Interface Flows in D1/D5 Holography

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Table of Contents

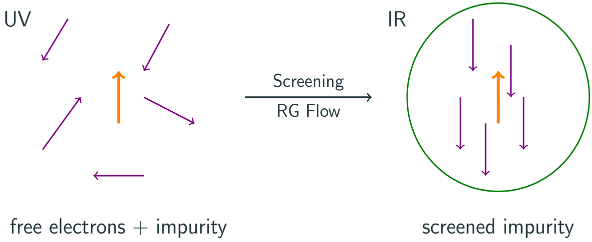
Kondo and Boundary RG Flows

Spin impurities and the Kondo effect

Heavy magnetic impurity interacts with conduction electrons

Ultraviolet. Free electrons with mild antiferromagnetic coupling to spin

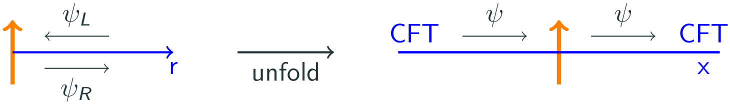
Infrared. Impurity is screened through binding with conduction electrons



$$H = \psi^\dagger i \nabla \psi + \lambda\, \delta(\vec{r})\, \vec{S} \cdot \vec{J}, \qquad \vec{J} = \frac{1}{2} \psi^\dagger T \psi$$

Kondo Model as CFT Affleck & Ludwig '91

S-wave approximation



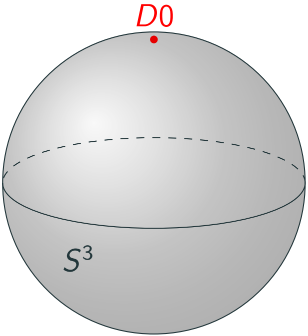
System described by action

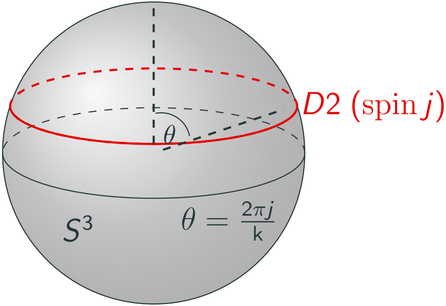
$$\mathcal{I} = \mathcal{I}_{\mathrm{WZW}}(\hat{\mathfrak{su}}(2)_k) + \lambda \int_{\partial\Sigma} dt\, \vec{S} \cdot \vec{J}(t),$$

where $\vec{S}$ is in spin-S irreducible representation of $\mathfrak{su}(2)$.

Conformally Invariant Boundary Conditions of $\hat{\mathfrak{su}}(2)_k$

- Labeled by set of primaries $j = \underbrace{0, \frac{1}{2}, \ldots, \frac{k}{2}}_{k+1}$
- Correspond to discrete set of conjugacy classes





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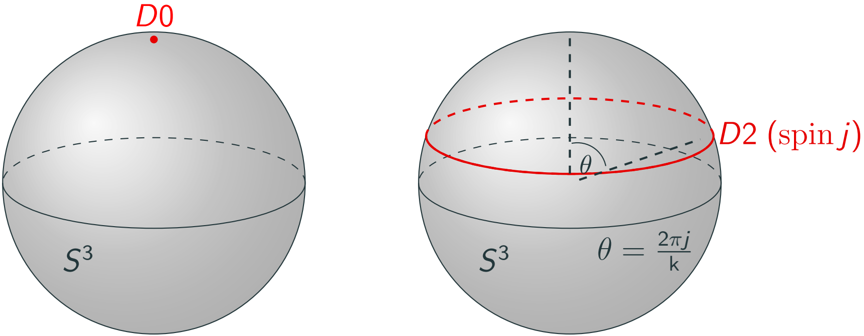
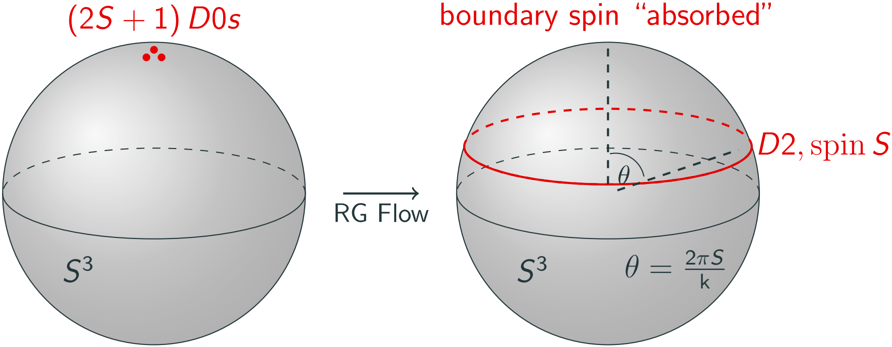


Both types of branes preserve $SO(3)$!

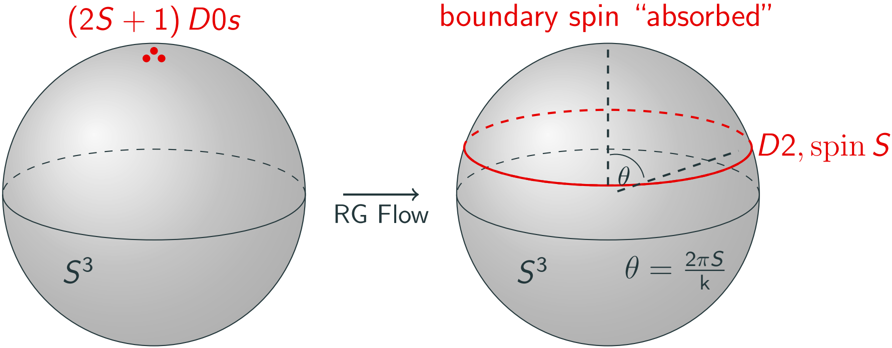
Non-abelian polarization

$(2S+1)$ pointlike Branes (spin-0) $\to$ 1 brane of spin S



$\hat{\mathfrak{su}}(2)_k$ $\quad S = 0, \frac{1}{2}, \dots, \frac{k}{2}$

Goal



Geometric implementation in AdS/CFT:
Kondo-like defect flows in D1/D5 system via non-abelian polarization.

Interface RG Flows in Holography

Type IIB on $M_{10} = \mathbb{R}^{1,1} \times \mathbb{R}^4 \times M_4$ (with $M_4 = \text{K3}$ or T^4):

	0	1	2	3	4	5	6	7	8	9
D5 (N_5)	•	•					•	•	•	•
D1 (N_1)	•	•								

Near-Horizon Limit. IIB string theory on $\text{AdS}_3 \times S^3 \times M_4$ supported by $F^{(3)}$ flux on AdS_3 & S^3

Symmetries

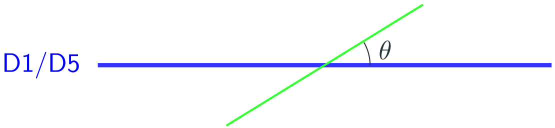
- $\mathcal{N} = (4, 4)$ small superconformal algebra.
- Bosonic part: $\mathfrak{so}(2, 2) \times \mathfrak{so}(4)$.

(p,q) String Interfaces Bachas & Petropoulos '00

	AdS_3			S^3			M_4			
	t	x	z	θ	ϕ	χ	6	7	8	9
D5 (N_5)	•	•					•	•	•	•
D1 (N_1)	•	•								
(p, q)	•	–	•							

$p =$ fundamental string charge, $q =$ D1-brane charge

- Interface preserves $\mathfrak{so}(2,1) \times \mathfrak{su}(2)$ & 8 superconformal charges
- Interface with $q \neq 0$ shifts central charge of CFT

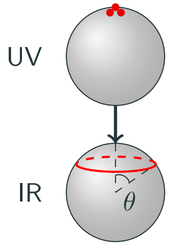


$\theta = \tan^{-1}(g_s p/q)$

Flows from brane polarization

Deform branes by non-abelian polarization: Myers '99
Coordinates on S^3 become non-commutative $\Longrightarrow$ fuzzy S^2 inside S^3.

- (p, q) strings puff up into D3 branes
- BPS flow solutions for general (p, q) obtained from κ symmetry projector (along lines of Gomis &al. '99).



	AdS$_3$			S^3			M_4			
	t	x	z	$\theta(z)$	ϕ	χ	6	7	8	9
D3 (p, q)	•	$(-)$	•		•	•				

Flows from brane polarization

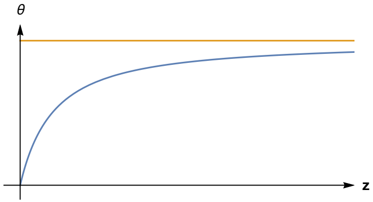
- In D3-brane description, $\theta = \theta(z)$

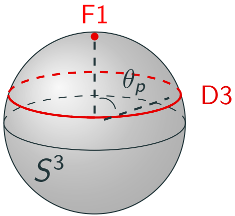
$$I = T_{\mathrm{D3}} \int d^4\xi e^{-\Phi} \sqrt{-\det(\hat{g} + F)} + T_{\mathrm{D3}} \int (C^{(2)} \wedge F + \frac{1}{2} F \wedge F)$$

- Simplest case: when D3 branes carry no D1 charge, solution is given by

$$z = z_0 \frac{\sin\theta}{\theta_p - \theta} \qquad \theta_p = \pi \frac{p}{N_5}$$

where z is the radial coordinate in Poincaré patch.





Backreacted Supergravity Solutions of Interface Fixed Points

Half BPS Supergravity Solutions Chiodaroli, Gutperle, Krym & Hung 2009-2010

Asymptotically $AdS_3 \times S^3 \times M_4$ $\frac{1}{2}$-BPS solutions

$$ds_{10}^2 = f_1^2 ds^2_{AdS_2} + f_2^2 ds^2_{S^2} + f_3^2 ds^2_{M_4} + \rho^2\, dz\, d\bar{z}$$

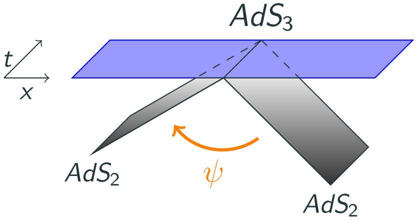
where $f_i = f_i(z, \bar{z})$, $\rho = \rho(z, \bar{z})$

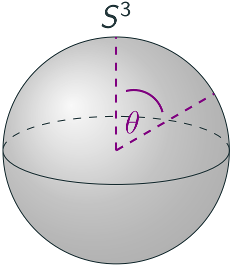
Preserves the desired Symmetries

- $\mathfrak{so}(2,1) \times \mathfrak{so}(3)$
- 8 super(conformal) symmetries

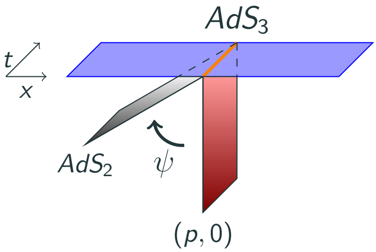
Empty $AdS_3 \times S^3 \times M_4$ Chiodaroli, Gutperle, Krym 2009

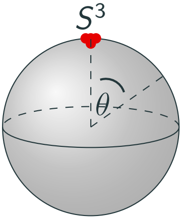
$$ds^2_{10} = \cosh^2\psi\, ds^2_{AdS_2} + \sin^2\theta\, ds^2_{S^2} + ds^2_{M_4} + d\psi^2 + d\theta^2$$





(p,q) Interface Solutions





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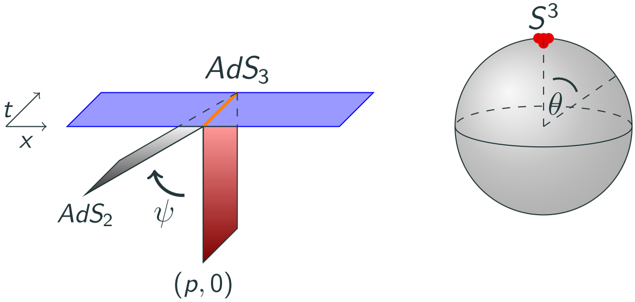
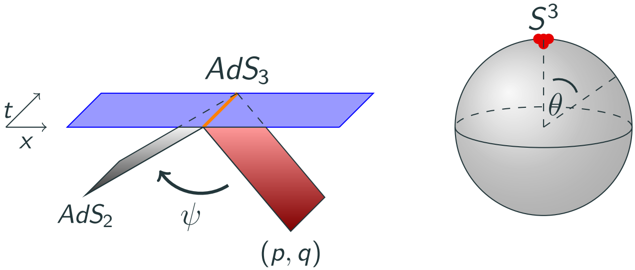
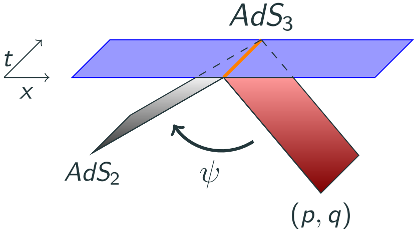
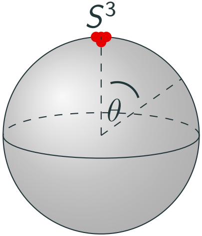


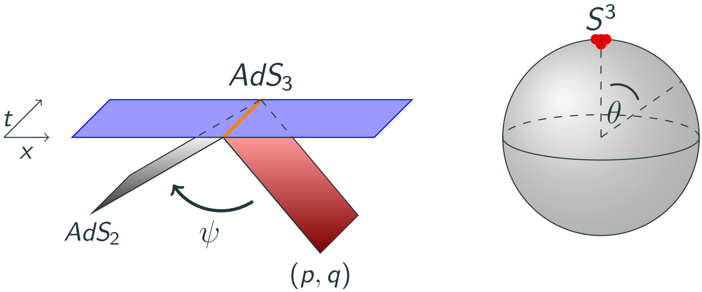


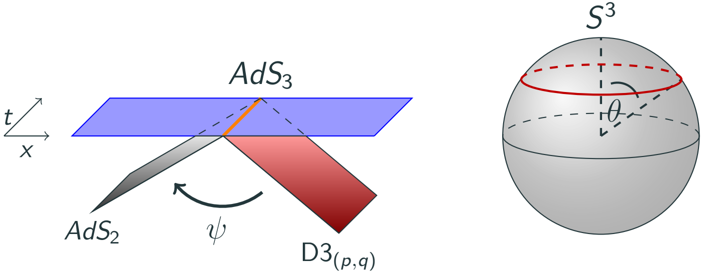
Interface Solutions and RG Flow





Interface Solutions and RG Flow

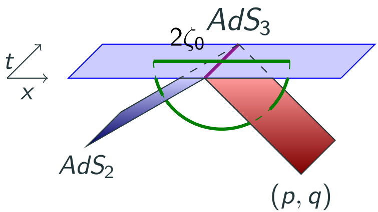


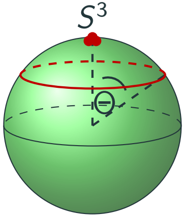


- Boundary entropy $s = \log g \overset{\text{fold}}{\longleftrightarrow}$ interface entropy.

$$\text{BCFT:} \qquad g = \langle 0 | \mathcal{B} \rangle\rangle$$

- Compute s as interface contribution to entanglement entropy
  Calabrese, Cardy '04
- Gravity dual is semi-classical $\implies$ use Ryu-Takayanagi formula
  Ryu, Takayanagi '06





(p, q) g-Theorem

Simplest case: pure F1 interfaces $(p, 0)$

$$\log g = \frac{c}{6}\left(\log\kappa + 1 - \frac{1}{\kappa}\right)$$

$$(p,0): \qquad \kappa = \frac{T(4N_1,\ p) + T(0,\ p)}{T(4N_1,\ p) - T(0,\ p)}$$

$$D3_{(p,0)}: \qquad \kappa = \frac{T(4N_1,\ p\frac{\sin\theta}{\theta}) + T(0,\ p\frac{\sin\theta}{\theta})}{T(4N_1,\ p\frac{\sin\theta}{\theta}) - T(0,\ p\frac{\sin\theta}{\theta})}$$

- g-theorem satisfied for all (p, q) interfaces
- g-factor contains contribution not visible in the probe brane limit

Interfaces in the D1/D5 CFT

Type IIB on $M_{10} = \mathbb{R}^{1,1} \times \mathbb{R}^4 \times M_4$ (with $M_4 = \text{K3}$ or T^4):

	0	1	2	3	4	5	6	7	8	9
D5 (N_5)	•	•					•	•	•	•
D1 (N_1)	•	•								

Gauge theory description. $\mathrm{U}(N_1) \times \mathrm{U}(N_5)$ gauge theory with bifundamental hypermultiplet. Consider Higgs branch. Gives:

Instanton description. D5 brane has a coupling $\int C^{(2)} \wedge \mathrm{Tr}(F \wedge F)$.
$\Longrightarrow$ D1 branes can be dissolved as $\mathrm{U}(N_5)$ gauge instantons on M_4.

Low energy dynamics. 2d $\mathcal{N} = (4,4)$ SCFT: Non-linear sigma model on the moduli space of instantons on M_4. Strominger & Vafa '96

Interfaces in D1/D5 CFT

Type IIB on $M_{10} = \mathbb{R}^{1,1} \times \mathbb{R}^4 \times M_4$:

	0	1	2	3	4	5	6	7	8	9
D5 (N_5)	•	•					•	•	•	•
D1 (N_1)	•	•								
F1 (p)	•		•							

- preserves $\mathcal{N} = 4,\ d = 1$ supersymmetry
- realized in gauge theory as Wilson line. Sources jump in background electric field, changing the CFT on one side while preserving the central charge. This case is an interface, not a defect.

Wilson line interfaces in D1/D5 CFT

- Wilson line $\leftrightarrow$ long string connecting distant D3 brane to D1/D5 system.
- After mixing, lowest-lying fermions have Lagrangian Tong & Wong '14

$$L_\eta = \eta^\dagger (i\partial_0 + \Omega_A \partial_t Z^A)\eta$$

where η is in the fundamental of $\mathrm{U}(N_5)$, Z^A is the coordinate on $\mathcal{M}$, and Ω_A is a $\mathrm{U}(N_5)$ connection on $M_4 \times \mathcal{M}$.

- This can be rewritten as the insertion of

$$W = \mathrm{Tr}_F \mathcal{P} \exp\left(i \int dt\, \partial_t Z^A \Omega_A(y_0, Z) \right)$$

with y_0 the location of the Wilson line in M_4.

Summary and Outlook

Summary

- Studied holographic duals of interface RG flows in the D1/D5 theory
- Probe brane limit: BPS RG flows for general (p, q) string defects
- Classical IIB Supergravity description representing backreaction for fixed points
- g-factor, including CFT contributions, in semi-classical limit of gravity

Outlook

- More detailed study from CFT point of view $\rightsquigarrow$ deformation
- Interfaces carrying D5/NS5 charges
- Generalizations to other top-down theories, especially

$$\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$$

Thank you for your attention!

Probe brane solutions

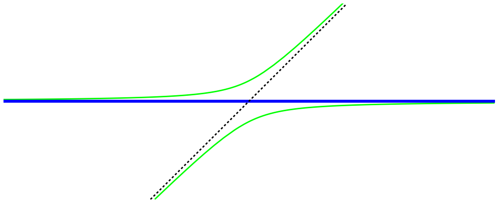
- (p, q) string interface dual to (p, q) strings in near-horizon geometry
- When D1 fields are abelian, behavior determined by DBI-CS action

$$I = qT_{\mathrm{D1}} \int d^2\xi e^{-\Phi} \sqrt{-\det(\hat{g} + F)} + qT_{\mathrm{D1}} \int (C^{(2)} + F)$$

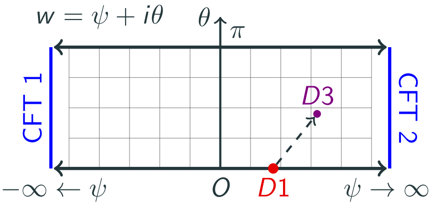
F1 charge p encoded in electric field F_{tx}.

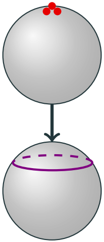
- Solutions are the near-horizon limit of:

$$x_1(x_2) = \frac{qT_{\mathrm{D1}}}{pT_{\mathrm{F1}}}(x_2 - \frac{g_s\alpha' N_5}{x_2}) \, .$$



(p, q) Interface Flows



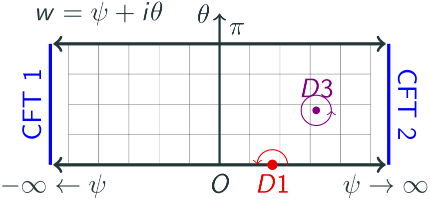


- Boundary RG flow $\Rightarrow$ CFTs remain unchanged

- Invariance of charges
  $\Rightarrow$ location of $D3$ in terms of location of D1 and #(D1-branes)

Imposing Branes

Solutions depend on harmonic functions $a,\ b,\ u,\ v$ and their duals $\tilde{a},\ \tilde{b},\ \tilde{u},\ \tilde{v}$



$$Q_{D3} = \int_{\mathcal{C}} dC^{(4)} = C_{T^4}\big|_{cycle}$$

$$C_{T^4} = \frac{1}{2}\left(\frac{b\tilde{b}}{a} - \tilde{u}\right)$$

Imposing Branes

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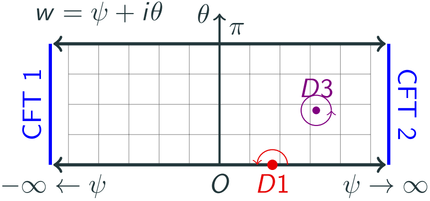
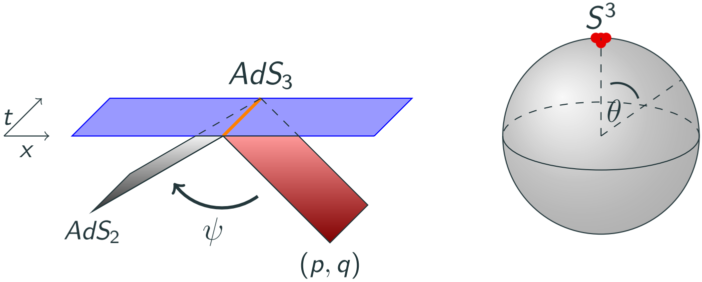


$$Q_{D3} = \int_{\mathcal{C}} dC^{(4)} = C_{T^4}|_{cycle}$$

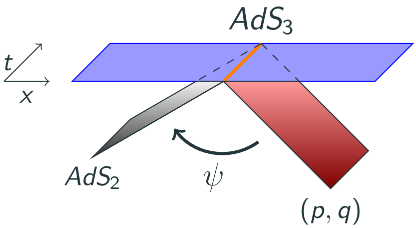
$$C_{T^4} = \frac{1}{2}\left(\frac{b\tilde{b}}{a} - \tilde{u}\right)$$

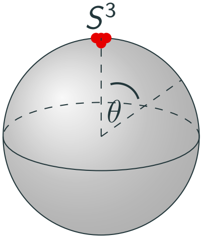
$$Q_{D1} = 4\pi\left(\int_{\mathcal{C}} \frac{4u}{a}\frac{au - b^2}{au + \tilde{b}^2} i(\partial_w c^{(1)} - \chi\partial_w b^{(1)})dw + \int_{\mathcal{C}} 4C_{T^4}dw\right) + c.c.$$

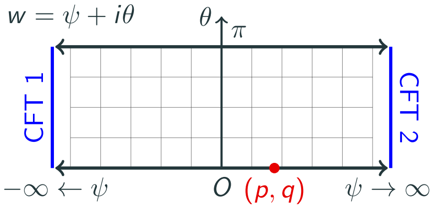
F1/D1 Interface Solution



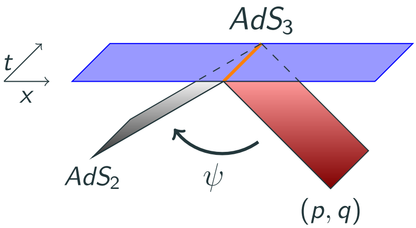
F1/D1 Interface Solution

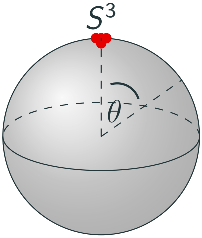


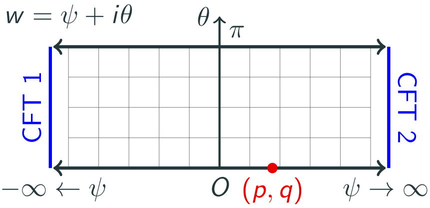


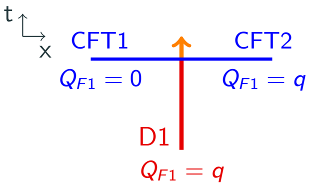


F1/D1 Interface Solution

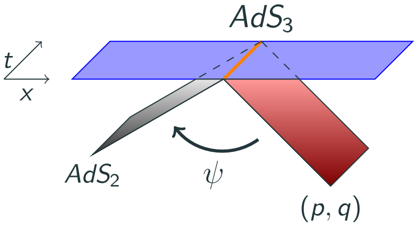


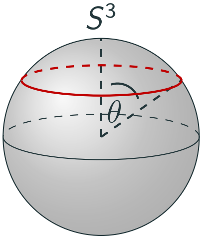


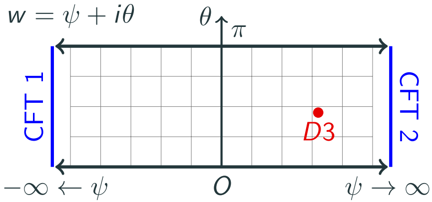




D3 Defect (dissolved D1/F1-branes)







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