Physics 105 Problem Set 9 Solutions

Problem 9.1, (*10 Points*)

Our two wave equations are,

$$Z_1(x,t) = 10\cos(kx + \omega t) \quad (1)$$

$$Z_2(x,t) = 10\cos(kx + \omega t + \pi/3) \quad (2)$$

We can show that this is a travelling wave by showing first, that it satisfies the wave equation and second, that its phase changes with time. Plugging Z_1 into the wave equation we get,

$$\frac{\partial^2 Z_1}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 Z_1}{\partial t^2} \quad (3)$$

Taking the derivatives we find this holds if,

$$v = \pm\frac{\omega}{k} \quad (4)$$

Note that just because a function satisfies the wave equation, it does not mean it is a travelling wave. For example, take at a standing wave of the form $A\sin(kx)\sin(\omega t)$ which also satisfies the wave equation. The to see how the phase moves lets look the motion of a peak. A peak occurs when,

$$\frac{\partial Z_1}{\partial x} = -k\sin(kx + \omega t) = 0 \quad (5)$$

$$\Rightarrow (kx + \omega t) = 2\pi n \quad (6)$$

Where n is an integer. For a given n we have,

$$x = -\frac{\omega t}{k} + \frac{2\pi n}{k} \quad (7)$$

Therefore and $v = -\frac{\omega}{k}$ and wave moves in the *negative* x direction. Also,

$$\lambda = \frac{2\pi}{k} \qquad f = \frac{\omega}{2\pi} \quad (8)$$

Now we want to find,

$$Z_3 = Z_1 + Z_2 = A_3\cos(kx + wt + \delta_3) \quad (9)$$

For the amplitude A_3 and phase δ_3 of the combined wave with $A_{1,2}$ being the amplitude of Z_1 or Z_2 and δ_2 the phase of Z_2 we get,

$$A_3 = 2A_{1,2}\cos(\frac{\delta_2}{2}) = 10\sqrt{3} \quad (10)$$

$$\delta_3 = \frac{\delta_2}{2} = \frac{\pi}{6} \tag{11}$$

Problem 9.2, (*10 Points*)
We are making a guitar with 24 frets and a string of length $L = 65\,\text{cm}$.
a) How far should the 12th and 24th frets be? What are the locations of all frets(optional)? Let x_n be the position of the n^{th} fret and f_n be the frequency produced by the lowest mode of the n^{th} fret.

$$f_o = \frac{v}{2L} \tag{12}$$

More generally we have,

$$f_n = \frac{v}{2x_n} \tag{13}$$

Therefore,

$$f_{12} = 2f_o = \frac{v}{2x_{12}} \quad \Rightarrow \quad x_{12} = \frac{L}{2} = \quad 32.5\,\text{cm} \tag{14}$$

$$f_{24} = 4f_o = \frac{v}{2x_{24}} \quad \Rightarrow \quad x_{24} = \frac{L}{4} = \quad 16.25\,\text{cm} \tag{15}$$

We also know that each fret should increase the frequency of the previous fret by the same proportion. This leads us to the equation,

$$f_{a+n} = f_a 2^{\frac{n}{12}} \tag{16}$$

And thus,

$$x_n = \frac{v}{f_o 2^{(\frac{n}{12}+1)}} = \frac{L}{2^{(\frac{n}{12})}} = \frac{65}{2^{(\frac{n}{12})}}\,\text{cm} \tag{17}$$

b) What is the speed of the wave? We have $f = 440\,\text{Hz}$, $L = .579\,\text{m}$, and $\lambda = 2L = 1.158\,\text{m}$. Then

$$v = \lambda f = 509.35\,\text{m/s} \tag{18}$$

c) What frequency do you get if there is a node at $2/3\,L$? Here we have $n = 3$ thus for the frequency we get,

$$f = n\frac{v}{\lambda} = nf_o = 1175.3\,\text{Hz} \tag{19}$$

We thank Michael Leung for preparing this document.