Physics 305, Fall 2008
Problem Set 7

due Thursday, November 19

1. **Particle in a time dependent box (10 pts):** A particle is in the ground state of a one-dimensional infinite square well with walls at $x = 0$ and $x = L$. At time $t = 0$, the width of the well is suddenly increased to $2L$. Find the probability that the particle will be found in the nth stationary state of the expanded well.

2. **Time dependent harmonic oscillator (20 pts):** Consider a harmonic oscillator with the Hamiltonian
$$H(t) = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{m\omega(t)^2}{2}x^2 \ ,$$
with
$$\omega(t) = \omega_0 + \delta\omega \ \sin\Omega t \ ,$$
and the frequencies ω_0 and $\delta\omega$ are independent of time with $\delta\omega \ll \omega_0$. Assume that this oscillator is in its ground state at time $t = 0$. At first order in perturbation theory, what excited state(s) will be populated? Calculate the probability of the oscillator being excited at time t, again to lowest nonvanishing order in perturbation theory. Use the formulation of the harmonic oscillator (with $\omega = \omega_0$) in terms of raising and lowering operators in solving all parts of this problem. (You may assume in the last steps of this problem that $|\Omega/2 - \omega_0| \ll |\Omega/2 + \omega_0|$.)

   (This problem is a rough model of a process that is important in nonlinear optics, where a laser with frequency Ω alters the dielectric properties of a material and thereby produces two photons of frequency $\omega_0 \approx \Omega/2$.)

3. **Hydrogen in an electric field (30 pts):** A hydrogen atom, initially in its ground state, is placed in a spatially-uniform electric field $\mathbf{E}(t)$ that has the time dependence
$$\mathbf{E}(t) = \begin{cases} 0 & \text{if } t \le 0 \\ \mathbf{E}_0 e^{-\gamma t} & \text{if } t > 0 \ . \end{cases}$$
What is the probability, to lowest nonvanishing order in $|\mathbf{E}_0|$, that for $t \to \infty$ this hydrogen atom is in an excited state with principal quantum number $n = 2$? (Ignore the coupling to photons, so the excited states are assumed to be stable.)

4. **The Theory of Linear Response (20 pts):** Thus far, most of our attention has been on the behavior of quantum states in response to time dependent perturbations. Another very interesting class of questions concerns the dependence of an expectation value on such a perturbation. Consider a Hamiltonian of the form $H = H_0 + H_1(t)$ where $H_1(-\infty) = 0$. In the distant past, we prepare the system in the state $|\Psi_0\rangle$. Show that at time t, to first order in $H_1(t)$,
$$\langle\Psi(t)|\mathcal{O}|\Psi(t)\rangle - \langle\Psi_0(t)|\mathcal{O}|\Psi_0(t)\rangle = \frac{1}{i\hbar}\int_{-\infty}^{t} dt' \, \langle\Psi_0(t')|[\mathcal{O}, H_1(t')]|\Psi_0(t')\rangle \ .$$
By $\Psi_0(t)$, we mean what the state would be at time t if H_1 were turned off. By $\Psi(t)$, we mean the actual state at time t.