same charges as the local currents
- Has the *same* structure as a 2D $\mathcal{N} = (0,2)$ multiplet, in particular

$$\overline{D}_{+}(\mathcal{R}_{--}^{\text{int.}(2)} + 2i\partial_{-}\mathcal{R}_{--}^{\text{int.}(0)}) = 0,$$

thus the *integrated, twisted EM tensor* is $\overline{Q}_{+}$-closed.

# Conclusion

- Matrix factorizations in 3D Landau-Ginzburg models understood
- Construction of bulk-boundary supercurrent multiplets for the preserved SUSY structures
- Cohomology and integrated currents in the bulk-boundary multiplets
- In the paper: Explicitly computed all the multiplets in the Landau-Ginzburg model
- In the paper: Further verification using operator quantization in the explicit LG model

# Outlook

- Utilize this construction in several applications:
  - More detailed study of cohomology and possibly dualities
  - Coupling to supergravity, SUSY on curved manifolds with boundary
- Generalizations:
  - Other dimensions and supersymmetries
  - Gauge theories

# Appendix

# Bulk S multiplet

$$
\begin{aligned}
\mathcal{S}_\mu = {} & j_\mu - i\theta\left(S_\mu + \tfrac{i}{\sqrt{2}}\gamma_\mu\overline{\omega}\right) - i\overline{\theta}\left(\overline{S}_\mu - \tfrac{i}{\sqrt{2}}\gamma_\mu\omega\right) + \tfrac{i}{2}\theta^2\overline{Y}_\mu + \tfrac{i}{2}\overline{\theta}^2 Y_\mu \\
& - (\theta\gamma^\nu\overline{\theta})\left(2T_{\nu\mu} - \eta_{\mu\nu}A - \tfrac{1}{4}\epsilon_{\mu\nu\rho}H^\rho\right) - i\theta\overline{\theta}\left(\tfrac{1}{4}\epsilon_{\mu\nu\rho}F^{\nu\rho} + \epsilon_{\mu\nu\rho}\partial^\nu j^\rho\right) \\
& + \tfrac{1}{2}\theta^2\overline{\theta}\left(\gamma^\nu\partial_\nu S_\mu - \tfrac{i}{\sqrt{2}}\gamma_\mu\gamma_\nu\partial^\nu\overline{\omega}\right) + \tfrac{1}{2}\overline{\theta}^2\theta\left(\gamma^\nu\partial_\nu\overline{S}_\mu + \tfrac{i}{\sqrt{2}}\gamma_\mu\gamma_\nu\partial^\nu\omega\right) \\
& - \tfrac{1}{2}\theta^2\overline{\theta}^2\left(\partial_\mu\partial^\nu j_\nu - \tfrac{1}{2}\partial^2 j_\mu\right), \\
\chi_\alpha = {} & -i\lambda_\alpha(y) + \theta_\beta\left[\delta_\alpha{}^\beta D(y) - (\gamma^\mu)_\alpha{}^\beta\left(H_\mu(y) - \tfrac{i}{2}\epsilon_{\mu\nu\rho}F^{\nu\rho}(y)\right)\right] \\
& + \tfrac{1}{2}\overline{\theta}_\alpha C - \theta^2(\gamma^\mu\partial_\mu\overline{\lambda})_\alpha(y), \\
\mathcal{Y}_\alpha = {} & \sqrt{2}\omega_\alpha + 2\theta_\alpha B + 2i\gamma^\mu_{\alpha\beta}\overline{\theta}^\beta Y_\mu + \sqrt{2}i(\theta\gamma^\mu\overline{\theta})\epsilon_{\mu\nu\rho}(\gamma^\nu\partial^\rho\omega)_\alpha \\
& + \sqrt{2}i\theta\overline{\theta}(\gamma^\mu\partial_\mu\omega)_\alpha + i\theta^2\gamma^\mu_{\alpha\beta}\overline{\theta}^\beta\partial_\mu B - \overline{\theta}^2\theta_\alpha\partial_\mu Y^\mu + \tfrac{1}{2\sqrt{2}}\theta^2\overline{\theta}^2\partial^2\omega_\alpha
\end{aligned}
$$

# Bulk constraints (1)

$$\overline{D}_+ \chi_\alpha^{B(0)} = 0,$$

$$\overline{D}_+ \chi_\alpha^{B(1a)} + 2i\partial_\perp \chi_\alpha^{B(0)} = 0,$$

$$\overline{D}_+ \chi_\alpha^{B(2)} = 0.$$

$$\mathrm{Im}(D_+ \chi_-^{B(0)} - \chi_+^{B(1a)}) = 0,$$

$$\overline{D}_+ \overline{\chi_-^{B(1a)}} + \chi_+^{B(2)} - 2i\partial_- \chi_+^{B(0)} - 2i\partial_\perp \chi_-^{B(0)} = 0,$$

$$\mathrm{Im}(D_+ \chi_-^{B(2)} - 2i\partial_- \chi_+^{B(1a)} - 2i\partial_\perp \chi_-^{B(1a)}) = 0.$$

# Bulk constraints (2)

$$\chi_\alpha^{B(1b)} = 0,$$

$$\chi_\alpha^{B(2)} + 2i\partial_- \chi_\alpha^{B(0)} = 0$$

$$\chi_\alpha^{B(0)} = \overline{D}_+ \mathcal{R}_{\alpha-}^{B(0)} - \overline{\mathcal{R}_{\alpha+}^{B(1)}},$$

$$-\chi_\alpha^{B(1a)} = \overline{D}_+ \mathcal{R}_{\alpha-}^{B(1)} + \mathcal{R}_{\alpha+}^{B(2)} + 2i\partial_\perp \mathcal{R}_{\alpha-}^{B(0)} + 2i\partial_- \mathcal{R}_{\alpha+}^{B(0)},$$

$$0 = \overline{D}_+ \overline{\mathcal{R}}_{\alpha\beta}^{B(1)},$$

$$\chi_\alpha^{B(2)} = \overline{D}_+ \mathcal{R}_{\alpha-}^{B(2)} + 2i\partial_\perp \overline{\mathcal{R}_{\alpha-}^{B(1)}} + 2i\partial_- \overline{\mathcal{R}_{\alpha+}^{B(1)}}.$$

# Boundary constraints (1)

$$
\begin{aligned}
0 &= \chi_-^{\partial(2)} + 2i\partial_-\chi_-^{\partial(0)}, \\
0 &= \overline{D}_+\chi_+^{\partial(1a)} - 2i\chi_+^{B(0)}|_\partial, \\
0 &= \mathrm{Im}(D_+\chi_-^{\partial(0)} - \chi_+^{\partial(1a)}), \\
0 &= \mathrm{Im}(D_+\chi_-^{\partial(2)} - 2i\partial_-\chi_+^{\partial(1a)} + 2i\chi_-^{B(1a)}|_\partial), \\
\chi_-^{\partial(0)} &= \overline{D}_+\mathcal{R}_{--}^{\partial(0)}, \\
\chi_+^{\partial(1a)} &= -\mathcal{R}_{++}^{\partial(2)} + 2i\mathcal{R}_{+-}^{B(0)}|_\partial - 2i\partial_-\mathcal{R}_{++}^{\partial(0)},
\end{aligned}
$$

# Boundary constraints (2)

$$\chi_{-}^{\partial(1a)} = 2i\mathcal{R}_{--}^{B(0)}|_{\partial},$$

$$\chi_{-}^{\partial(2)} = \overline{D}_{+}\mathcal{R}_{--}^{\partial(2)} - 2i\overline{\mathcal{R}_{--}^{B(1)}}|_{\partial},$$

$$\chi_{+}^{\partial(2)} = -2i\overline{\mathcal{R}_{+-}^{B(1)}}|_{\partial},$$

$$\chi_{+}^{\partial(0)} = 0,$$

$$\chi_{\alpha}^{\partial(1b)} = 0.$$